Name: ______________________________

MAC 2312 — Homework 4

Directions: Complete the following problems for a homework grade. Solutions *must* be presented in a neat and professional manner in order to receive credit, answers given without showing work will not be eligible to receive partial credit, and *work for the problems* ***must*** *be done on scratch paper and not on this handout!* **Date Due:** Tuesday, November 8.

1. Go to the #thanksgiving_st00fs channel in our course's SLACK room (see course homepage for the URL) and say something about Thanksgiving. Do you like it? Do you not like it? What do you eat? Are you going somewhere? Tell us things!

2. Sketch the curve given by the parametric equations $x(t) = t^2$, $y(t) = t^3 - 4t$, $-4 \le t \le 4$, by plotting points. Indicate with an arrow the direction in which the curve is traced as t increases.

t									
x									
y									

3. Let C be the parametric curve given by $x(t) = \cos(t)$, $y(t) = \sin(t)$, $0 \leq t < 2\pi$.

   (a) Find dy/dx. **Hint:** You'll want to simplify here.

   (b) Find an equation of the tangent line to C at the point $\left(3, e^{16}\right)$.

   (c) Find the points on C where the tangent is horizontal or vertical.

   (d) Find the intervals on which C is increasing and decreasing.

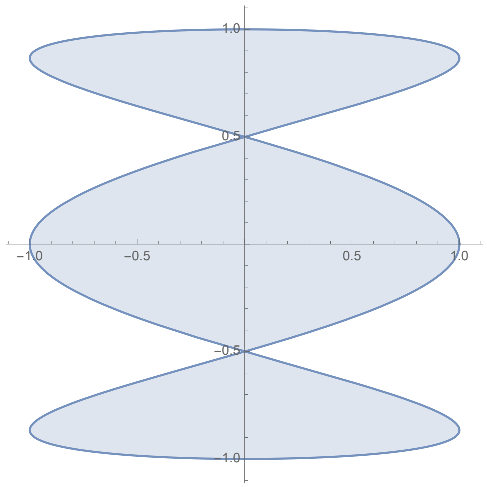
   (e) Find d^2y/dx^2. **Hint:** You'll want to simplify here, too!

   (f) Find the intervals on which C is concave up and concave down.

4. Repeat parts (a) through (f) of number 3 for the curve defined by $x(t) = 1 + \sqrt{t}$, $y(t) = e^{t^2}$.

5. Show that the curve $x(t) = t^3 - t$, $y(t) = t^2$ has two tangents at $(0, 1)$, and sketch its graph.

6. Find the area bounded by y-axis and the *Lissajous figure* $x(t) = \cos 3t$, $y(t) = \sin t$, $0 \le t \le 2\pi$ whose graph is shown below.



7. Find the length of the curve $x(t) = t^8$, $y(t) = t^4$, $1 \le t \le 3$. **Hint:** You'll most likely need u-sub *and* trig sub to compute the integral you eventually get, which should look like

