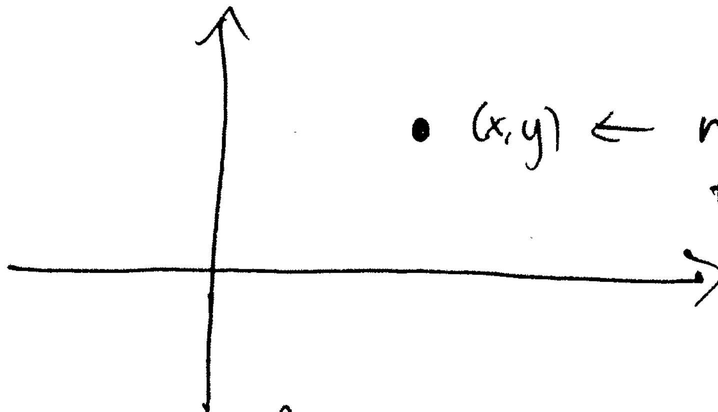
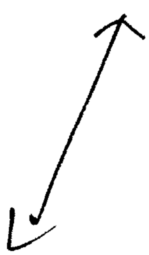
§12.1 - Three Dimensions

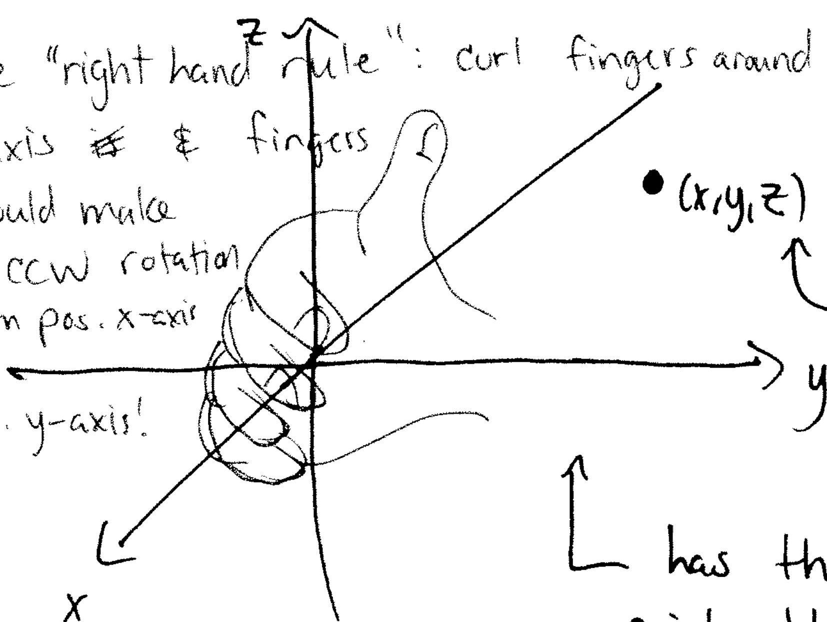
Recall: $\mathbb{R}^2$ = collection of ordered pairs of reals



• (x,y) ← need exactly two coords (hence $\mathbb{R}^{\boxed{2}}$!!)

has two coordinate axes & every pt obtained by specifying two coords (1 for each axis).





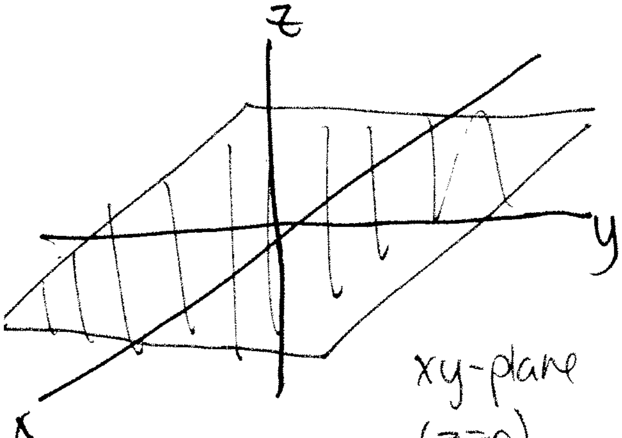
use "right hand rule": curl fingers around z-axis & fingers should make 90° ccw rotation from pos. x-axis to pos. y-axis!

