Name: ______________________________

MAC 2312 — Homework 6

Directions: Complete the following problems for a homework grade. Solutions *must* be presented in a neat and professional manner in order to receive credit, answers given without showing work will not be eligible to receive partial credit, and *work for the problems **must** be done on scratch paper and not on this handout!* **Date Due:** Thursday, December 1.

1. Go to the #music_recs channel in our course's SLACK room (see course homepage for the URL) and make a music recommendation! The more obscure, the better!

2. Suppose f is a continuous positive decreasing function for $x \geq 1$ and that $a_n = f(n)$. Rank the following three quantities in *increasing* order:

$$\int_1^6 f(x)\,dx \qquad \sum_{i=1}^{5} a_i \qquad \sum_{i=2}^{6} a_i.$$

3. Use the integral test to determine whether each of the following series converges.

(a) $\displaystyle\sum_{n=1}^{\infty} (n-4)^{-1/3}$

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{n^2}{(n^3+9)^{5/2}}$

(c) $\displaystyle\sum_{n=12}^{\infty} ne^{-n^2}$

(d) $\displaystyle\sum_{n=146}^{\infty} \frac{1}{n(\ln n)^2}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^4+1}$

(f) $\displaystyle\sum_{n=13}^{\infty} \frac{1}{n(n+2)}$

4. For which values of p are the following series convergent?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^p}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^p}$

(c) $\displaystyle\sum_{n=1}^{\infty} n(1+n^2)^p$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\ln n[\ln(\ln n)]^p}$

5. Use a comparison test (either *the* comparison test or the limit comparison test) to determine whether each of the following series converges or diverges.

(a) $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$

(b) $\sum_{n=5}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$

(c) $\sum_{k=3}^{\infty} \frac{6^k}{5^k-1}$

(d) $\sum_{n=2}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3+4n+3}}$

(e) $\sum_{k=1}^{\infty} \frac{(\ln k)^{12}}{k^{9/8}}$

(f) $\sum_{j=1}^{\infty} \frac{j!}{j^3}$

(g) $\sum_{k=2}^{\infty} \frac{4k^2+14k}{3k^4-5k^2-17}$

(h) $\sum_{n=3}^{\infty} \frac{1}{(n+1)!}$

6. Use either the root test or the ratio test to examine each of the following series for convergence. For those which are convergent: Are they absolutely convergent? (If the root/ratio tests are inconclusive, examine convergence using any other technique at your disposal.)

(a) $\sum_{n=1}^{\infty} n^2$

(b) $\sum_{k=2}^{\infty} \left(\frac{k}{k+10}\right)^k$

(c) $\sum_{n=1}^{\infty} \frac{1}{n^n}$

(d) $\sum_{r=1}^{\infty} \left(1+\frac{1}{r}\right)^{-r}$

(e) $\sum_{k=2}^{\infty} k^{-2}$

(f) $\sum_{n=1}^{\infty} \frac{n!}{n \ln n}$

(g) $\sum_{k=4}^{\infty} \left(1-\frac{1}{k}\right)^{-k^2}$

(h) $\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$

7. Determine whether each of the following series converges or diverges (using any technique you know).

(a) $\frac{1+(-1)^1}{1} + \frac{1+(-1)^2}{2} + \frac{1+(-1)^3}{3} + \cdots$

(b) $\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}}}$

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{n^n}{n!}$

(d) $\sum_{k=1}^{\infty} 2^{-k^2}$

(e) $\sum_{n=3}^{\infty} \frac{1}{n^{3/2} - (\ln n)^4}$

(f) $\sum_{k=2}^{\infty} (-1)^k \cos\left(\frac{\pi}{k}\right)$

(g) $\sum_{j=1}^{\infty} \frac{j!}{(2j)!}$

(h) $\frac{1}{5} + \frac{1}{8} + \frac{1}{11} + \frac{1}{14} + \frac{1}{17} + \cdots$

(i) $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}}$

(j) $\sum_{n=1}^{\infty} \frac{1}{e^{\sqrt{n}}}$

(k) $\sum_{j=2}^{\infty} \frac{2^j + 4^j}{7^j}$

(l) $\sum_{n=91}^{\infty} \frac{n}{4^{-n} + 5^{-n}}$

(m) $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\sqrt{n+1} - \sqrt{n}\right)$

(n) $\sum_{k=1}^{\infty} (0.8)^{-k} k^{-0.8}$

(o) $\sin\left(\frac{1}{1^2}\right) + \sin\left(\frac{1}{2^2}\right) + \sin\left(\frac{1}{3^2}\right) + \cdots$

(p) $\sum_{k=12}^{\infty} \frac{(-1)^k}{\sqrt{k^3 - k^2}}$

8. (a) Is there any value k such that $\sum_{n=1}^{\infty} \frac{2^n}{n^k}$ converges? (**Hint**: There is.)

(b) Use the fact that $\lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right)^n = \frac{1}{e}$ to show that $\sum_{k=1}^{\infty} \frac{k!}{k^k}$ converges.

(c) Given that $2^{n^2} = (2^n)^n$ and that $n! \le n^n$, prove that $\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}$ diverges.