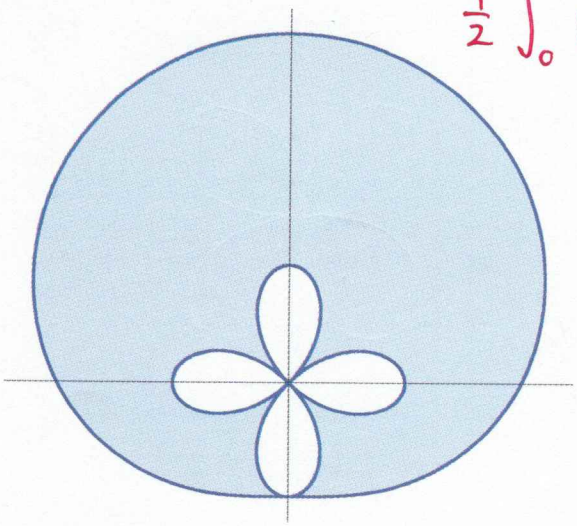
KEY

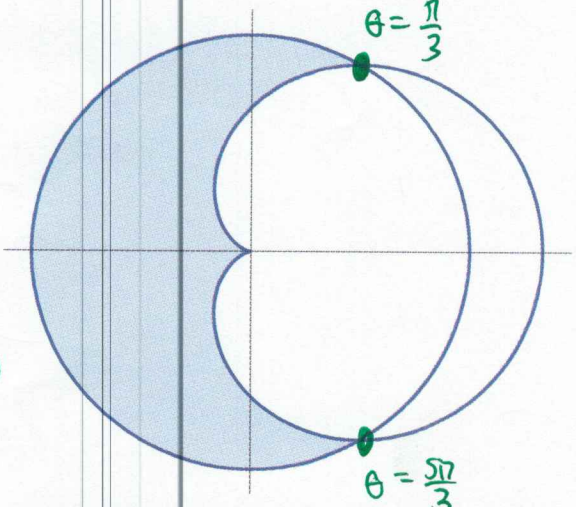
12. Find the area of the following shaded region, where the outer curve is given by $r = 2 + \sin\theta$ and where the inner curve is given by $r = \cos(2\theta)$.

$\frac{1}{2}\int_0^{2\pi} (2+\sin\theta)^2 - (\cos 2\theta)^2 \, d\theta$

$= 4\pi.$



13. For each of the following, determine (i) the arc lengths of each of the two curves; and (ii) the area of the corresponding shaded region. **Hint:** To find the areas, you'll need to know the intersections of the two curves!



(a) Circle: $r = 3$; Cardioid: $r = 2(1 + \cos t)$

$3 = 2(1+\cos\theta)$

$\Downarrow$

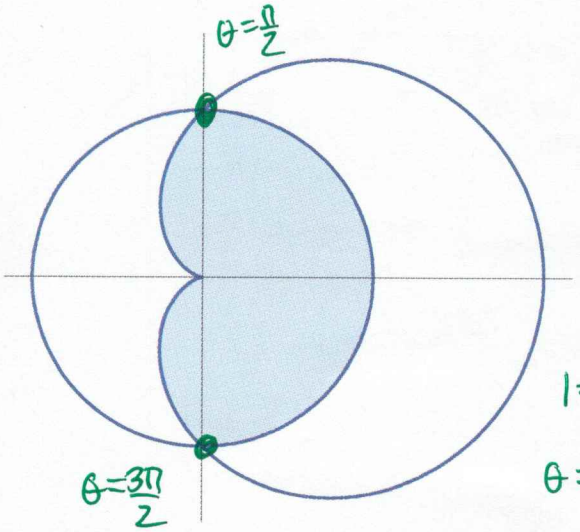
$\cos\theta = \frac{1}{2}$

$\Downarrow$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$= \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (3)^2 - (2(1+\cos\theta))^2 \, d\theta$

$= \frac{9\sqrt{3}}{2} + 2\pi$



(b) Circle: $r = 1$; Cardioid: $r = 1 + \cos t$

$1 = 1+\cos\theta$

$\Downarrow$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$= \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1 - (1+\cos\theta)^2 \, d\theta$

$= 2 - \frac{\pi}{4}$