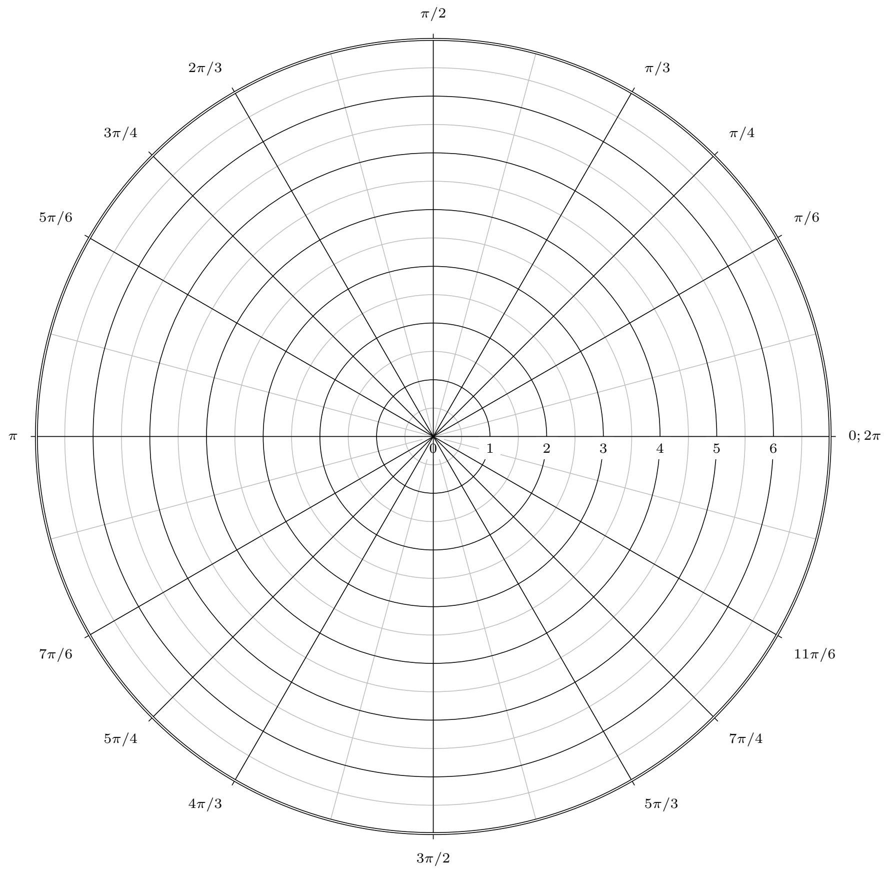
$$\int_1^3 4t^3\sqrt{(2t^4)^2 + 1}\, dt \quad \text{(or something equivalent).}$$

8. (a) Set up an integral that represents the surface area of the solid formed when the portion of the cycloid $x(t) = 2(t - \sin t)$, $y(t) = 2(1 - \cos t)$, $0 < t < 4\pi$, is revolved about the x-axis.

   (b) Find the exact surface area obtained by rotating the curve $x = 2\cos^3 t$, $y = 2\sin^3 t$, $0 \le t \le \pi/2$, about the x-axis.

9. The *speed* s of a particle p traveling along the path $x = f(t)$, $y = g(t)$ is defined to be $s = \sqrt{[f'(t)]^2 + [g'(t)]^2}$. If p moves around an elliptical track according to the equations $x = \cos t$, $y = \sin 2t$, $0 \le t < 2\pi$, when is its speed the greatest? When is it the least?

10. (a) Plot the points $P_1(5, \pi/6)$, $P_2(-5, \pi/6)$, $P_3(5, \pi/6)$, and $P_4(-5, -\pi/6)$.



(b) Describe the set of points P whose polar coordinates (r, θ) satisfy $0 \le r \le 2$ and $0 \le \theta \le \pi$.

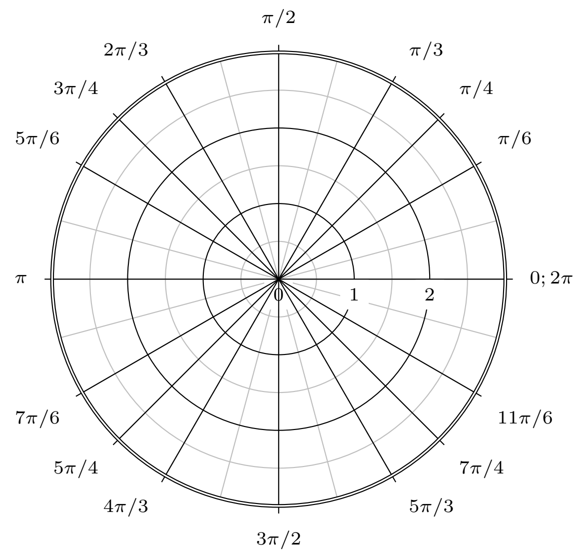
(c) Convert the following points to polar coordinates:

$$\left(2, 2\sqrt{3}\right), \quad \left(-2, -2\sqrt{3}\right), \quad (-1, 1), \quad (1, -1), \quad (3, 7).$$

(d) Convert the following points to rectangular/Cartesian coordinates:

$$(3, \pi/4), \quad (5, -\pi/6), \quad (-5, \pi/6), \quad (2, 0), \quad (2, 3).$$

11. (a) Sketch the curve $r(\theta) = 2\sin 2\theta$. **Hint:** It should be a rose with some number of petals.



