Exam Review

1. Let $\mathbf{u} = \langle 3, -2, 2\rangle$ and $\mathbf{v} = \mathbf{i} - \mathbf{j} - 3\mathbf{k}$. Compute each of the following or state that it does not exist; if it does not exist, clearly explain why.

   (a) $|\mathbf{u} + 2\mathbf{v}|$

   (b) $|\mathbf{u} - \mathbf{v}| \times \mathbf{u}$

   (c) $\text{proj}_{\mathbf{u}}\, \mathbf{v}$

   (d) $\mathbf{v} \times \mathbf{u}$

   (e) $|\mathbf{u} \times \mathbf{v}|$

   (f) $(\mathbf{u} \cdot \mathbf{v}) \cdot (\mathbf{i} + \mathbf{j})$

   (g) The angle between $\mathbf{v}$ and $\mathbf{u} - \text{proj}_{\mathbf{v}}\, \mathbf{u}$.

2. Let $\mathbf{r}(t) = \langle \cos t, \sin(\sin(\pi/2 - t)), t^2\rangle$. Find the equation of the tangent line to $\mathbf{r}$ at $t = 0$.

3. (a) Find the equation of the plane containing the points $P(1, 3, 5)$, $Q(-1, -1, 7)$, and $R(0, -2, 1)$.

   (b) Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 4$ with the plane from part (a).

4. Let $\mathbf{r}(t) = \langle e^t \cos t, e^t, e^t \sin t\rangle$. Find each of the following. **Simplify fully!**

   (a) The domain of $\mathbf{r}$

   (b) The velocity, acceleration, and speed of a particle with position vector $\mathbf{r}(t)$

   (c) The unit tangent vector $\mathbf{T}(t)$ of $\mathbf{r}$

   (d) The unit normal vector $\mathbf{N}(t)$ of $\mathbf{r}$

   (e) The binormal vector $\mathbf{B}(t)$ of $\mathbf{r}$. What is $|\mathbf{B}(t)|$?

   (f) The curvature $\kappa(t)$ of $\mathbf{r}$

   (g) The tangential component of the acceleration $\mathbf{a}(t)$

   (h) The normal component of $\mathbf{a}(t)$