§14.1 - Functions of several variables

- Previously, we had vector functions with one input (a parameter):

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle \Rightarrow \vec{r} : t \longmapsto \langle f(t), g(t), h(t) \rangle.$$

Def: A <u>function of two real variables</u> is a rule which assigns to each ordered pair of real numbers in a set D a unique real number $f(x,y)$.

↳ • D = <u>domain</u>

- $\{f(x,y) : (x,y) \text{ in } D\}$ = <u>range</u>.
- x, y = <u>independent vars</u>; $z = f(x,y)$ = <u>dependent var</u>.

Ex: Let $f(x,y) = \dfrac{\sqrt{x+y+1}}{x-1}$ & $g(x,y) = x \cdot \ln(y^2 - x)$.

(a) Find $f(3,2)$ & $g(3,2)$

(b) Find & sketch domain of f & g.

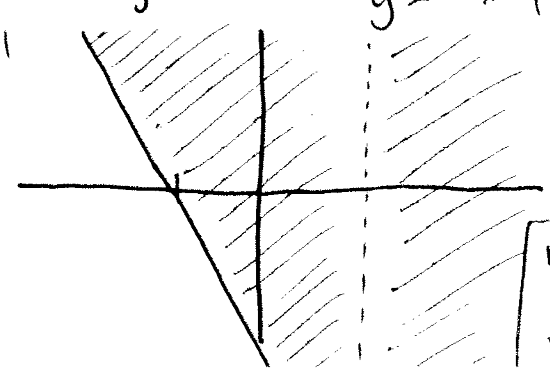
$\boxed{f}$

(a) $f(3,2) = \dfrac{\sqrt{3+2+1}}{3-1} = \dfrac{\sqrt{6}}{2}$

(b) <u>Domain:</u> • $x \neq 1$

• $x+y+1 \geq 0 \Rightarrow y \geq -x-1$

$y=-x-1$ (incl)



may have to check pts to

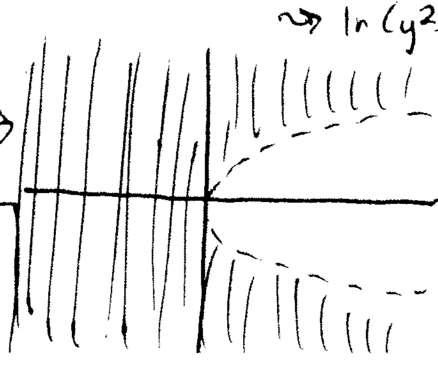
$\boxed{g}$

(a) $g(3,2) = 3 \cdot \ln(2^2 - 3)$
$= 3\ln(1) = 0.$

(b) <u>Domain:</u> • $\ln \square \Rightarrow \square > 0$

$\rightsquigarrow \ln(y^2 - x) \Rightarrow y^2 - x > 0$

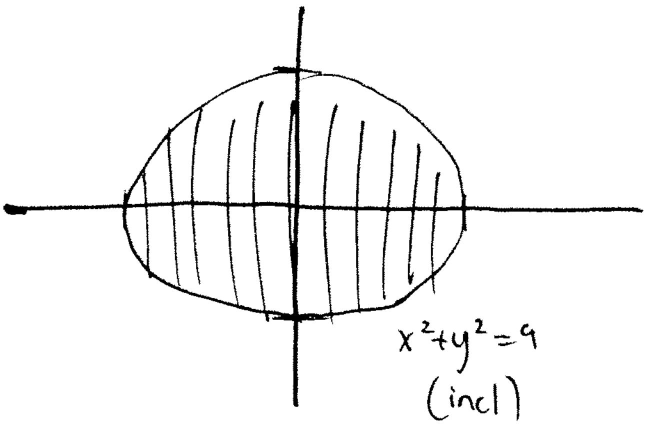
$\Rightarrow y^2 > x.$



$y^2 = x$ (not incl)

Ex: Find the domain and range of $g(x,y)=\sqrt{9-x^2-y^2}$

• Domain: $9-x^2-y^2 \geq 0 \Rightarrow y^2 \leq 9-x^2 \Rightarrow x^2+y^2 \leq 9$.



