Trig Integrals

Integrals of the form $\int \sin^n(x)\cos^m(x)\,dx$ **for** $n, m > 0$

Case 1. Either n or m is odd.

- Factor a term from the **odd** power.
- Use trig identities to rewrite everything in terms of the **even-power** term.
- Use u-substitution with u equal to the even-power term.

Case 2. Both n and m are even.

- Use ≥ 1 of the following trig identities to rewrite the integrand into something simpler:

$$\begin{aligned}\cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta)\\ &= 2\cos^2(\theta) - 1\\ &= 1 - 2\sin^2(\theta)\\ \sin(2\theta) &= 2\sin(\theta)\cos(\theta)\end{aligned}$$

$$\begin{aligned}\cos^2(\theta) &= \frac{1+\cos(2\theta)}{2}\\ \sin^2(\theta) &= \frac{1-\cos(2\theta)}{2}\end{aligned}$$

Example: Compute $\int \cos^2(x)\sin^2(x)\,dx$

Solution: First, we use the identities $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$ and $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$ to rewrite the integrand

$$\int \cos^2(x)\sin^2(x)\,dx = \int \left(\frac{1+\cos(2x)}{2}\right)\left(\frac{1-\cos(2x)}{2}\right)dx.$$

Now, do some algebra:

$$\int \left(\frac{1+\cos(2x)}{2}\right)\left(\frac{1-\cos(2x)}{2}\right)dx = \frac{1}{4}\int 1 - \cos^2(2x)\,dx.$$

Next, rewrite the right-hand side using the trig identity for $\cos^2$:

$$\cos^2(2x) = \frac{1-\cos(4x)}{2} \implies \frac{1}{4}\int 1-\cos^2(2x)\,dx = \frac{1}{4}\int 1 - \frac{1-\cos(4x)}{2}\,dx.$$

From here, you can solve the integral using elementary methods. $\square$

Integrals of the form $\int \tan^n(x)\sec^m(x)\,dx$ **for** $n, m > 0$

Case 1. n is odd.

- Factor out $\sec(x)\tan(x)$
- Rewrite **tan** as sec using the identity $\tan^2(x) = \sec^2(x) - 1$.
- Use the substitution $u = \sec(x)$ & $du = \sec(x)\tan(x)\,dx$.

Case 2. m is even.

- Factor out $\sec^2(x)$.
- Rewrite **sec** as **tan** using the identity $\tan^2(x) = \sec^2(x) - 1$.
- Use the substitution $u = \tan(x)$ & $du = \sec^2(x)\,dx$.

NOTE 1: We'll do an example of this in class in the next section.

NOTE 2: We'll ignore any other types of integrals in this section (though there may be a couple on your homework).