KEY

3. Let C be the parametric curve given by $x(t) = \cos(t)$, $y(t) = \sin(t)$, $0 \leq t < 2\pi$.

   (a) Find dy/dx. **Hint:** You'll want to simplify here.

   $\longrightarrow -\cot(t) = \frac{-\cos t}{\sin t}$

   (b) Find an equation of the tangent line to C at the point $(3, e^{16})$.

   TYPO HERE!

   (c) Find the points on C where the tangent is horizontal or vertical.

   Horizontal: $\frac{\pi}{2}, \frac{3\pi}{2}$ Vertical: $0, \pi,$

   (d) Find the intervals on which C is increasing and decreasing.

   Increasing: $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$ Decreasing: $(\frac{\pi}{2}, \frac{3\pi}{2})$

   (e) Find d^2y/dx^2. **Hint:** You'll want to simplify here, too!

   $\csc(t)\cot(t) = \frac{\cos t}{\sin^2 t}$

   (f) Find the intervals on which C is concave up and concave down.

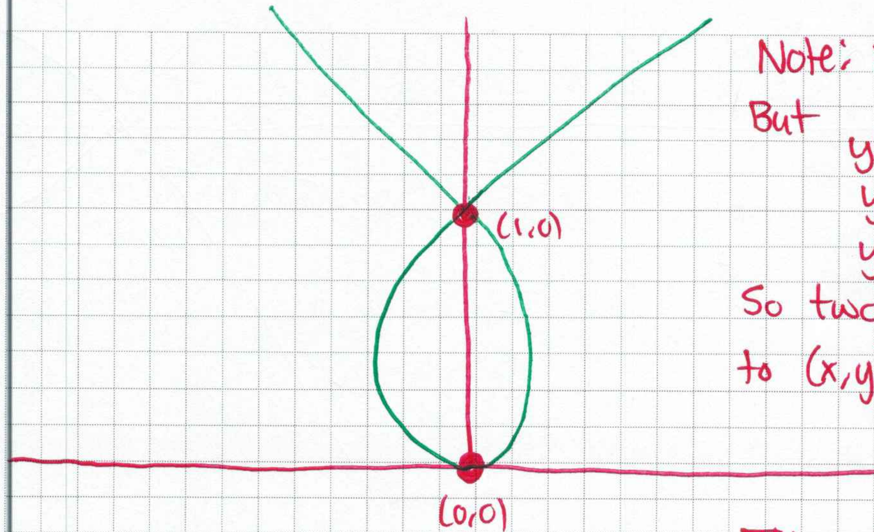
   $C\uparrow$: $(\pi, 2\pi)$ $C\downarrow$: $(0, \pi)$

4. Repeat parts (a) through (f) of number 3 for the curve defined by $x(t) = 1 + \sqrt{t}$, $y(t) = e^{t^2}$.

(a) $4t^{3/2}e^{t^2} \rightarrow$ (b) $y - e^{16} = 32e^{16}(x-3) \rightarrow$ (d) Increasing everywhere

(c) Horiz. at $t=0 \rightarrow$ (e) $8t^2e^{t^2} \rightarrow$ (f) $C\uparrow$ everywhere

5. Show that the curve $x(t) = t^3 - t$, $y(t) = t^2$ has two tangents at $(0, 1)$, and sketch its graph.



Note: $x(t)=0 \Rightarrow t=-1,0,1.$
But
$y(-1)=1$
$y(0)=0$
$y(1)=1.$
So two t-vals correspond to $(x,y)=(0,1)$, and $\left.\frac{dy}{dx}\right|_{t=-1} \neq \left.\frac{dy}{dx}\right|_{t=1}$
This shows that there are two tangents there!