• Range: By def,

range $= \{z \mid z = \sqrt{9-x^2-y^2},\ (x,y) \in D\}$

$\downarrow$

Note: • range ≥ 0 b/c inside pos sq rt.

• range ≤ 3 b/c $(0,0)$ gives minimum

$\Rightarrow$ range $= \{z : 0 \leq z \leq 3\} = [0,3]$

(b) Sketch the graph. $\rightarrow$ The graph of two-var function f w/ domain D is set of all pts $(x,y,z) \in \mathbb{R}^3$ s.t. $z = f(x,y)$ & $(x,y) \in D$.

• $z = \sqrt{9-x^2-y^2}$

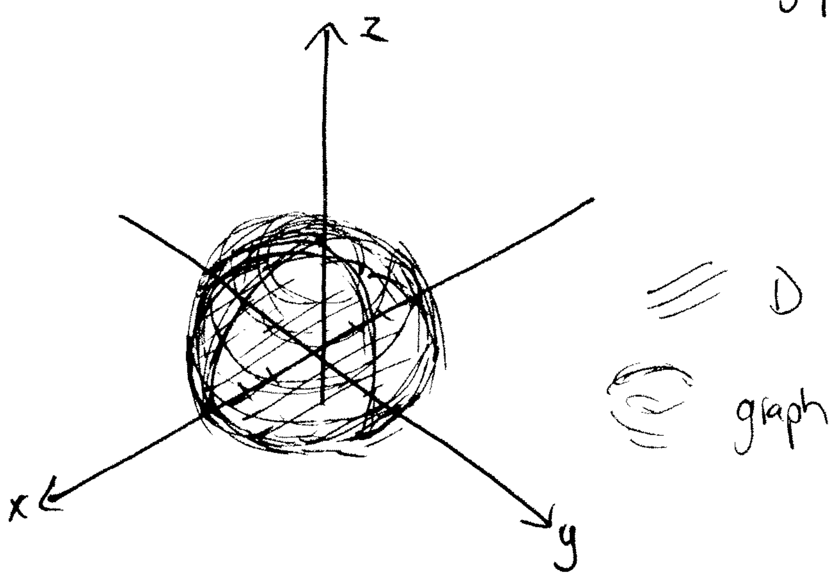
$\Downarrow$

$z^2 = 9 - x^2 - y^2$

$\Downarrow$

$x^2+y^2+z^2 = 9.$ $\rightarrow$

• Also, D = Disk of rad. 3 in xy-plane

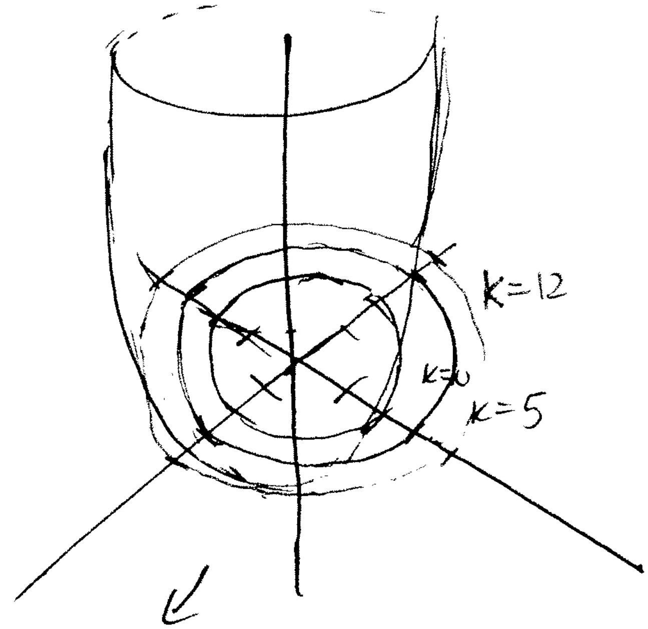


Level curves

One way to draw 2var functions is via contour maps on which points of constant elevation are joined via Contour lines or level curves.

