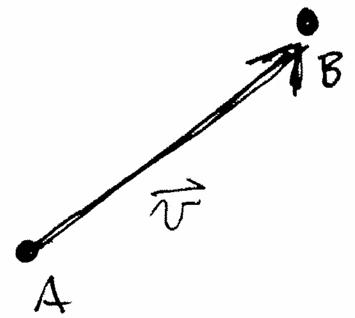
§12.2 - Vectors

• Intuitively, vectors have <u>magnitude</u> and <u>direction</u>.

↳ represent as directed arrows between two pts:

Regardless of <u>location</u>, two vectors w/ same length & same direction are considered equal.



A = initial pt = "tail"

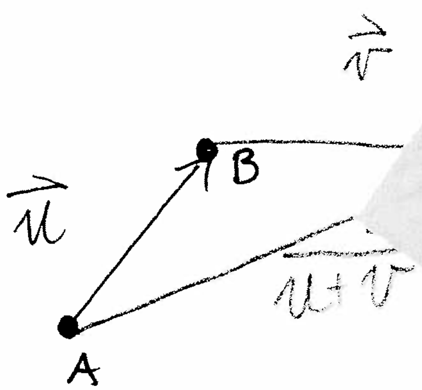
B = terminal pt = "head"

$\vec{0}$ = zero vector = vector w/ zero length

‖ only vec w/ no direction

Can write $\vec{v} = \overrightarrow{AB}$.

<u>Addition</u>: If $\vec{u}$ & $\vec{v}$ are vectors w/ initial pt of $\vec{v}$ at terminal pt of $\vec{u}$, then $\overrightarrow{u+v}$ … the vector with initial pt = initial pt of $\vec{u}$ … = terminal pt of $\vec{v}$.



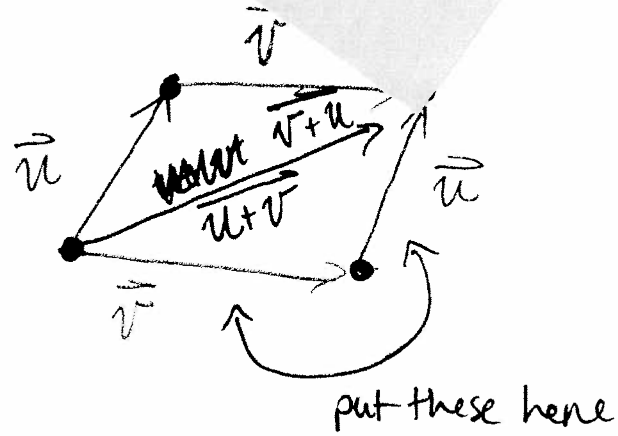
$= \overrightarrow{AC}$

• Quiz Tuesday
• Will post notes, surveys, and emails this wknd

<u>Prac</u>: 1-44, (45-48, 61-64)*

<u>Also: Parallelogram law!</u>

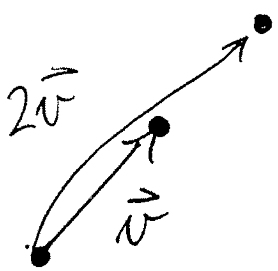
$\overrightarrow{u+v} = \overrightarrow{v+u}$ and is equal to diagonal of this parallelogram.

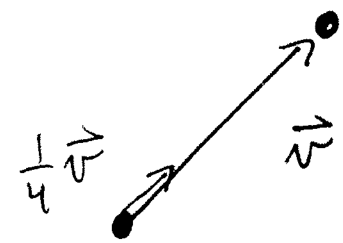


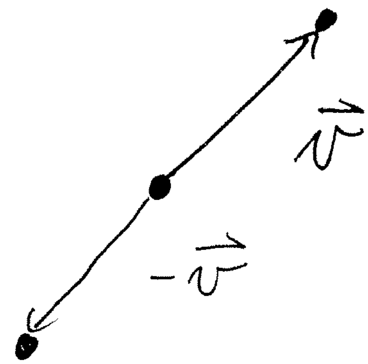
<u>Scalar Multiplication</u>: For $c \in \mathbb{R}$, $c>0$, the scalar multiple $\overrightarrow{cv}$ is the vector w/ same direction of $\vec{v}$ & length $c \cdot \text{length}(\vec{v})$

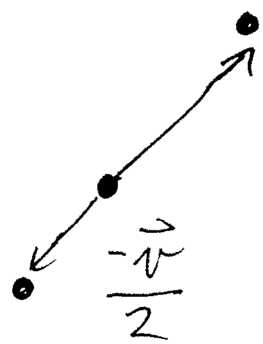
If $c<0$, opposite direction & length $|c| \cdot \text{length}(\vec{v})$.

If $c=0$ or $\vec{v}=\vec{0}$ vector, $\overrightarrow{cv} = \vec{0}$.



