Name: ______________________________

MAC 2312 — Homework 1

Directions: Complete the following problems (front and back) for a homework grade. Answers given without showing work will not be eligible to receive partial credit. Problems *must* be neatly written up and presented in a professional manner in order to receive credit.
Date Due: *Either* September 15, 2016 *or* September 20, 2016.

1. (a) Navigate to our course homepage at

   `http://www.math.fsu.edu/~cstover/teaching/fa16_2312/`

   (b) Read and familiarize yourself with the three resources listed under *Supplementary Resources* on the General Info tab.

   (c) Follow the instructions for using Slack messenger.

   Note: This may require that I approve your email address, so to avoid some last minute glitch where I don't get to your approval on-time, please don't wait to do this!

   (d) Navigate to the channel `#three_things_about_me` in the left column under Channels (its browser url should be something like `https://fall2016-calc2.slack.com/messages/three_things_about_me/`) and post three random things about yourself.

   Note: This will be visible to everyone who signs into our class's chat room, so you definitely want to keep this PG-13, safe for work, and non-incriminatory. ☺

2. Compute each of the following integrals, noting that each can be found using "old knowledge" (i.e., techniques you knew before learning IBP).

   (a) $\int \tan(x)\,dx$

   (b) $\int \sec(x)\,dx$ **Hint**: $\dfrac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)} = 1.$

3. (a) Use the product rule for derivatives to derive the identity for integration by parts:

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx.$$

(b) Evaluate each of the following using integration by parts.

i. $\displaystyle\int (2x+2)e^{-x}\,dx$

ii. $\displaystyle\int_{\sqrt{\pi}}^{\pi^2} (x^2+2x+1)e^{-x}\,dx$

iii. $\displaystyle\int x^3\sqrt{1+x^2}\,dx$ **Hint**: If you try to integrate $\sqrt{1+x^2}$, you'll have a bad day.

iv. $\displaystyle\int \arcsin(3x)\,dx$

v. $\displaystyle\int 2x\arctan(x)\,dx$ **Hint**: $\displaystyle\frac{a^2}{1+a^2} = \frac{(1+a^2)-1}{1+a^2}$

vi. $\displaystyle\int (\arcsin(x))^2\,dx$

vii. $\displaystyle\int x\tan^2(x)\,dx$ **Hint**: $\tan^2(x) = \sec^2(x) - 1$.

(c) Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves $y = \cos(\pi x/2)$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis.

(d) i. Use integration by parts to prove that $\displaystyle\int f(x)\,dx = xf(x) - \int xf'(x)\,dx$.

ii. Using the above formula along with the definition $\displaystyle\mathrm{LI}(x) \stackrel{\text{def}}{=} \int \frac{1}{\ln(x)}\,dx$, calculate

$$\int \ln(\ln(x))\,dx.$$

4. (a) Compute each of the following trig integrals.

   i. $\int \sec^2(\theta) \tan^3(\tan(\theta))\, d\theta$ **Hint**: Let $u = \tan(\theta)$; then, see Example 7 in §7.2.

   ii. $\int \sin^5(\phi) \cos^3(\phi)\, d\phi$

   iii. $\int \sin(5x) \cos(8x)\, dx$

   iv. $\int \sin^2(x) \cos^8(x)\, dx$

(b) Evaluate

$$\int \sin(x) \cos(x)\, dx$$

by four methods: (i) the substitution $u = \cos(x)$; (ii) the substitution $u = \sin(x)$; (iii) the identity $\sin(2x) = 2\sin(x)\cos(x)$; and (iv) integration by parts.

(c) Find the average value of the function $f(x) = \sin^2(x)\cos^2(x)$ on the interval $[-\pi, 32\arctan(19)]$.

(d) Assuming m and n are positive integers (i.e., $m, n \in \mathbb{Z}^{>0}$), prove each of the following identities:

   i. $\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\, dx = 0$

   ii. $\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\, dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$

   iii. $\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\, dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$

5. (a) Evaluate each of the following integrals.

   i. $\int \frac{x^3}{\sqrt{x^2+4}}\,dx$

   ii. $\int \sqrt{4-x^2}\,dx$

   iii. $\int \frac{dt}{t^5\sqrt{9t^2-16}}$

   iv. $\int \frac{y^2}{(3+4y-4y^2)\sqrt{3+4y-4y^2}}\,dy$

   v. $\int \frac{k^2}{(k^2+a^2)^{3/2}}\,dk$

(b) Find the area of the region bounded by the hyperbola $16x^2 - 9y^2 = 144$ and the line $x = 3\sqrt{2}$.

