13.4 - Motion in space

- If a particle's position ~~through~~ in time is given by $\vec{r}(t)$, then:

$$\underbrace{\vec{v}(t)}_{\text{velocity}} = \vec{r}\,'(t) \qquad \underbrace{\vec{a}(t)}_{\text{acceleration}} = \vec{r}\,''(t)$$

$$v = \text{speed} = |\text{velocity}| = |\vec{v}(t)| = |\vec{r}\,'(t)|.$$

v ← not a vector.

- We often rewrite acceleration as two components:
  - $\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \dfrac{\vec{v}(t)}{v} \Rightarrow \boxed{\vec{v} = v\vec{T}.}$ (1)
  - $\boxed{\vec{a} =} \vec{v}\,'(t) \underset{(1)}{=} \dfrac{d}{dt}\left(v\vec{T}\right) = \boxed{v\vec{T}' + v'\vec{T}}$ (2)
  - $\kappa = \dfrac{|\vec{T}'|}{|\vec{r}\,'|} = \dfrac{|\vec{T}'|}{v} \Rightarrow \boxed{|\vec{T}'| = \kappa v.}$ (3)
  - $\vec{N} = \dfrac{\vec{T}'}{|\vec{T}'|} \Rightarrow \vec{T}' = \vec{N}\,|\vec{T}'| \underset{(3)}{\Rightarrow} \boxed{\vec{T}' = \kappa v\vec{N}.}$

So, in (2):

$$\vec{a} = v\vec{T}' + v'\vec{T} \underset{(3)}{=} \underbrace{v(\kappa v\vec{N})}_{\substack{\text{normal component}\\ a_N}} + \underbrace{v'}_{\substack{\text{tangential component}\\ a_T}}\vec{T} = a_N\vec{N} + a_T\vec{T}$$

where
$a_T = v'$
$a_N = \kappa v^2$

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Can rewrite WRT $\vec{r}$:

$$a_T = v' = \frac{\vec{r}\,'\cdot\vec{r}\,''(t)}{|\vec{r}\,'(t)|} \qquad \left[= \frac{\vec{v}\cdot\vec{a}}{v} \right]$$

$$a_N = \kappa v^2 = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|r'(t)|}$$

Ex: Particle moving w/ position function $\langle t^2, t^2, t^3 \rangle$ has acceleration components....

- $\vec{r}\,'(t) = \langle 2t, 2t, 3t^2 \rangle$
- $|\vec{r}\,'(t)| = \sqrt{8t^2 + 9t^4}$
- $\vec{r}\,''(t) = \langle 2, 2, 6t \rangle$
- $\vec{r}\,'(t) \times \vec{r}\,''(t) = \langle 6t^2, -6t^2, 0 \rangle$
- $| \uparrow | = \sqrt{72t^4}$

$$\Rightarrow \quad a_T = \frac{\vec{r}\,'\cdot\vec{r}\,''}{|\vec{r}\,'|} = \frac{4t + 4t + 18t^3}{\sqrt{8t^2 + 9t^4}}$$

$$a_N = \frac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|} = \frac{\sqrt{72t^4}}{\sqrt{8t^2+9t^4}}.$$