Name: ~KEY~

MAC 2313 — Homework 1

Directions: Complete the following problems for a homework grade. Solutions *must* be presented in a neat and professional manner in order to receive credit, answers given without showing work will not be eligible to receive partial credit, and *work for the problems* ***must*** *be done on scratch paper and not on this handout!* **Date Due:** Monday, January 30.

1. (a) Navigate to our course homepage at

   `http://www.math.fsu.edu/~cstover/teaching/sp17_2313/`

   (b) Read and familiarize yourself with the three resources listed under *Supplementary Resources* on the GENERAL INFO tab.

   (c) Follow the instructions for using SLACK messenger.

   Note: This may require that I approve your email address, so to avoid some last minute glitch where I don't get to your approval on-time, please don't wait to do this!

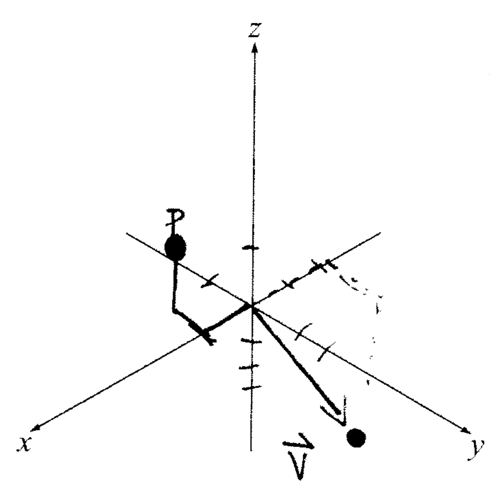
   (d) Navigate to the channel `#random` in the left column under CHANNELS (its browser url should be something like `https://spring2017-calc3.slack.com/messages/random/`) and introduce yourself.

   Note: This will be visible to everyone who signs into our class's chat room, so you definitely want to keep this PG-13, safe for work, and non-incriminatory. ☺

2. Plot $(1,-1,1)$ and $\langle -2,2,-3\rangle$ on the axes provided below. **Note**: The arrowheads are pointing towards the *positive* values on each axis!

$P = (1,-1,1)$

$\vec{V} = \langle -2,2,-3 \rangle$



3. Write the vector from $(0, 1, 2)$ to $(1, \pi, -4)$ in ~~terms of the standard basis~~ vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

$$\vec{V} = \langle 1-0, \pi-1, -4-2 \rangle = \boxed{\vec{i} + (\pi-1)\vec{j} - 6\vec{k}}$$

4. Consider the vectors $\mathbf{u} = \langle -2, 3, 1 \rangle$, $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, and $\mathbf{w} = \langle 0, -1, -1 \rangle$. Determine whether each of the following quantities is a scalar or a vector, and express each vector with respect to $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$. ✱ Not written with respect to $\vec{i}, \vec{j}, \vec{k}$.

(a) $\mathbf{v} + \mathbf{w}$

$\langle 1, 0, 0 \rangle$

(b) $3\mathbf{v}$

$\langle 3, 3, 3 \rangle$

(c) $(\mathbf{u} \cdot \mathbf{w})\,\mathbf{v}$ $\langle -4, -4, -4 \rangle$

(d) The unit vector in the same direction as $\mathbf{u} + \mathbf{w}$

$\vec{u} + \vec{w} = \langle -2, 2, 0 \rangle$; unit vector $= \langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \rangle$

(e) $(\mathbf{w} \cdot \mathbf{w})\,\mathbf{w}$

$\langle 0, -2, -2 \rangle$

(f) $\mathbf{w} \times \mathbf{u}$ $\langle 2, 2, -2 \rangle$

(g) $\mathbf{u} \times \mathbf{w}$ $\langle -2, -2, 2 \rangle$

(h) The angle between $\mathbf{u} + \mathbf{w}$ and $\mathbf{u} \times \mathbf{w}$.

