KEY

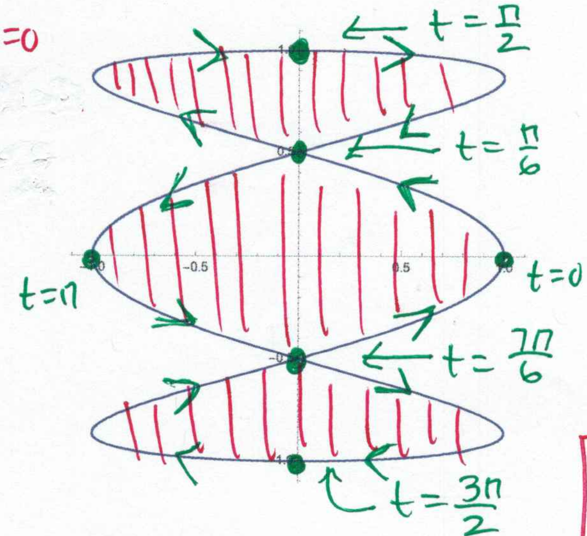
6. Find the area bounded by y-axis and the *Lissajous figure* $x(t) = \cos 3t$, $y(t) = \sin t$, $0 \le t \le 2\pi$ whose graph is shown below.

Note: $x = 0 \Rightarrow \cos 3t = 0$

$\Rightarrow 3t = \frac{\pi}{2}$, $3t = \frac{3\pi}{2}$

$\Rightarrow t = \frac{\pi}{6}$, $t = \frac{\pi}{2}$

& $t = \frac{7\pi}{6}$, $t = \frac{3\pi}{2}$ by symmetry



So:

$A = 4(\text{Area in Quad 1})$

$= 4\int_0^{\pi/2} x \frac{dy}{dt}\,dt$

$= 4\int_0^{\pi/6} \cos(3t)\cos t\,dt + 4\int_{\pi/2}^{\pi/6} \cos(3t)\cos t\,dt$

$= 4\left(\frac{3\sqrt{3}}{16} + \frac{3\sqrt{3}}{16}\right) = \frac{3\sqrt{3}}{2}$

7. Find the length of the curve $x(t) = t^8$, $y(t) = t^4$, $1 \le t \le 3$. **Hint:** You'll most likely need u-sub *and* trig sub to compute the integral you eventually get, which should look like

$$\int_1^3 4t^3\sqrt{(2t^4)^2 + 1}\,dt \quad \text{(or something equivalent).}$$

① $u = 2t^4 \quad du = 8t^3\,dt \Rightarrow \frac{du}{8} = t^3\,dt \rightarrow \frac{4}{8}\int\sqrt{u^2+1}\,du$

Now: $u = \tan\theta \Rightarrow du = \sec^2\theta\,d\theta \rightsquigarrow \frac{1}{2}\int\sqrt{\sec^2\theta}\,\sec^2\theta\,d\theta = \frac{1}{2}\int\sec^3\theta\,d\theta$: You guys are stuck here!

8. (a) Set up an integral that represents the surface area of the solid formed when the portion of the cycloid $x(t) = 2(t - \sin t)$, $y(t) = 2(1 - \cos t)$, $0 < t < 4\pi$, is revolved about the x-axis.

$A = \int_0^{4\pi} 2\pi\left(2(1-\cos t)\right)\sqrt{(2(1-\cos t))^2 + (2\sin t)^2}\,dt$

(b) Find the exact surface area obtained by rotating the curve $x = 2\cos^3 t$, $y = 2\sin^3 t$, $0 \le t \le \pi/2$, about the x-axis.

$A = \frac{24}{5}\pi\sin^5 t\Big]_{t=0}^{t=\pi/2} = \boxed{\frac{24}{5}\pi}$

9. The *speed* s of a particle p traveling along the path $x = f(t)$, $y = g(t)$ is defined to be $s = \sqrt{[f'(t)]^2 + [g'(t)]^2}$. If p moves around an elliptical track according to the equations $x = \cos t$, $y = \sin 2t$, $0 \le t < 2\pi$, when is its speed the greatest? When is it the least?

$s = \sqrt{(-\sin t)^2 + (2\cos 2t)^2} \Rightarrow \frac{ds}{dt} = \frac{2\cos(t)\sin(t) - 16\cos(2t)\sin(2t)}{2\sqrt{4\cos^2(2t) + \sin^2 t}}$

$= \sqrt{\sin^2 t + 4\cos^2(2t)}$

There was a typo which makes this hard to finish but you would: { Find crit. vals, put on # line, find incr / decr, and note max = where incr. changes to decr. & min = decr → incr.