MAC 2312 — Homework 6

———SOLUTIONS———

1. You don't have to go home but you can't (and shouldn't) stay here (...because there's nothing to see...).

2. **Solution:** $\sum_{i=2}^{6} a_i \leq \int_1^6 f(x)\,dx \sum_{i=1}^{5} a_i.$

3. (a) Diverge
   (b) Converge
   (c) Converge
   (d) Converge
   (e) Converge
   (f) Converge

4. **DO NOT WORRY ABOUT THIS PROBLEM FOR THE EXAM!**
   (a) No p.
   (b) $p > 1$.
   (c) $p < -1$.
   (d) No p.

5. Below, "regular" means the (regular) comparison test is "easier" (more straightforward?) while "limit" means the same thing for the limit comparison test.
   (a) Diverge — Limit
   (b) Diverge — Regular
   (c) Diverge — Either
   (d) Converge — Limit
   (e) Converge — Neither; skip this one!
   (f) Diverge — Either, but it's hardish
   (g) Converge — Either
   (h) Converge — Either, but it's hardish (again)

6. (a) Diverge — both inconclusive; $a_n \not\to 0$
   (b) Diverge — root inconclusive; $a_n \not\to 0$
   (c) Converge — ratio
   (d) Diverge — root inconclusive; $a_n \not\to 0$
   (e) Converge — both inconclusive; p-test
   (f) Diverge — ratio
   (g) Diverge — root (hard); also, $a_n \not\to 0$
   (h) Converge — ratio

7. (a) Diverge — comparison with $\sum 1/n$ works
   (b) Converge — p-test with $p = \sqrt{2} > 1$
   (c) Diverge — alternating series test; ratio test should work too
   (d) Converge — root test
   (e) Converge — limit comparison test with $b_n = 1/n^{3/2}$
   (f) Diverge — $a_n \not\to 0$
   (g) Converge — ratio test; (regular/limit) comparison test if you're clever
   (h) Diverge — write as $\sum \frac{1}{5+3(n-1)}$, then limit comparison with $b_n = 1/n$
   (i) Diverge — $a_n \not\to 0$; ratio test will also work; maybe alternating series too?
   (j) Converge — integral test (it's a tricky integral); note that root+ratio test are both inconclusive
   (k) Converge — geometric series times two
   (l) Diverge — comparison with $b_n = n/4^{-n}$ is easiest; ratio test will also work
   (m) Converge — alternating series test

(n) Diverge — l$a_n \not\to 0$

(o) Converge — write as $\sum a_n$ for $a_n = \sin 1/n^2$; $\sum |a_n|$ and use comparison test with $\sum 1/n^2$

(p) Converge — alternating series test

8. (a) **Skip this problem**

   (b) Repeat the same procedure we did in class today; **DEFINITELY KNOW HOW TO DO THIS ONE!**

   (c) This is a comparison test problem where I've essentially shown you which comparison(s) to do; try this, but if you get stuck, skip it.