$\frac{\pi}{2}$

(i) The unit vector in the same direction as $\mathbf{w} \times \mathbf{u} - \mathbf{u} \times \mathbf{w}$

unit In same direction as $\langle 4, 4, -4 \rangle$ is $\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \rangle$

(j) $|\mathbf{v} - \mathbf{w}|$

3

(k) $|\mathbf{v} \times \mathbf{u}|$ $\sqrt{38}$

(l) The unit vector orthogonal to both $\mathbf{u} + 3\mathbf{v}$ and $2\mathbf{w}$

$(\vec{u} + 3\vec{v}) \times 2\vec{w} = \langle -4, 2, -2 \rangle \Rightarrow$ unit vec $= \langle -\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \rangle$

(m) The area of the parallelogram spanned by $\mathbf{u} + \mathbf{i}$ and $-2\mathbf{w}$

$= |(\vec{u} + \vec{i}) \times (-2\vec{w})| = 2\sqrt{6}$

(n) $\text{comp}_{\mathbf{u}}\,\mathbf{v}$ and $\text{proj}_{\mathbf{u}}\,\mathbf{v}$

proj $= \langle -\frac{2}{7}, \frac{3}{7}, \frac{1}{7} \rangle$

comp $= |\text{proj}| = \sqrt{\frac{2}{7}}$

5. Let $\mathbf{a} = \langle a_1, a_2, a_3\rangle$, $\mathbf{b} = \langle b_1, b_2, b_3\rangle$, $\mathbf{c} = \langle c_1, c_2, c_3\rangle$, $\mathbf{u} = \langle u_1, u_2, u_3\rangle$, and $\mathbf{v} = \langle v_1, v_2, v_3\rangle$ be arbitrary vectors in $\mathbb{R}^3$. Prove each of the following.

(a) $\mathbf{i}\cdot\mathbf{j} = \mathbf{j}\cdot\mathbf{k} = \mathbf{k}\cdot\mathbf{i} = 0$

$\vec{i}\cdot\vec{j} = \langle 1,0,0\rangle\cdot\langle 0,1,0\rangle = 1(0)+0(1)+0(0) = 0.$

Same for others.

(b) $\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = \mathbf{k}\cdot\mathbf{k} = 1$

$\vec{i}\cdot\vec{i} = \langle 1,0,0\rangle\cdot\langle 1,0,0\rangle = 1(1)+0(0)+0(0) = 1$

Same for others.

(c) The vector $\mathbf{b} - \text{proj}_{\mathbf{a}}\,\mathbf{b}$ is orthogonal to $\mathbf{a}$

Let $\vec{z} = \text{proj}_{\vec{a}}\vec{b}$. Then $(\vec{b}-\vec{z})\cdot\vec{a} = \vec{b}\cdot\vec{a} - \vec{z}\cdot\vec{a}$. But

$\vec{z}\cdot\vec{a} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}\right)\cdot\vec{a} = \frac{|\vec{a}|^2}{|\vec{a}|^2}(\vec{b}\cdot\vec{a}) = \vec{b}\cdot\vec{a}$, so $\vec{b}\cdot\vec{a} - \vec{z}\cdot\vec{a} = \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{a}$
$= 0.$

(d) $\underbrace{|\mathbf{a}+\mathbf{b}|^2}_{①} + \underbrace{|\mathbf{a}-\mathbf{b}|^2}_{②} = \underbrace{2|\mathbf{a}|^2}_{③} + \underbrace{2|\mathbf{b}|^2}_{④}$

① $= (a_1+b_1)^2 + (a_2+b_2)^2 + (a_3+b_3)^2$; ② $= (a_1-b_1)^2 + (a_2-b_2)^2 + (a_3-b_3)^2$

③ $= 2(a_1^2+a_2^2+a_3^2)$; ④ $= 2(b_1^2+b_2^2+b_3^2)$. Lots of cancellation happens w/ ① + ② ....

(e) $\underbrace{\mathbf{a}\times(\mathbf{b}\times\mathbf{c})}_{①} = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$

① $= \langle -a_2b_2c_1 - a_3b_3c_1 + a_2b_1c_2 + a_3b_1c_3,\ a_1b_2c_1 - a_1b_1c_2 - a_3b_3c_2 + a_3b_2c_3,$
$a_1b_3c_1 + a_2b_3c_2 - a_1b_1c_3 - a_2b_2c_3\rangle$. The right side should be the same. (^_^)

(f) $(\mathbf{u}\times\mathbf{v})\cdot(\mathbf{a}\times\mathbf{b}) = (\mathbf{u}\cdot\mathbf{a})(\mathbf{v}\cdot\mathbf{b}) - (\mathbf{v}\cdot\mathbf{a})(\mathbf{u}\cdot\mathbf{b})$

(g) $(\mathbf{u}\times\mathbf{v})\times\mathbf{w} + (\mathbf{v}\times\mathbf{w})\times\mathbf{u} + (\mathbf{w}\times\mathbf{u})\times\mathbf{v} = \mathbf{0}$

I'll let you guys do these so I don't spoil the excitement. :P

6. Find each of the following.

(a) The equation of the line passing through $(2,1,-3)$ and $(6,-1,-5)$.