$\mathbb{R}^3$ = Collection of ordered triples

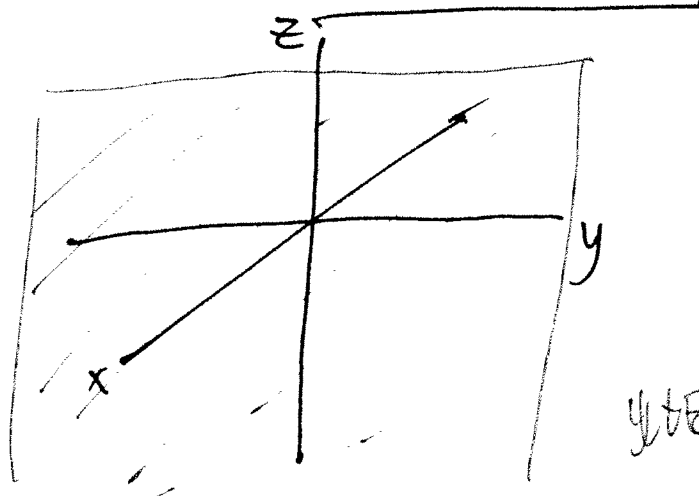
• (x,y,z) ← need 3 coords

has three coord. axes & every point obtained by specifying <u>three</u> coords (1 for each axis).

- In $\mathbb{R}^3$, there are also three coordinate planes:

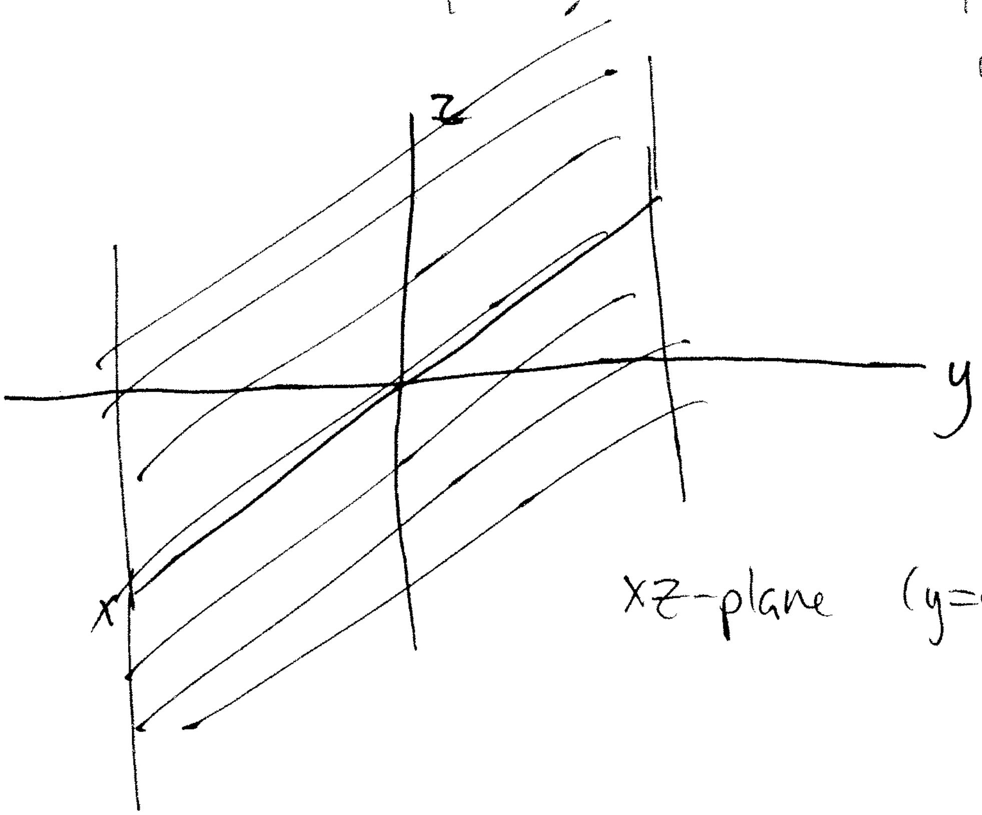


xy-plane
($z=0$)