Def: The level curves of a function f of two variables are the curves $f(x,y)=k$ for $k=$ const. in range(f).

Ex:



$x^2+y^2-4 = f(x,y)$

↳ Consider x^2+y^2-4

- If $k=0$:

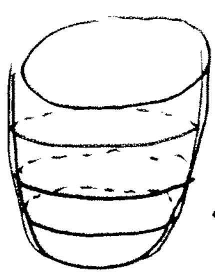
  $x^2+y^2-4=0$

  ↳ $x^2+y^2=4$

- $k=5$

  ↳ $x^2+y^2-4=5$

  $x^2+y^2=9$



← 5

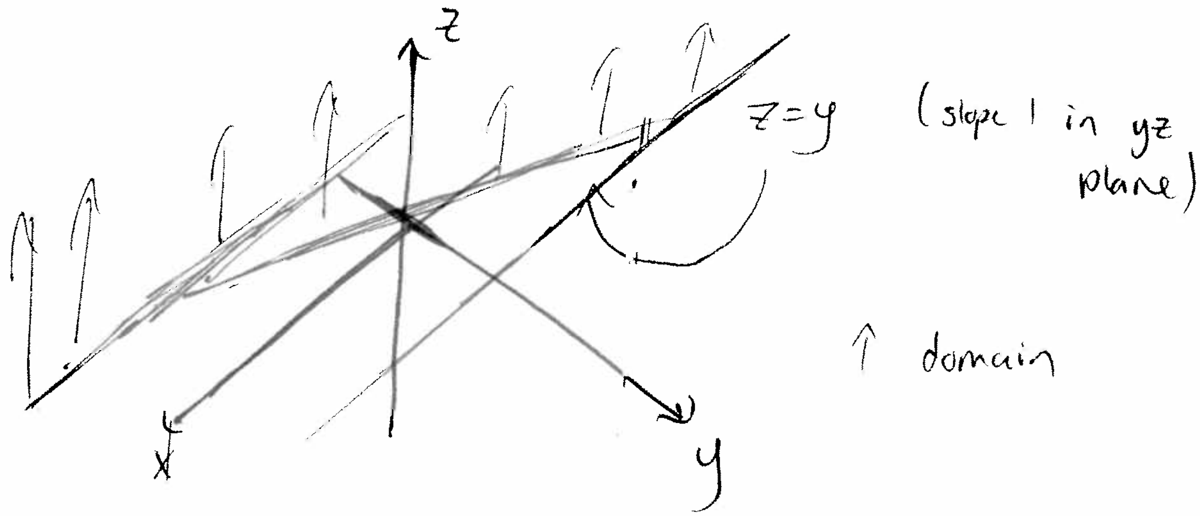
← all pts have elevation (z-value) 0.

1

Three or more variables

- A *function of three vars* assigns a real # $f(x,y,z)$ to every triple (x,y,z) of reals in some domain $D \subset \mathbb{R}^3$.

  Ex: $f(x,y,z)$ has domain $z-y>0 \Rightarrow z>y$.
  $= \ln(z-y)+xy\sin z$



Ex: level surfaces of $f(x,y,z)=x^2+y^2+z^2$?

$x^2+y^2+z^2=k$ is sphere of radius $\sqrt{k}$

- A *function of n variables* assigns real number to an n-tuple $(x_1,\dots,x_n)$ in some domain in $\mathbb{R}^n$.
  - ↳ • $f(x_1,\dots,x_n)$ views f as function of n real #'s or as function of single pt $(x_1,\dots,x_n)$.
    • $f(\langle x_1,\dots,x_n\rangle)$ views f as vector function.

these come up later.