§ 13.1 – Vector Functions & Space Curves

<u>Def:</u> A <u>vector-valued function</u> is a function from $\mathbb{R}$ to a set of vectors.

↳ If vectors are 3D, a vector function $\vec{r}$ has the form $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$, $f, g, h : \mathbb{R} \to \mathbb{R}$.

<u>Ex:</u> $\vec{r}(t) = \langle \cos(t), e^t, \sqrt{\sqrt{t}+1} \rangle$

↑ component functions

└ domain = all reals for which defined:

$\cos(t)$ ↑ all $\mathbb{R}$

e^t ↑ all $\mathbb{R}$

$\sqrt{\sqrt{t}+1}$ ↑ $t \geq 0$ ← intersection $= \{t : t \geq 0\} = \mathrm{dom}(\vec{r})$.

<u>Limits:</u>

$$\lim_{t \to a} \vec{r}(t) = \langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \rangle.$$

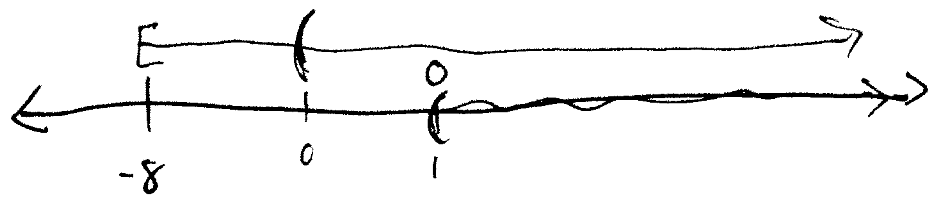
<u>Ex:</u> $\vec{r}(t) = (1+t^3)\vec{i} + te^{-t}\vec{j} + \left(\frac{\sin t}{t}\right)\vec{k} \Rightarrow \lim_{t \to 0} \vec{r}(t) = \langle 1, 0, 1 \rangle$.

Ex: $\lim_{t \to 1} \left\langle \frac{t^2 - t}{t-1}, \sqrt{t+8}, \frac{\sin \pi t}{\ln t} \right\rangle$ & its domain.

(#4)

- **Domain:**
  - $\frac{t^2-t}{t-1}$: $t \neq 1$
  - $\sqrt{t+8}$: $t \geq -8$
  - $\frac{\sin \pi t}{\ln t}$: $t > 0$, $t \neq 1$

  $\Rightarrow$



$\uparrow$ intersection $=$ $(0, 1) \cup (1, \infty)$

- **Limit:** $\left\langle 1, 3, \lim_{t \to 1} \frac{\sin \pi t}{\ln t} \right\rangle = \langle 1, 3, -\pi \rangle$

$$\searrow$$

$$= \lim_{t \to 1} \frac{\pi \cos \pi t}{\frac{1}{t}} = -\pi$$

Def: $\vec{r}(t)$ **continuous** at $t = a$ if $\lim_{t \to a} \vec{r}(t) = \vec{r}(a)$.

$$\Updownarrow$$

$f(t), g(t), h(t)$ continuous at $t = a$.