[illegible]

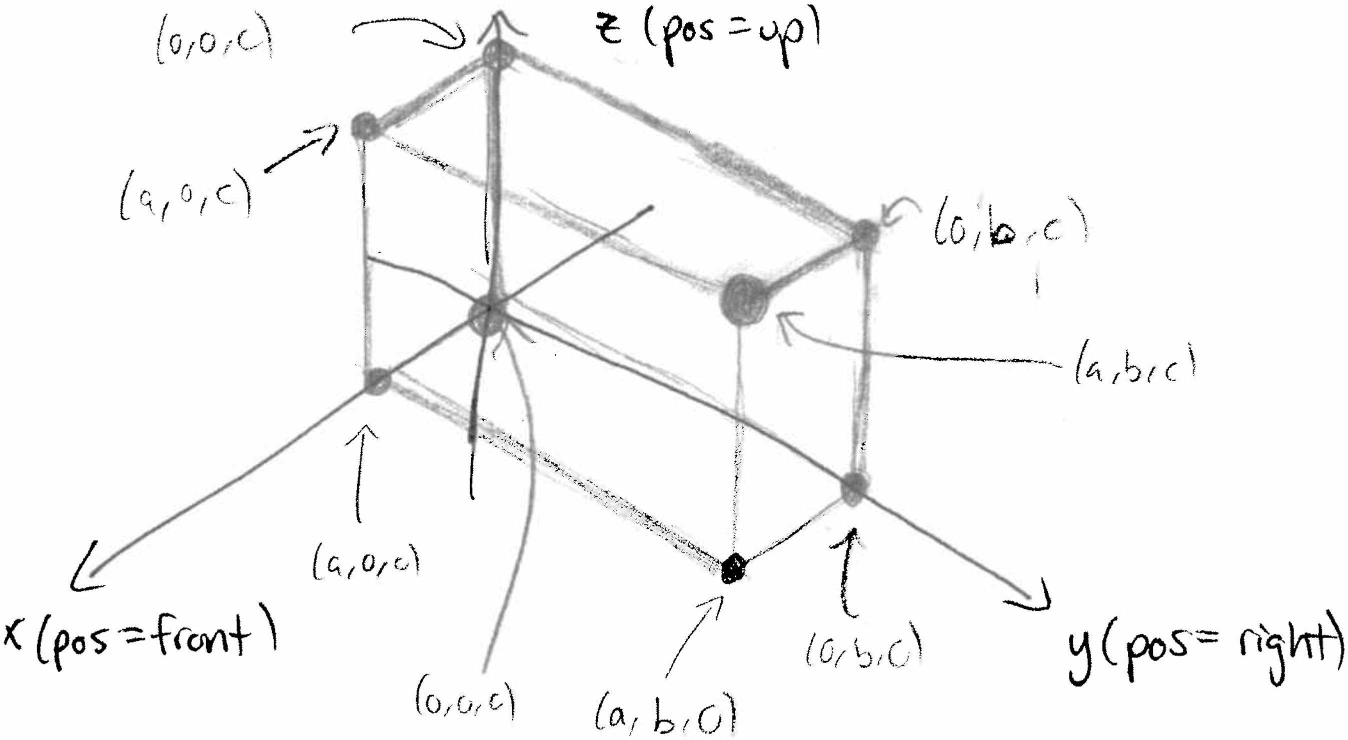
yz-plane
($x=0$



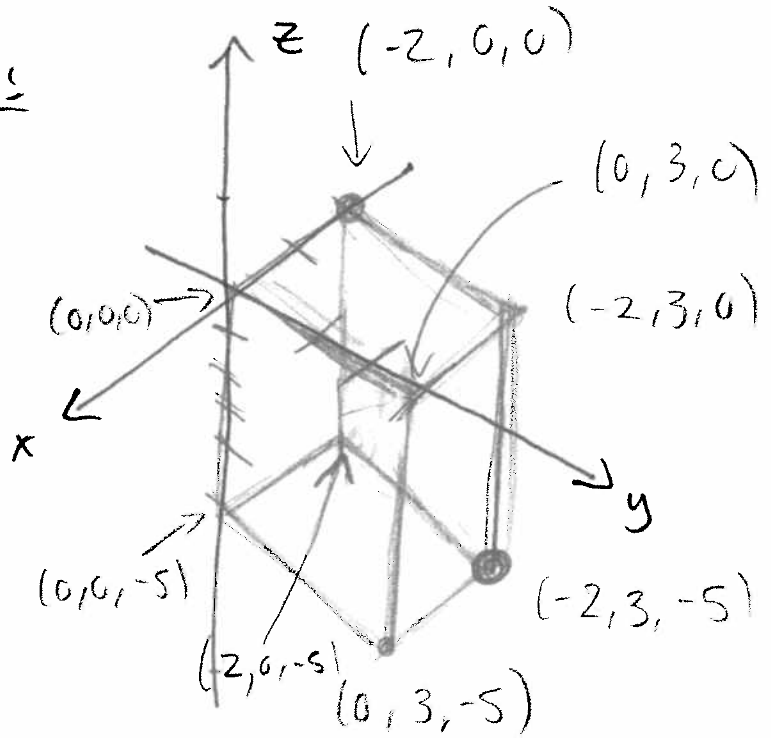
xz-plane ($y=0$)

(show mathematica pic?)

Ex:

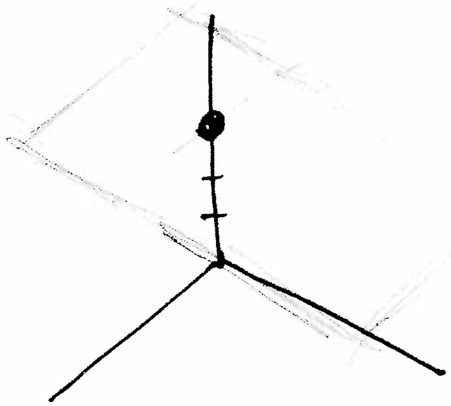

(0,0,c)
z (pos = up)
(a,0,c)
(0,b,c)
(a,b,c)
(a,0,0)
x (pos = front)
(0,0,0)
(a,b,0)
(0,b,0)
y (pos = right)

Ex:

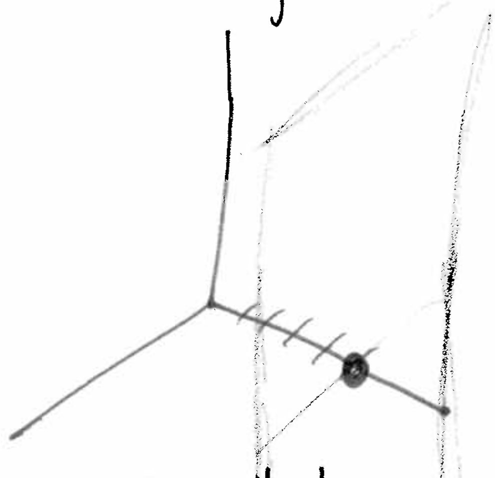

z
(-2,0,0)
(0,3,0)
(0,0,0)
(-2,3,0)
x
y
(0,0,-5)
(-2,3,-5)
(2,0,-5)
(0,3,-5)

Ex: what surfaces in $\mathbb{R}^3$ do the following represent?

