Feb 28, 2017

Exam 2

MAC 2313—CALCULUS III, SPRING 2017

(NEATLY!) PRINT NAME: ______________________________

Read all of what follows carefully before starting!

1. This test has **4 problems** (12 parts total) and is worth **100 points**. *Please be sure you have all the questions before beginning!*
2. The exam is closed-note and closed-book. You may **not** consult with other students, and **no** calculators may be used!
3. Show all work clearly in order to receive full credit. Points will be deducted for incorrect work, and unless otherwise stated, no credit will be given for a correct answer without supporting calculations. **No work = no credit!** (unless otherwise stated)
4. You may use appropriate results from class and/or from the textbook as long as you fully and correctly state the result and where it came from.
   - If you use a result/theorem, you have to state *which* result you're using and explain *why* you're able to use it!
5. You **do not** need to simplify results, unless otherwise stated.
6. There is scratch paper at the end of the exam; you may also use the backs of pages or get more scratch paper from me.

Question	1 (20 pts)	2 (15 pts)	3 (30 pts)	4 (35 pts)	Total (100 pts)
Points					

Do not write in these boxes! If you do, you get 0 points for those questions!

1. Let $f(x, y) = e^{x+xy}$.

   (a) (*10 pts*) Find the equation of the tangent plane to f at the point $(1, 0, e)$.

SOLUTION:

Part (b) is on the next page

(b) (*10 pts*) Prove that $f(x, y)$ is a solution to the *partial differential equation*

$$xf_y - f_x = f_{yy} - \frac{f_{xx}}{1+y}.$$

Hint: This means that if you compute f_x, f_y, f_{xx} and f_{yy}, they should satisfy that equation.

SOLUTION:

2. Let $f(w, x, y, z) = w + e^x - \sin y + z\cos z - xyz + 4$, where

$$w = r^2 s - t \qquad x = t\sin s + r \qquad y = r(t - s) \qquad z = e^{e^t} + r - s.$$

(a) (*5 pts*) Find f_{zyxw}, citing any results you may use to simplify your computations.

SOLUTION:

Part (b) is on the next page

(b) (*10 pts*) Find $\dfrac{\partial f}{\partial t}$.

SOLUTION:

3. Let $g(x, y) = xe^y$, let $\theta = \pi/6$, and let $\mathbf{u}$ be the unit vector given by θ.

(a) (*5 pts*) Is g differentiable? Why or why not?

(b) (*10 pts*) Using any technique you know (e.g. witchcraft, voodoo magic, limits, shortcuts we learned in class,...), find the directional derivative of g in the direction of $\mathbf{u}$.