(c) Compute each of the following integrals using both of the methods stated.

   i. $\int \frac{x^3}{(9-x^2)^{5/2}}\,dx$ using
   (1) the substitution $x = 3\sin(\theta)$, and (2) the substitution $u^2 = 9 - x^2$.

   ii. $\int \frac{1}{x^4(9+x^2)^{1/2}}\,dx$ using
   (1) the substitution $x = 3\tan(\theta)$, and (2) the substitution $u^2 = \frac{9+x^2}{x^2}$.

   iii. $\int \frac{1}{x\sqrt{x^2-1}}\,dx$ using
   (1) the substitution $x = \sec(\theta)$, and (2) the substitution $u^2 = x^2 - 1$.

6. (a) Write out the form of the partial fraction decomposition of the following functions but *do not* determine the numerical values of the coefficients (see Example 7 in §7.4).

i. $\dfrac{t^6+1}{t^6+t^3}$

ii. $\dfrac{x^5+x^3+1}{(x^2-x)(x^4+2x^2+1)}$

(b) Evaluate each of the following integrals.

i. $\displaystyle\int \frac{10}{(y-1)(y^2+9)}\,dy$

ii. $\displaystyle\int \frac{x^3+2x^2+3x-2}{(x^2+2x+2)^2}\,dx$

iii. $\displaystyle\int \frac{8+x}{3x^3+13x^2+18x+8}\,dx$ **Hint**: $x=-1$ is a root of the denominator.

iv. $\displaystyle\int \frac{2+v^4}{v^3+9v}\,dv$

v. $\displaystyle\int \frac{e^{2z}}{e^{2z}+3e^z+2}\,dz$

(c) i. If $t=\tan(x/2)$, sketch a right triangle to prove that

$$\cos\left(\frac{x}{2}\right)=\frac{1}{\sqrt{1+t^2}} \qquad \text{and} \qquad \sin\left(\frac{x}{2}\right)=\frac{t}{\sqrt{1+t^2}}.$$

ii. Show that $\cos(x)=\dfrac{1-t^2}{1+t^2}$, $\sin(x)=\dfrac{2t}{\sqrt{1+t^2}}$, and $dx=\dfrac{2}{1+t^2}\,dt$.

iii. Use the above substitutions to evaluate

$$\int_{\pi/3}^{\pi/2} \frac{1}{1+\sin(x)-\cos(x)}\,dx$$

by transforming the integrand into a rational function of t.

7. Solve each of the following integrals using any of the methods you know.

(a) $\int (4y+2)^{\sqrt{\pi}}\, dy$

(b) $\int t\sin(t)\cos(t)\, dt$

(c) $\int_0^4 \frac{x-1}{x^2-4x-5}\, dx$

(d) $\int e^{x+e^x}\, dx$

(e) $\int \frac{\ln(x)}{x\,(1+(\ln(x))^2)^{3/2}}\, dx$

(f) $\int \left(\ln(x) + \frac{3x^2-2}{x^3-2x-8}\right) dx$

(g) $\int \sin\left(\sqrt{t\sqrt{2}}\right)\, dt$

(h) $\int \frac{\sec(\theta)\tan(\theta)}{\sec^2(\theta)-\sec(\theta)}\, d\theta$

(i) $\int \theta\tan^2(\theta)\, d\theta$

(j) $\int_0^1 x\sqrt{2-\sqrt{1-x^2}}\, dx$

(k) $\int \frac{dx}{x\sqrt{4x^2-1}}$

(l) $\int \cos(x)\cos^3(\sin(x))\, dx$

(m) $\int_{\pi/4}^{\pi/3} \frac{\ln(\tan(x))}{\sin(x)\cos(x)}\, dx$

(n) $\int \frac{dx}{\sqrt{x}(2+\sqrt{x})^4}$

(o) $\int \frac{1}{(x-2)(x^2+4)}\, dx$

(p) $\int \frac{dy}{\sqrt{4y^2-4y-3}}$