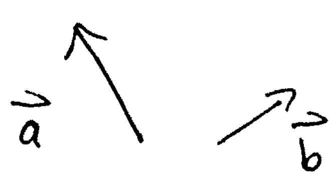






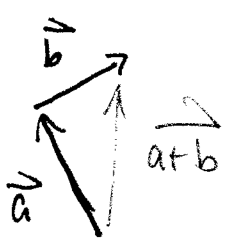
- write $\vec{u} - \vec{v}$ for $\vec{u} + (-\vec{v})$

<u>Ex:</u> Given

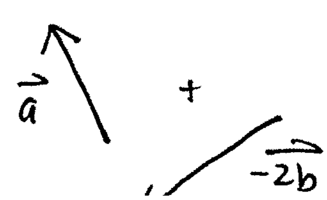


, draw (a) $\overrightarrow{a+b}$ and (b) $\overrightarrow{a-2b}$.

(a)



(b) $\overrightarrow{a-2b} = \overrightarrow{a+(-2b)}$



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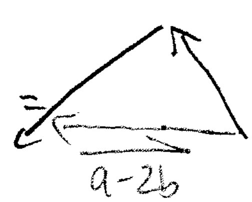
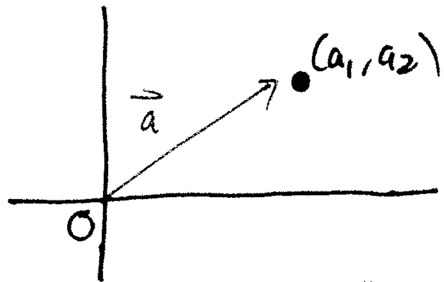
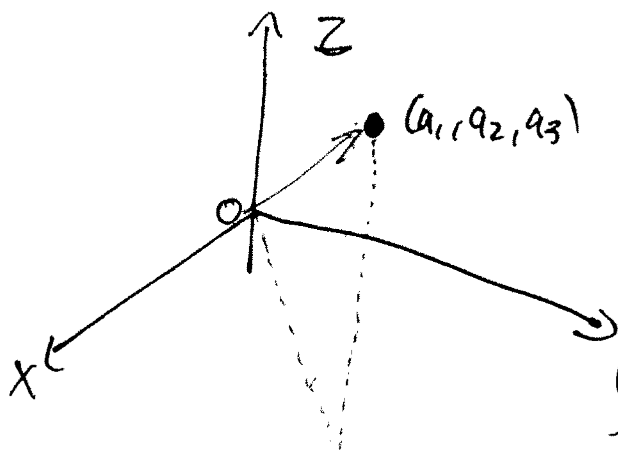


Components

Put vector at origin and see where terminal pt is.

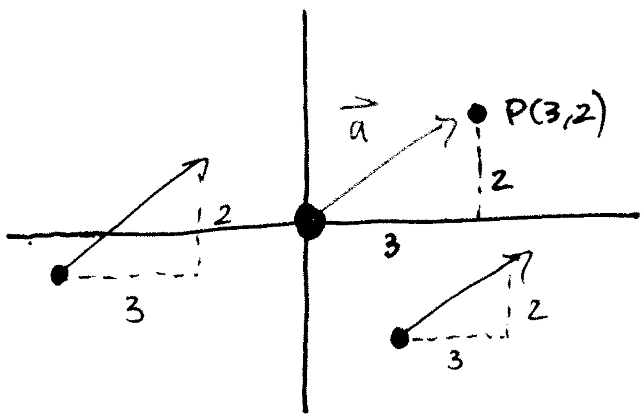


$\vec{a} = \langle a_1, a_2 \rangle$



$\vec{a} = \langle a_1, a_2, a_3 \rangle$

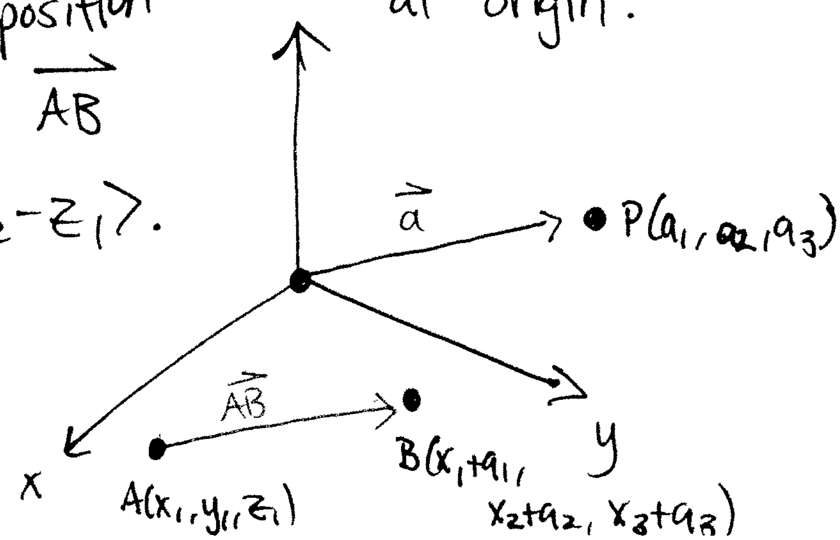
This can be confusing!