Part (c) is on the next page

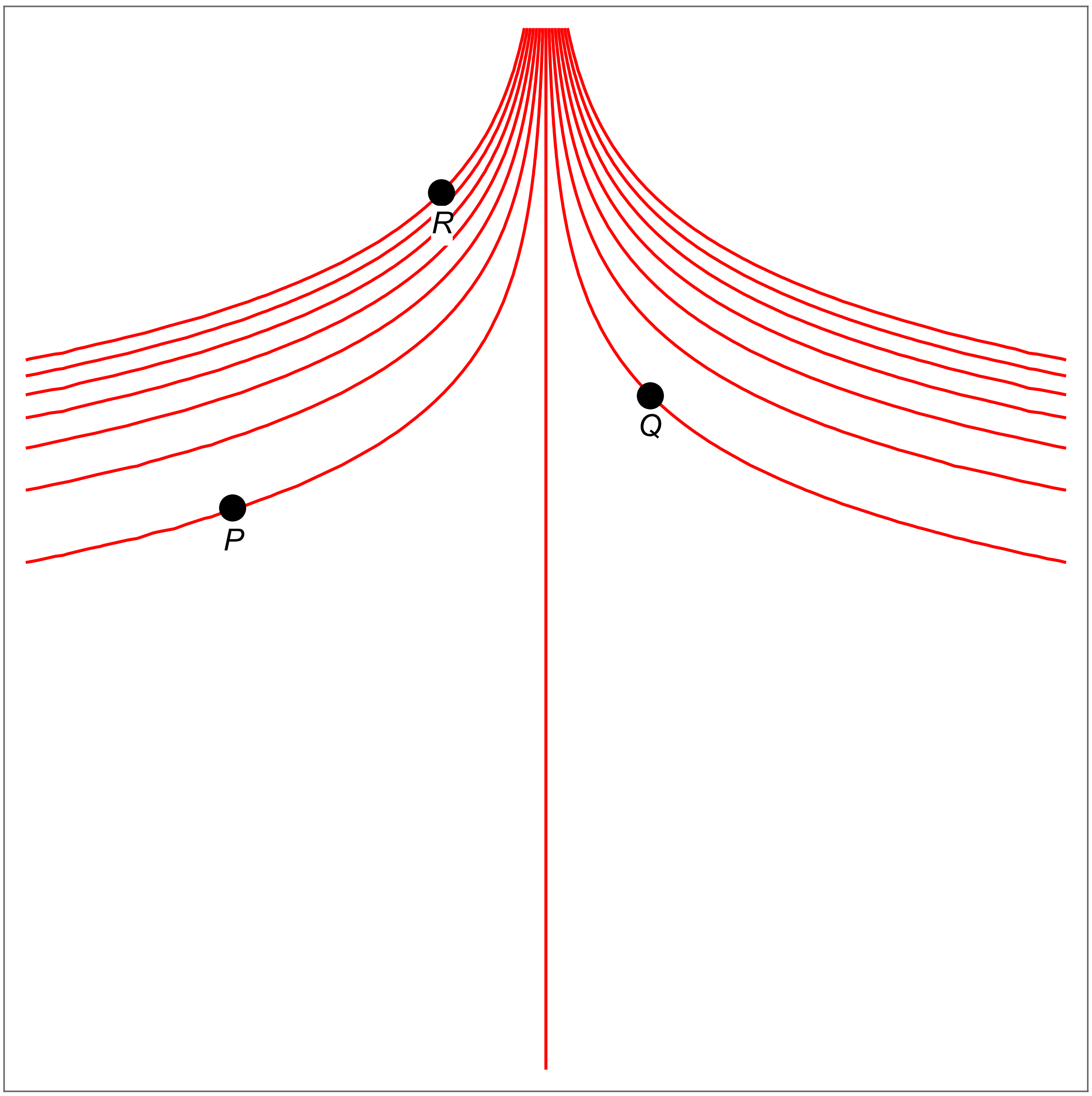
(c) (*10 pts*) At the point $P(1, 1)$, in what direction does g have the maximum rate of change? What is this maximum rate of change?

SOLUTION:

Part (d) is on the next page

(d) (*5 pts*) Below is a contour plot of $g(x, y) = k$ for various values of k (a constant).

At each of the three points P, Q, and R, draw a vector in the same direction as ∇g. *I don't care about the magnitude of the vectors you draw.*



4. Let $f(x, y) = x^2 + 3y^2$.

(a) (*5 pts*) Is $(3, 4)$ a critical point of f? How do you know?

(b) (*10 pts*) The point $P(0, 0)$ is a critical point for f. Use the second derivative test to determine if P is a local maximum, a local minimum, or a saddle point for f.

Part (c) is on the next page

(c) (*10 pts*) Use Lagrange multipliers to find the extreme values of f on the ellipse $g(x, y) = 1$, where

$$g(x, y) = \frac{x^2}{4} + y^2.$$

SOLUTION:

Part (d) is on the next page

(d) (*10 pts*) Find absolute maxima and minima of f on the region Σ, where

$$\Sigma = \left\{ (x, y) \text{ in } \mathbb{R}^2 \text{ such that } \frac{x^2}{4} + y^2 \leq 1 \right\}.$$

Note: If you use a result/theorem, you have to state *which* result you're using and explain *why* you're able to use it!

SOLUTION:

Scratch Paper