Various Laws/Theorems/Rules About Sequences

Limit Laws for Sequences

If $\{a_n\}$ and $\{b_n\}$ are *convergent* sequences and if c is a constant, then:

(i) $\lim_{n\to\infty} (a_n \pm b_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n$.

(ii) $\lim_{n\to\infty} ca_n = c \lim_{n\to\infty} a_n$

(iii) $\lim_{n\to\infty} (a_n b_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$

(iv) $\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$

(v) $\lim_{n\to\infty} a_n^p = \left(\lim_{n\to\infty} a_n\right)^p$ if $p > 0$ and $a_n > 0$.

Miscellaneous Rules and Theorems

Squeeze Theorem:

If $a_n \leq b_n \leq c_n$ and if $\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} c_n$, then $\lim_{n\to\infty} b_n = L$.

Continuous Image of Convergent Sequence is Convergent:

If $\lim_{n\to\infty} a_n = L$ and the function f is continuous at L, then $\lim_{n\to\infty} f(a_n) = f(L)$.

Convergence of $\{r^n\}$:

The sequence $\{r^n\}$ is convergent if $-1 < r \leq 1$ and divergent for all other values of r. Moreover,

$$\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{if} \quad -1 < r < 1 \\ 1 & \text{if} \quad r = 1 \end{cases}$$