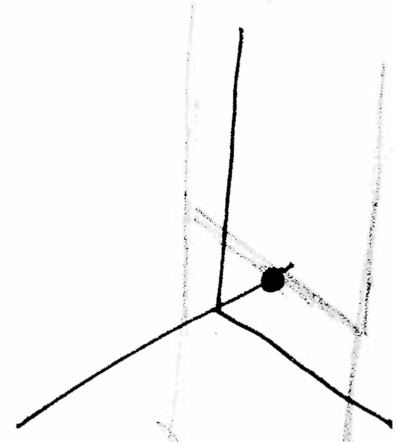
(a) $z=3$ (b) $y=5$ (c) $x=-1$



plane ∥ to xy-plane



plane ∥ to xz-plane



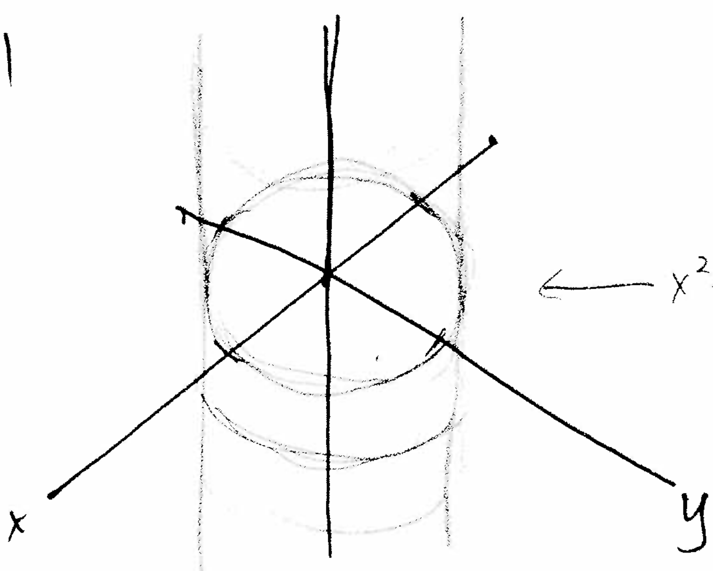
plane ∥ to yz-plane

Note: this doesn't match $\mathbb{R}^2$!

Ex: Discuss: (a) $x^2+y^2=1$ in $\mathbb{R}^3$

(b) $x^2+y^2=1$ & $z=3$ in $\mathbb{R}^3$

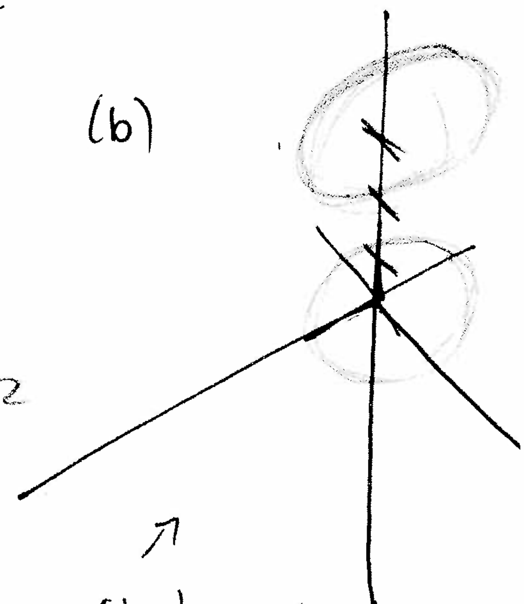
(a)



← $x^2+y^2=1$ in $\mathbb{R}^2$

<u>HOLLOW</u> cylider ∥ to z-axis

(b)



↗ single circle

DISTANCE

The distance $|P_1P_2|$ between pts $P_1(x_1, y_1, z_1)$ & $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

LOOK AT PROOF ON PG 813!

<u>Ex:</u> The distance from $P(2,-1,7)$ to $Q(1,-3,5)$

$$= \sqrt{(1-2)^2 + (-3-(-1))^2 + (5-7)^2}$$

$$= \sqrt{1+4+4} = \sqrt{9} = 3.$$

Ex: (Sphere) of radius r

By def, sphere is all pts $P(x,y,z)$ whose distance from Center $C(h,k,l)$ is r. So:

$$\sqrt{(x-h)^2 + (y-k)^2 + (z-l)^2} = r$$

$$\Rightarrow (x-h)^2 + (y-k)^2 + (z-l)^2 = r^2.$$

$$\Rightarrow x^2+y^2+z^2 = r^2 \text{ if } C = \text{origin}.$$

<u>Ex:</u> $x^2+y^2+z^2+4x-6y+2z+6=0 \ldots$

:)