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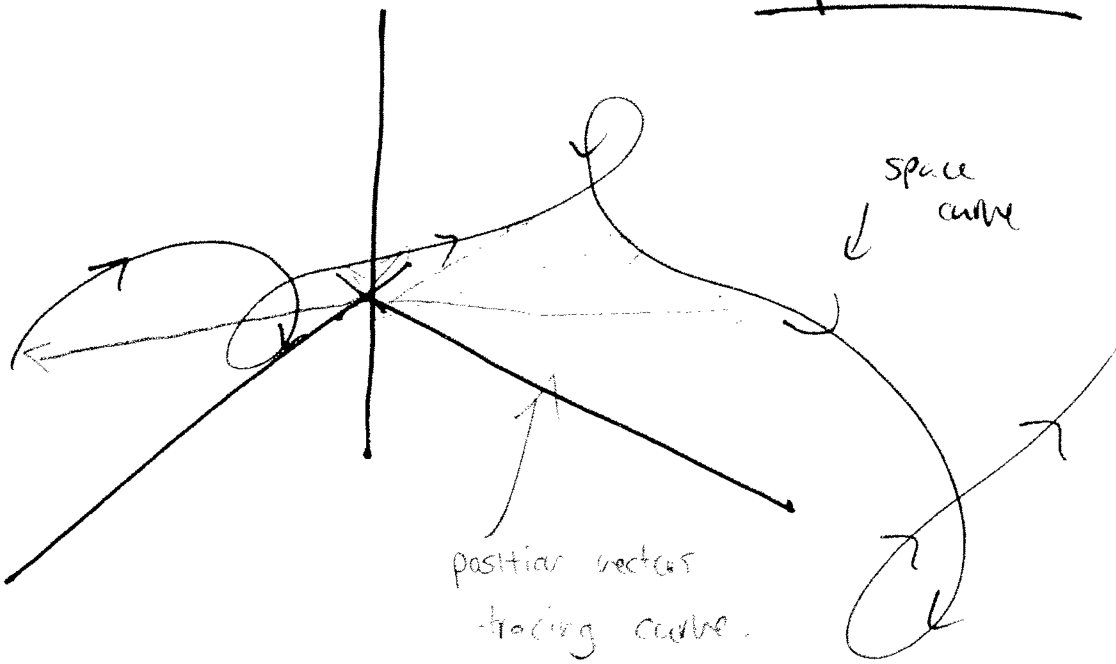
Space curves

Def: The set of all pts (x,y,z) satisfying $x=f(t)$, $y=g(t)$, $z=h(t)$ (where t varies) is called a <u>space curve</u>.



Ex: Describe: ① $\vec{r}(t) = \langle 1+t,\ 2+5t,\ -1+6t \rangle$

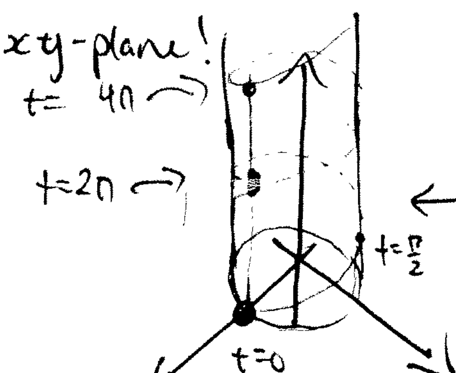
↳ line parallel to $\langle 1,5,6 \rangle$ thru $(1,2,-1)$.

② $\vec{r}(t) = \cos(t)\vec{i} + \sin(t)\vec{j} + t\vec{k}$

$x^2+y^2=1 \Rightarrow$ curve lies on the circular cylinder $x^2+y^2=1$.

$z=t \Rightarrow$ curve spirals upward as t incr.

