§11.5 - Alternating Series

Def: An <u>alternating series</u> is a series whose terms alternate positive to negative!

<u>Ex:</u> $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$

$-\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{4}{5} - \frac{5}{6} + \cdots = \sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$

Alternating series is always of the form

$a_n = (-1)^{n-1} b_n$ or $a_n = (-1)^n b_n$

for b_n a positive number!

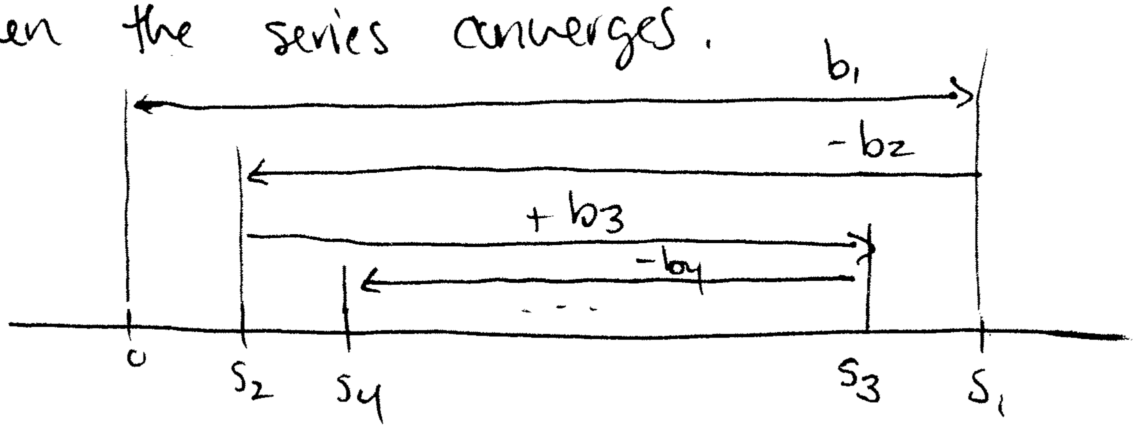
The Alternating Series Test

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$, $b_n > 0$, satisfies

① $b_{n+1} \leq b_n \quad \forall n$ ← "Eventually true" is okay!

② $\lim_{n \to \infty} b_n = 0$,

then the series converges.



Ex: Discuss Convergence / Divergence:

① $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$

↳ Note that $\displaystyle \frac{1}{n+1} < \frac{1}{n} \Rightarrow b_{n+1} < b_n \ \forall n$

& $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n} = 0$ $\Big] \Rightarrow$ conv

② $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3n}{4n-1}$

↳ $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{3n}{4n-1} = \frac{3}{4} \neq 0.$

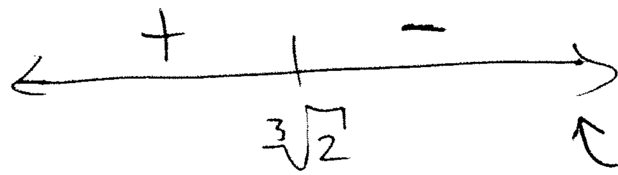
<u>DIVERGE</u>

③ $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+1}$

- For increasing, test function $f(x) = \dfrac{x^2}{x^3+1}$:

$$f'(x) = \frac{x(2-x^3)}{(x^3+1)^2} \longrightarrow \text{Crit pts } 0, \sqrt[3]{2}, -1$$

not in range $(1,\infty)$

$\Rightarrow$ 

plug in $f'(x)$

$\Rightarrow f$ decreasing on $(\sqrt[3]{2}, \infty)$ ← This is good enough!

- Also, $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n^2}{n^3+1} = 0$, <u>CONVERGE</u>