These are all the same vector!

- different vectors called <u>representations</u> of $\vec{a} = \langle 3, 2 \rangle$
- $\vec{a}$ called the <u>position vector</u> b/c it starts at origin.

<u>Def:</u> Given the points ~~[illegible]~~ $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$ the position vector corresponding to rep. $\overrightarrow{AB}$ is $\vec{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$.



<u>Ex:</u> Find vector represented by the directed line segment from A $(2,-3,4)$ to $B(-2,1,1)$.

↳ $\vec{a} = \langle -2-2,\ 1-(-3),\ 1-4 \rangle$

$= \langle -4, 4, -3 \rangle$

<u>Algebra w/ components</u>

- Let $|\vec{a}|$ or $\|\vec{a}\|$ be the length of vector $\vec{a}$. Then

$$\vec{a} = \langle a_1, a_2 \rangle \Rightarrow |a| = \sqrt{a_1^2 + a_2^2}$$

$$\vec{a} = \langle a_1, a_2, a_3 \rangle \Rightarrow |a| = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

- <u>Also:</u> If $\vec{a} = \langle a_1, a_2, a_3 \rangle$ & $\vec{b} = \langle b_1, b_2, b_3 \rangle$, then:

$$\overrightarrow{a \pm b} = \langle a_1 \pm b_1,\ a_2 \pm b_2,\ a_3 \pm b_3 \rangle$$

$$\overrightarrow{ca} = \langle ca_1, ca_2, ca_3 \rangle.$$

- Same in 2 dimensions!
- Also, has all usual properties (add or mult. in any order, ...)

↳ see table on pg 819.

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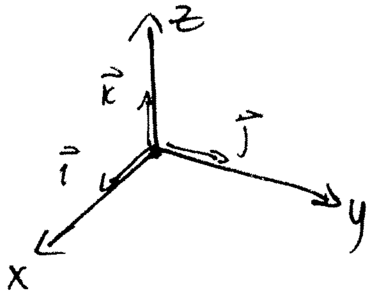
- let V_2 = coll. of vectors in $\mathbb{R}^2$

  V_3 = ... $\mathbb{R}^3$.

<u>Standard Basis vectors in $\mathbb{R}^3$</u>

$\vec{i} = \langle 1,0,0\rangle \qquad \vec{j} = \langle 0,1,0\rangle \qquad \vec{k} = \langle 0,0,1\rangle$



↳ all have length 1 & point along axes.

Also, they help decompose vectors:

For any $\vec{a} = \langle a_1, a_2, a_3\rangle$, $= \langle a_1,0,0\rangle + \langle 0,a_2,0\rangle + \langle 0,0,a_3$

$$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$$

<u>Ex:</u> $\langle 2,-4,\pi\rangle = 2\vec{i} - 4\vec{j} + \pi\vec{k}$

<u>Ex:</u> Write $\overrightarrow{2a+3b}$ WRT $\vec{i},\vec{j},\vec{k}$ where $\vec{a} = \langle 1,2,-3\rangle$ &

$\vec{b} = 4\vec{i} + 7\vec{k}$.

$$\overrightarrow{2a+3b} = 2(\vec{i} + 2\vec{j} - 3\vec{k}) + 3(4\vec{i} + 0\vec{j} + 7\vec{k})$$

$$= (2\vec{i} + 12\vec{i}) + (4\vec{j} + 0\vec{j}) + (-6\vec{k} + 21\vec{k})$$

$$= 14\vec{i} + 4\vec{j} + 15\vec{k}.$$

:)

Unit vectors

A unit vector is a vector of length 1.

Ex: $\vec{i}, \vec{j}, \vec{k}, \quad \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \rangle$.

Given $\vec{a} \neq \vec{0}$, there is a unique unit vector w/ same direction as $\vec{a}$:

$$\vec{u} = \frac{\vec{a}}{|\vec{a}|}.$$

Prove this has len 1!

$$= \underbrace{\boxed{\frac{1}{|\vec{a}|}}}_{\text{Scalar}} \vec{a}$$

Ex: $2\vec{i} - \vec{j} - 2\vec{k}$ has ~~unit vector~~ length $\sqrt{4+1+4} = 3$,

so $\frac{1}{3}(2\vec{i} - \vec{j} - 2\vec{k})$ is unit vec w/ same direction!

$$\| \\ \frac{2}{3}\vec{i} - \frac{1}{3}\vec{j} - \frac{2}{3}\vec{k}.$$