§16.7–§16.9 Review Problems

Name: ______________________________

Note: This isn't for a grade and there is no due date. Also, if your goal is to focus on the new material, you should do problems 2–5 first.

1. Evaluate the surface integral
$$\iint_F \mathbf{F} \cdot d\mathbf{S},$$
where $\mathbf{F}(x, y, z) = xy\,\mathbf{i} + 3x^2\,\mathbf{j} + xz\,\mathbf{k}$ and where F is the surface $z = xe^y$, $0 \le x \le 1$, $0 \le y \le 1$ with upward orientation.

2. Use Stokes' theorem to evaluate each of the following.

   (i) $\iint_F \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle 2x \cos 2z, e^y \sin z, x^2 y e^y \rangle$ and F is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \ge 0$, oriented upward.

   (ii) $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = (x + y^2)\,\mathbf{i} + (y + z^2)\,\mathbf{j} + (z + x^2)\,\mathbf{k}$ and where C is the triangle with vertices $(2, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$.

3. Verify that Stokes' theorem is true for the vector field $\mathbf{F} = \langle y, z, x \rangle$ and the surface F equal to the hemisphere $x^2 + y^2 + z^2 = 1$, $z \ge 0$, oriented upward.
Hint: $\sin^2(x) = \frac{1}{2}(1 - \cos 2x)$. Also you can parametrize F as
$$\mathbf{r}(u, v) = \langle \cos(u) \sin(v), \sin(u) \sin(v), \cos(v) \rangle \qquad (0 \le u \le 2\pi, 0 \le v \le \pi/2),$$
which is like spherical coordinates with $\rho = 1$ *except* that $dA = du\,dv$ without a pesky $\sin(v)$ Jacobian-thing in there.

4. Use the divergence theorem to compute each of the following.

   (i) The flux of $\mathbf{F} = \langle x^2 \sin y, x \cos y, -xz \sin y \rangle$ across the "fat sphere" F equal to $x^8 + y^8 + z^8 = 8$.

   (ii) $\iint_F \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = (\cos z e^{e^z} + xy^2)\,\mathbf{i} + z \cos x \sin xze^{-z}\,\mathbf{j} + (\sin y + x^2 z)\mathbf{k}$ and where F is the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.
   Hint: Use cylindrical coordinates.

5. Verify that the divergence theorem is true for the vector field $\mathbf{F} = \langle x^2, xy, z \rangle$ and the solid E bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.