Exam Review

1. Sketch each of the following regions and write each as the type designated. **There should be no integrals in your answers!**

   (a) $D = \{(x, y) : -\sqrt{4 - x^2} \le y \le \sqrt{4 - x^2}, -2 \le x \le 2\}$; as a polar rectangle

   (b) $T_1 =$ the triangle in $\mathbb{R}^2$ with vertices $(0, 0)$, $(0, 3)$, $(3, 3)$; as a Type I region

   (c) $T_2 =$ the same triangle as in (a); as a Type II region

   (d) $D =$ the annular region in the upper half plane, bounded between the lines $y = x$ and $y = -x$ and the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$; as a polar rectangle

2. (a) Use double integrals to find the volume of each of the solids described below.

      i) The solid that lies under the paraboloid $z = x^2 + y^2$ and above the region D in the xy-plane bounded by the line $y = 2x$ and the parabola $y = x^2$.

      ii) The solid under the surface $z = xy$ and above the triangle with vertices $(1, 1)$, $(6, 1)$, and $(1, 3)$.

      iii) The solid inside the sphere $x^2 + y^2 + z^2 = 9$ and outside the cylinder $x^2 + y^2 = 3$.

   (b) For each of the regions described in (a), set up and evaluate the integral you would use to get the volume of the region using *the other* order of integration.

3. Express each of the following regions in polar coordinates.

   (a) The bottom half of the disk of radius 3, centered at $(0, 0)$.

   (b) The left half of the disk of radius 1, centered at $(-1, 0)$.

   (c) The area in the upper half plane inside the circle $x^2 + y^2 = 1$, between the lines $y = x$ and $y = -x$, and outside the 4-petaled rose $(x^2 + y^2)^3 = (x^2 - y^2)^2$. **Hint**: $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

   (d) The annular region lying between the circles $x^2 + y^2 = 2$ and $x^2 + y^2 = 5$.

4. Find the surface area of the part of the paraboloid $z = 16 - x^2 - y^2$ that lies under the plane $z = 9$ and above the plane $z = 4$.

5. Evaluate each of the following triple integrals, changing coordinate systems as necessary.

   (a) $\iiint_E 6xy\, dV$, where E lies under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.

(b) $\iiint_E dV$, where E is the solid enclosed by the cylinder $x^2+z^2=4$ and the planes $y=-1$ and $y+z=4$.

(c) $\iiint_E (x+y+z)\,dV$, where E is the solid in the first octant that lies under the paraboloid $z=4-x^2-y^2$.

(d) $\iiint_H (9-x^2-y^2)\,dV$, where H is the solid hemisphere $x^2+y^2+z^2 \le 9$, $z \ge 0$.

6. (a) Use a triple integral to find the volume of the tetrahedron T bounded by the planes $x=0$, $y=0$, $z=0$, and $x+y+z=2$

   (b) Set up and evaluate the other five triple integrals to representing the volume of T (so altogether, you should have evaluated six integrals—one each for $dxdydz$, $dxdzdy$, $dydxdz$, $dydzdx$, $dzdxdy$, and $dzdydx$—and you should get the same volume for each).

   Hint: See the "Dummit" `.pdf` file mentioned in the *Links, etc.* tab on the course webpage; in particular, se pp10–12 for a really good explanation of how to do problems like this one.

7. Rewrite each of the rectangular iterated integrals in the coordinate system specified. **Do not integrate!**

   (a) $\displaystyle\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} e^{x^2+y^2}\,dx\,dy$; in polar coordinates

   (b) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{3-x^2-y^2} \sqrt{x^2+y^2}\,dz\,dy\,dx$; in cylindrical coordinates.

   (c) $\displaystyle\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_{0}^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\,dz\,dx\,dy$; in spherical coordinates.

8. (a) Compute the Jacobian for each of the transformations listed.
      i) From rectangular coordinates to polar coordinates in $\mathbb{R}^2$.
      ii) From rectangular coordinates to cylindrical coordinates in $\mathbb{R}^3$.
      iii) From rectangular coordinates to spherical coordinates in $\mathbb{R}^3$.

   (b) Use your answer for each of the transformations in part (a) to derive the formula for double and/or triple integrals (double integrals for $\mathbb{R}^2$ and triple in $\mathbb{R}^3$) of a function $f(x,y)$ and/or $f(x,y,z)$ (the former for $\mathbb{R}^2$ and the latter for $\mathbb{R}^3$) over a region D in $\mathbb{R}^2$ and/or E in $\mathbb{R}^3$.
   the result from part (a) to derive the formula for triple integration in spherical coordinates.