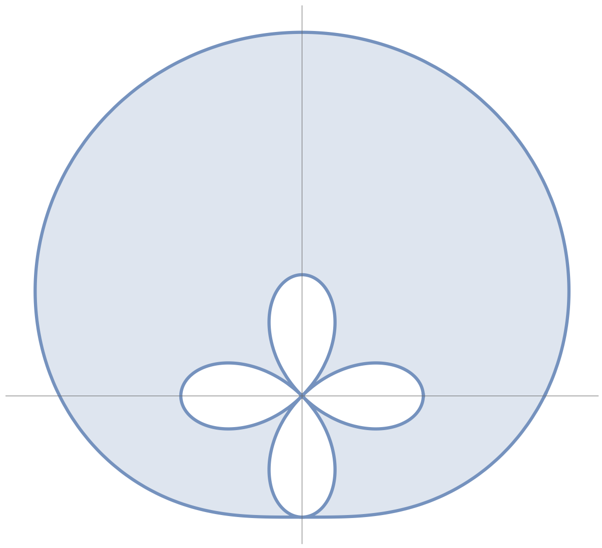
(b) Find the equation of the line tangent to $r(\theta)$ at the point $(\sqrt{3}, \pi/3)$.

(c) Find the points where the tangent line to the curve $r(\theta)$ are horizontal and/or vertical.

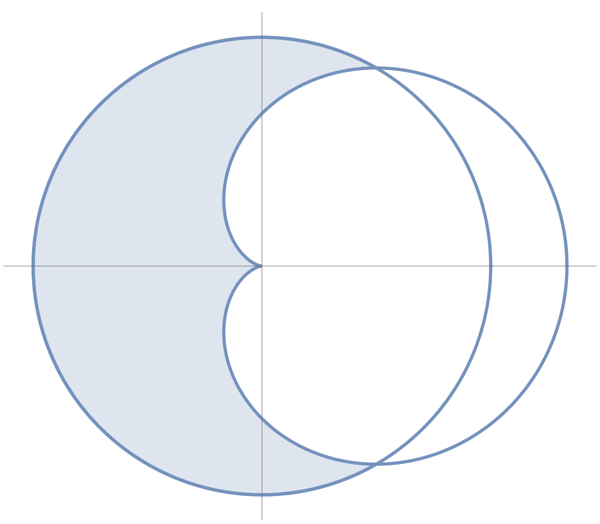
(d) Find the area bounded by the any one petal of the given curve.

(e) Write the integral corresponding to the (arc) length of the portion of the graph of $r(\theta)$ consisting of any *two* petals.

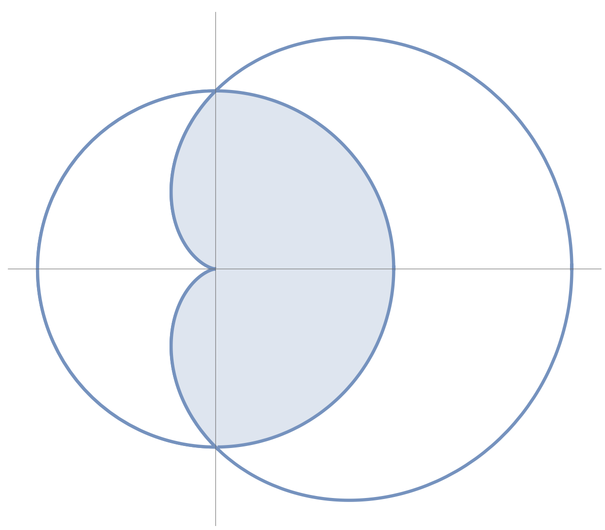
12. Find the area of the following shaded region, where the outer curve is given by $r = 2 + \sin\theta$ and where the inner curve is given by $r = \cos(2\theta)$.



13. For each of the following, determine (i) the arc lengths of each of the two curves; and (ii) the area of the corresponding shaded region. **Hint:** To find the areas, you'll need to know the intersections of the two curves!



(a) Circle: $r = 3$; Cardioid: $r = 2(1 + \cos t)$



(b) Circle: $r = 1$; Cardioid: $r = 1 + \cos t$