Name: ______________________

MAC 2313 — Homework 1

Directions: Complete the following problems for a homework grade. Solutions *must* be presented in a neat and professional manner in order to receive credit, answers given without showing work will not be eligible to receive partial credit, and *work for the problems **must** be done on scratch paper and not on this handout!* **Date Due:** Monday, January 30.

1. (a) Navigate to our course homepage at

   `http://www.math.fsu.edu/~cstover/teaching/sp17_2313/`

   (b) Read and familiarize yourself with the three resources listed under *Supplementary Resources* on the General Info tab.

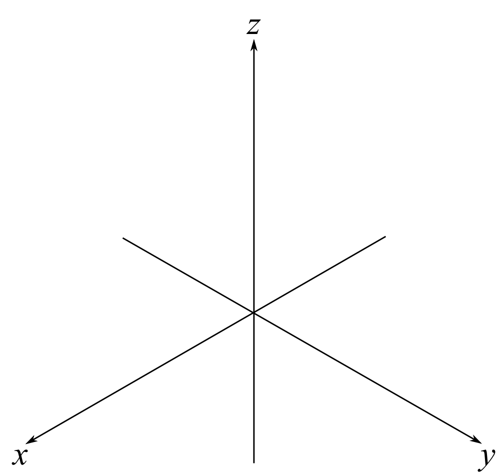
   (c) Follow the instructions for using Slack messenger.

   Note: This may require that I approve your email address, so to avoid some last minute glitch where I don't get to your approval on-time, please don't wait to do this!

   (d) Navigate to the channel `#random` in the left column under Channels (its browser url should be something like `https://spring2017-calc3.slack.com/messages/random/`) and introduce yourself.

   Note: This will be visible to everyone who signs into our class's chat room, so you definitely want to keep this PG-13, safe for work, and non-incriminatory. ☺

2. Plot $(1, -1, 1)$ and $\langle -2, 2, -3 \rangle$ on the axes provided below. **Note**: The arrowheads are pointing towards the *positive* values on each axis!



3. Write the vector from $(0, 1, 2)$ to $(1, \pi, -4)$ in terms of the standard basis vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

4. Consider the vectors $\mathbf{u} = \langle -2, 3, 1\rangle$, $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, and $\mathbf{w} = \langle 0, -1, -1\rangle$. Determine whether each of the following quantities is a scalar or a vector, and express each vector with respect to $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

   (a) $\mathbf{v} + \mathbf{w}$

   (b) $3\mathbf{v}$

   (c) $(\mathbf{u} \cdot \mathbf{w})\,\mathbf{v}$

   (d) The unit vector in the same direction as $\mathbf{u} + \mathbf{w}$

   (e) $(\mathbf{w} \cdot \mathbf{w})\,\mathbf{w}$

   (f) $\mathbf{w} \times \mathbf{u}$

   (g) $\mathbf{u} \times \mathbf{w}$

   (h) The angle between $\mathbf{u} + \mathbf{w}$ and $\mathbf{u} \times \mathbf{w}$.

   (i) The unit vector in the same direction as $\mathbf{w} \times \mathbf{u} - \mathbf{u} \times \mathbf{w}$

   (j) $|\mathbf{v} - \mathbf{w}|$

   (k) $|\mathbf{v} \times \mathbf{u}|$

   (l) The unit vector orthogonal to both $\mathbf{u} + 3\mathbf{v}$ and $2\mathbf{w}$

   (m) The area of the parallelogram spanned by $\mathbf{u} + \mathbf{i}$ and $-2\mathbf{w}$

   (n) $\text{comp}_{\mathbf{u}}\,\mathbf{v}$ and $\text{proj}_{\mathbf{u}}\,\mathbf{v}$

5. Let $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, $\mathbf{c} = \langle c_1, c_2, c_3 \rangle$, $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$, and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ be arbitrary vectors in $\mathbb{R}^3$. Prove each of the following.

   (a) $\mathbf{i} \cdot \mathbf{j} = \mathbf{j} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{i} = 0$

   (b) $\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1$

   (c) The vector $\mathbf{b} - \text{proj}_{\mathbf{a}}\, \mathbf{b}$ is orthogonal to $\mathbf{a}$

   (d) $|\mathbf{a} + \mathbf{b}|^2 + |\mathbf{a} - \mathbf{b}|^2 = 2|\mathbf{a}|^2 + 2|\mathbf{b}|^2$

   (e) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

   (f) $(\mathbf{u} \times \mathbf{v}) \cdot (\mathbf{a} \times \mathbf{b}) = (\mathbf{u} \cdot \mathbf{a})(\mathbf{v} \cdot \mathbf{b}) - (\mathbf{v} \cdot \mathbf{a})(\mathbf{u} \cdot \mathbf{b})$

   (g) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} + (\mathbf{v} \times \mathbf{w}) \times \mathbf{u} + (\mathbf{w} \times \mathbf{u}) \times \mathbf{v} = \mathbf{0}$

6. Find each of the following.

(a) The equation of the line passing through $(2, 1, -3)$ and $(6, -1, -5)$.

(b) The equation of the line passing through $(3, -1, 2)$ in the direction of $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$.

(c) The equation of the line through $(1, -1, 4)$ and perpendicular to both $\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $\langle 0, -3, 4\rangle$.

(d) The equation of the plane passing through $(1, 1, 1)$ with normal vector $\mathbf{n} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

(e) A unit normal vector to the plane $3x + y - z = 10$.

(f) The equation of the plane through the point $(3, -1, -1)$ and perpendicular to the vector $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

(g) The equation of the line through $(1, 1, 1)$ and orthogonal to the plane containing $(0, 1, 0)$, $(1, 0, 0)$, and $(0, 0, 1)$.

(h) The angle between the planes $x + y + z = 1$ and $-x - 6y + z = 4$.

(i) The (curve formed by the) intersection of the two-sheet hyperboloid $-x^2 + \frac{1}{4}y^2 - \frac{1}{2}z^2 = 1$ and the plane $z = x - \frac{1}{10}y$ (see figures below). **Simplify fully!**