$= \text{Through } (2,1,-3) \text{ \& } \parallel \text{ to } \langle 4,-2,-2\rangle = \boxed{\langle 2+4t,\ 1-2t,\ -3-2t\rangle}$

(b) The equation of the line passing through $(3,-1,2)$ in the direction of $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$.

$= \langle 3+2t,\ -1-3t,\ 2+4t\rangle$

(c) The equation of the line through $(1,-1,4)$ and perpendicular to both $\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $\langle 0,-3,4\rangle$.

$= \text{Through } (1,-1,4) \text{ \& } \parallel \text{ (cross prod)} = \langle 1,-4,-3\rangle = \boxed{\langle 1+t,\ -1-4t,\ 4-3t\rangle}.$

(d) The equation of the plane passing through $(1,1,1)$ with normal vector $\mathbf{n} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

$= \quad 2(x-1) + 1(y-1) - 2(z-1) = 0$

(e) A unit normal vector to the plane $3x + y - z = 10$.

$\text{Normal} = \langle 3,1,-1\rangle \Rightarrow \boxed{\text{unit normal} = \frac{1}{\sqrt{11}}\langle 3,1,-1\rangle}$

(f) The equation of the plane through the point $(3,-1,-1)$ and perpendicular to the vector $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

$1(x-3) - 2(y+1) + 1(z+1) = 0$

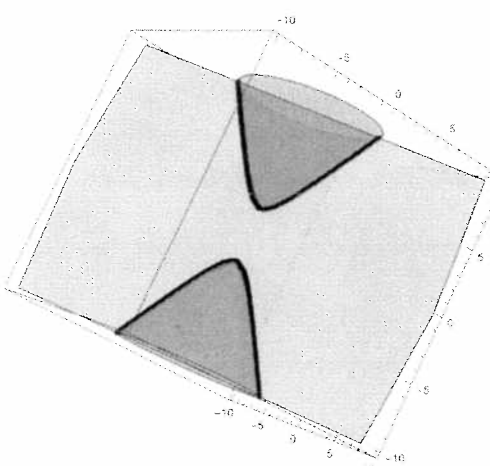
(g) The equation of the line through $(1,1,1)$ and orthogonal to the plane containing $\overset{P}{(0,1,0)}$, Q $(1,0,0)$, and $(0,0,1)$R ($\Rightarrow$ Contains $\vec{PQ} = \langle 1,-1,0\rangle$ & $\vec{PR} = \langle 0,-1,1\rangle$)

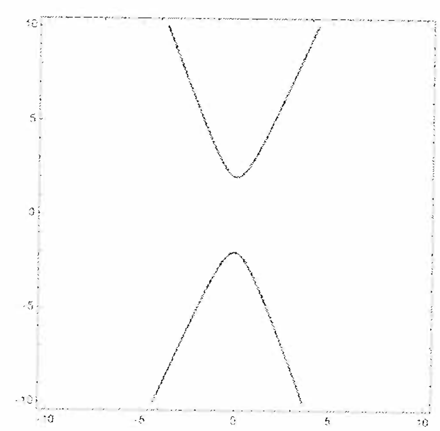
$\therefore$ Contains $(0,1,0)$ & $\perp$ to $\langle -1,-1,-1\rangle \Rightarrow \boxed{-1(x-0) - 1(y-1) - 1(z-0) = 0}$

(h) The angle between the planes $x + y + z = 1$ and $-x - 6y + z = 1$.

Find angle between $\langle 1,1,1\rangle$ & $\langle -1,-6,1\rangle$: $\boxed{\theta = \cos^{-1}\left(\frac{-6}{\sqrt{114}}\right)}$

(i) The (curve formed by the) intersection of the two-sheet hyperboloid $-x^2 + \frac{1}{4}y^2 - \frac{1}{2}z^2 = 1$ and the plane $z = x - \frac{1}{10}y$ (see figures below). **Simplify fully!**





Plug in $z = x - \frac{1}{10}y$ & reduce:

$-x^2 + \frac{1}{4}y^2 - \frac{1}{2}\left(x - \frac{1}{10}y\right)^2 = 1 \Rightarrow \boxed{-1.5x^2 + \frac{1}{10}xy + 0.245y^2 = 1}$

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