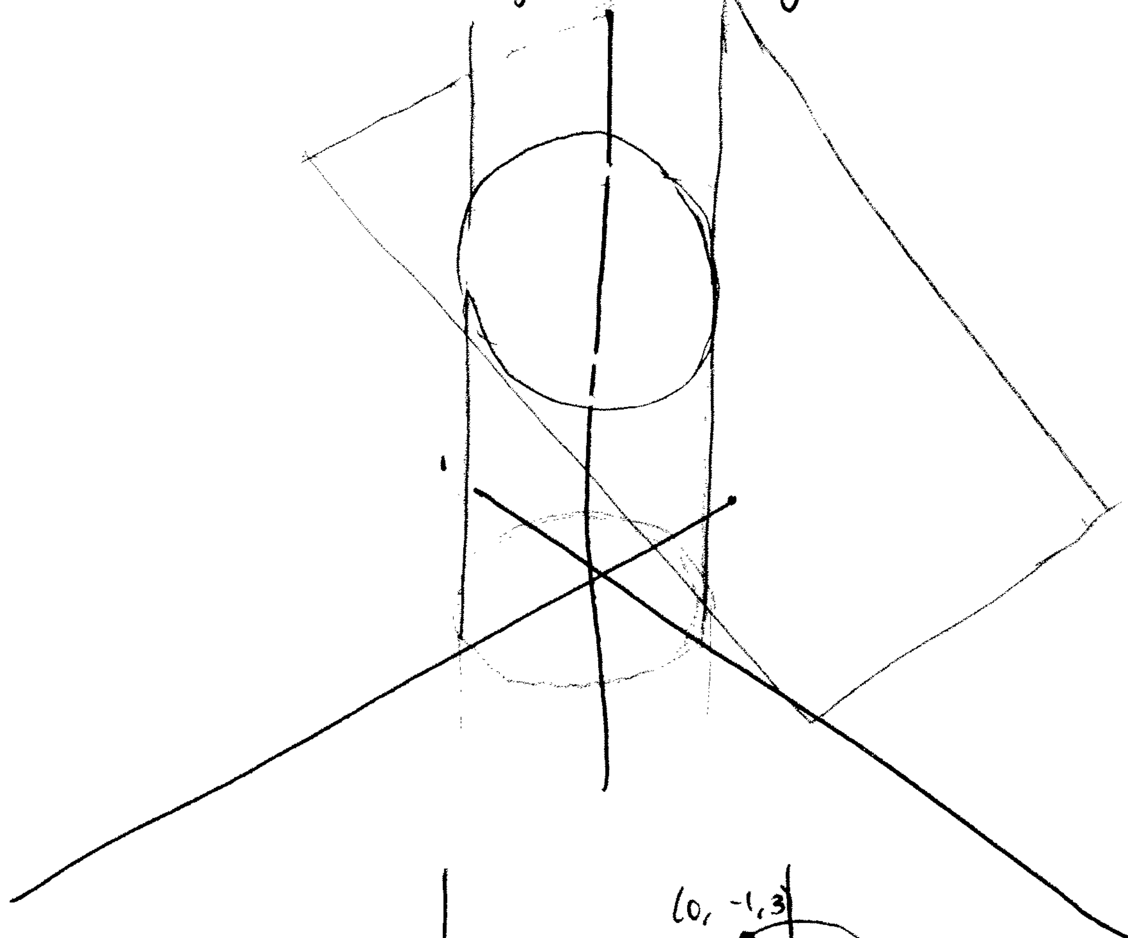
projection to xy-plane!

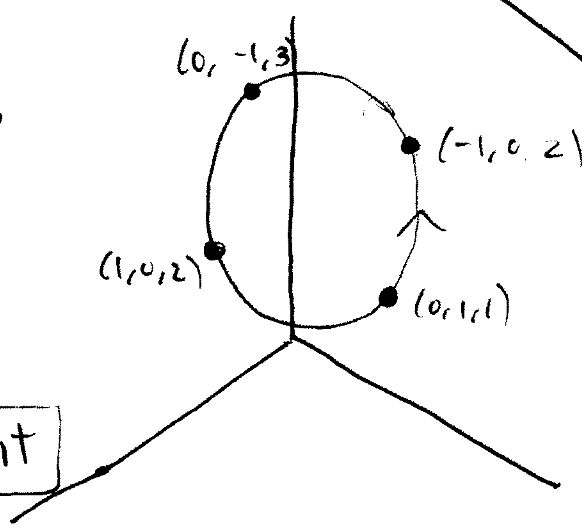


← $x^2+y^2=1$ in $\mathbb{R}^3$

:)

Ex: Find intersection of $x^2+y^2=1$ & $y+z=2$





Note:

Project to xy-plane $\Rightarrow x^2+y^2=1$

$\Rightarrow x=\cos t$ $\boxed{y=\sin t}$

$0\le t\le 2\pi$.

- plane = $y+z=2$

$\Rightarrow z=2-y \Rightarrow z=2-\sin t$

$\vec{r}(t) = \langle \cos t, \sin t, 2-\sin t \rangle$.