Exam Review Solutions

1. (a) $\sqrt{57}$

   (b) Does not exist—can't cross a scalar with a vector.

   (c) $\left\langle -\frac{3}{17}, \frac{2}{17}, -\frac{2}{17} \right\rangle$

   (d) $\langle -8, -11, 1 \rangle$

   (e) $\sqrt{186}$

   (f) Does not exist—can't dot a scalar with a vector.

   (g) $\frac{\pi}{2}$

2. **Note:** There was a typo in this problem that led to the tangent vector $\mathbf{r}'(t)$ being the zero vector at $t = 0$. For that reason, I told some people to do $t = \pi/4$ for practice. I'm including solutions of both here!

   Throughout, note that:

   - $\sin(\pi/2 - t) = \cos t$ and $\cos(\pi/2 - t) = \sin t$. This isn't necessary to notice, but it makes the derivatives easier and these solutions make use of it.
   - In light of the above, $\mathbf{r}'(t) = \langle -\sin t, -\sin t \cos(\cos t), 2t \rangle$.

   t=0:

   $\mathbf{r}'(0) = \langle 0, 0, 0 \rangle$ is parallel to the tangent line and $P = \mathbf{r}(0) = (1, \sin 1, 0)$ is on it, so the line is "point plus vector t," i.e.

   $$x = 1 + 0t \quad y = \sin 1 + 0t \quad z = 0 + 0t \Longrightarrow \boxed{x = 1 \quad y = \sin 1 \quad z = 0}.$$

   $t = \pi/4$:

   $\mathbf{r}'\left(\frac{\pi}{4}\right) = \left\langle -\frac{1}{\sqrt{2}}, \frac{\cos\left(\frac{1}{\sqrt{2}}\right)}{\sqrt{2}}, \frac{\pi}{2} \right\rangle$ is parallel to the tangent line and $P = \mathbf{r}\left(\frac{\pi}{4}\right) = \left(\frac{1}{\sqrt{2}}, \sin\left(\frac{1}{\sqrt{2}}\right), \frac{\pi^2}{16}\right)$ is on it, so the line (after simplifying) is:

   $$\boxed{x = -\frac{t-1}{\sqrt{2}} \quad y = -\frac{\cos\left(\frac{1}{\sqrt{2}}\right)}{\sqrt{2}} t + \sin\left(\frac{1}{\sqrt{2}}\right) \quad z = \frac{1}{16}\pi\left(\pi + 8t\right).}$$

3. (a) The plane containing P, Q, and R necessarily contains the vectors $\vec{PQ} = \langle -2, -4, 2\rangle$ and $\vec{PR} = \langle -1, -5, -4\rangle$, so the vector $\vec{PQ} \times \vec{PR}$ is perpendicular to it. Therefore,

$$\mathbf{n} = \vec{PQ} \times \vec{PR} = \langle 26, -10, 6\rangle$$

is normal to the plane. Using $\mathbf{n}$ and $P(1, 3, 5)$, we have the equation of the plane being

$$\boxed{26(x-1) - 10(y-3) + 6(z-5) = 0}. \tag{1}$$

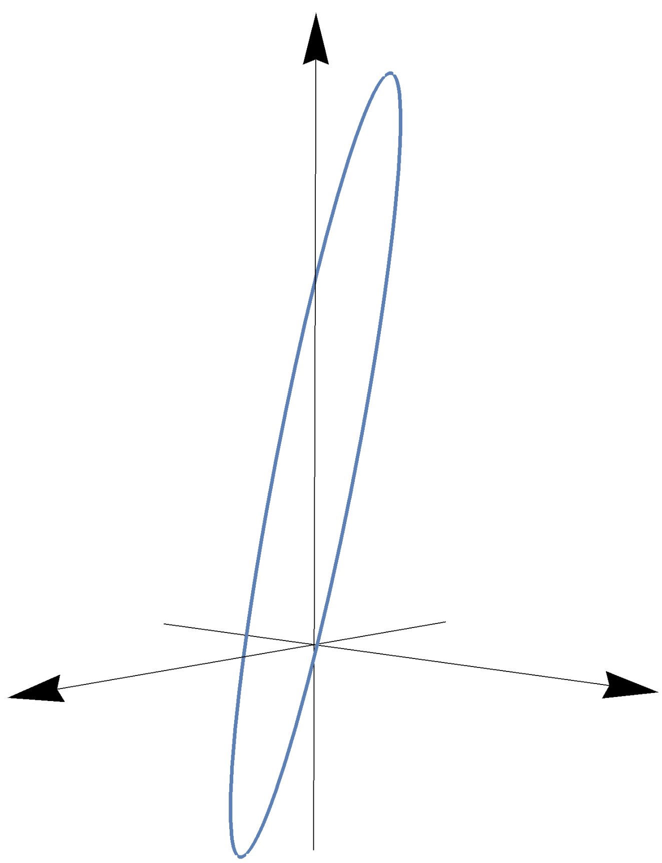
(b) As in class, we note that the we can parametrize the cylinder $x^2 + y^2 = 4$ using $x = 2\cos t$ and $y = 2\sin t$. Plugging these into (1) and solving for z yields

$$z = \frac{1}{3}(10\sin t - 26\cos t + 13),$$

and so the parametrization of this curve is

$$\boxed{\mathbf{r}(t) = \left\langle 2\cos t, 2\sin t, \frac{1}{3}(10\sin t - 26\cos t + 13)\right\rangle}.$$

This curve can be seen below.



4. Note that after simplification,

$$\mathbf{r}'(t) = \left\langle e^t \cos t - e^t \sin t, e^t, e^t \sin t + e^t \cos t \right\rangle$$

and

$$\mathbf{r}''(t) = \left\langle -2e^t \sin t, e^t, 2e^t \cos t \right\rangle.$$

(a) $\mathbb{R}$

(b) Velocity is $\mathbf{r}'(t) = \left\langle e^t \cos t - e^t \sin t, e^t, e^t \sin t + e^t \cos t \right\rangle$

Speed is $|\text{velocity}| = |\mathbf{r}'(t)| = \sqrt{3}e^t$

Acceleration is $\mathbf{r}''(t) = \left\langle -2e^t \sin t, e^t, 2e^t \cos t \right\rangle$.

(c) $\mathbf{T}(t) = \left\langle \dfrac{\cos t - \sin t}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{\sin t + \cos t}{\sqrt{3}} \right\rangle$

(d) $\mathbf{N}(t) = \left\langle -\dfrac{\sin t + \cos t}{\sqrt{2}}, 0, \dfrac{\cos t - \sin t}{\sqrt{2}} \right\rangle$

(e) $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t) = \left\langle \dfrac{\cos t - \sin t}{\sqrt{6}}, \sqrt{\dfrac{2}{3}}, \dfrac{\sin t + \cos t}{\sqrt{6}} \right\rangle$

(f) $\kappa(t) = \dfrac{\sqrt{2}}{3}e^{-t}$

(g) $a_T = \sqrt{3}e^t$

(h) $a_N = \sqrt{2}e^t$

Note: Simplifying $a_T\mathbf{T} + a_N\mathbf{N}$ yields

$$a_T\mathbf{T} + a_N\mathbf{N} = \left\langle -2e^t \sin t, e^t, 2e^t \cos t \right\rangle = \mathbf{a}(t),$$

as expected (because, remember: $\mathbf{a}(t)$ *should* equal $a_T\mathbf{T} + a_N\mathbf{N}$ for all t...).