Name: ______________________________

MAC 2313 — Homework 2

Directions: Complete the following problems for a homework grade. Solutions *must* be presented in a neat and professional manner in order to receive credit, answers given without showing work will not be eligible to receive partial credit, and *work for the problems* ***must*** *be done on scratch paper and not on this handout!* **Date Due:** Tuesday, February 28 (*Use this to study!!*)

1. Let $f(x, y) = 3x^2 + 2xy - y^3$, let $g(x, y) = -x^3 + x + y + 2y^2$, let $\mathbf{v} = \langle 3, 4\rangle$, and let $\mathbf{u}$ be the unit vector in the direction of $\mathbf{v}$. **Note**: On your exam, you *will* have to do questions like (b) and (d), and I *won't* give you the formulas!

   (a) Using any technique you know (e.g. witchcraft, voodoo magic, derivative rules,...), find f_x and f_y.

   (b) Use the limit definitions

   $$f_x(x, y) = \lim_{h\to 0} \frac{f(x + h, y) - f(x, y)}{h} \qquad f_y(x, y) = \lim_{h\to 0} \frac{f(x, y + y) - f(x, y)}{h}$$

   to find f_x and f_y. Does this match your answer for (a)?

   (c) Using any technique you know (e.g. witchcraft, voodoo magic, gradient thangs,...), find $D_{\mathbf{u}}f(x, y)$.

   (d) Use the limit definition

   $$D_{\mathbf{u}}f(x, y) = \lim_{h\to 0} \frac{f(x + ah, y + bh) - f(x, y)}{h}$$

   to find $D_{\mathbf{u}}f(x, y)$. Does this match your answer for (c)?

   (e) Are f and/or g differentiable? Why or why not?

2. Let $f(w, x, y, z) = x^2 + y^2 + z^2 + xy + yz + xz - 3xyz$, where $w = 2r^2 + s$, $x = 2s^2 + r - t$, $y = r^2 s + t$, and $z = se^r - 2\sqrt{t}$. Find each of the following. **Note**: w also has t's in it, it just has 0 of them....

(a) $\dfrac{\partial f}{\partial w}$

(b) $\dfrac{\partial f}{\partial x}$

(c) $\dfrac{\partial f}{\partial y}$

(d) $\dfrac{\partial f}{\partial z}$

(e) $\dfrac{\partial f}{\partial r}$

(f) $\dfrac{\partial f}{\partial s}$

(g) $\dfrac{\partial f}{\partial t}$

(h) $\dfrac{\partial^2 f}{\partial r\, \partial t}$

3. Let $f(x, y) = \cos(xy)$.

   (a) Find $\nabla f(x, y)$.

   (b) Find the equation of the tangent plane to $z = f(x, y)$ at the point $\left(\frac{\pi}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$.

   (c) How many critical points does f have, and how do you know?

   (d) Is $(0, 0, 1)$ a critical point of f? How do you know?

   (e) Could the point $(2, 3, f(2, 3))$ be a critical point of f? Why or why not?

   (f) Find f_{xx}, f_{xy}, f_{yx}, and f_{yy}.

   (g) Use (g) to find $D(x, y) = \det \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$.

   (h) $(0, 1, 1)$ *is* a critical point of f. Is it a local max, local min, or saddle point? How do you know?

   (i) Let $\Sigma = \{(x, y)$ in $\mathbb{R}^2$ such that $-1 \leq x \leq 1$ and $-1 \leq y \leq 1\}$. Does f attain an absolute maximum on Σ? An absolute minimum? How do you know?

   (j) Find the absolute maximum and absolute minimum of the function f on the closed triangular region Δ with vertices $(2, 0)$, $(0, 2)$, and $(0, -2)$.