September 20, 2016

Exam 1

MAC 2312—Calculus II, Fall 2016

(Neatly!) Print Name: KEY

Read all of what follows carefully before starting!

1. This test has **3 problems** (7 parts total), is worth **65 points**, and has **1 bonus problem**. *Please be sure you have all the questions before beginning!*
2. The exam is closed-note and closed-book. You may **not** consult with other students.
3. No calculators may be used on this exam!
4. Show all work clearly in order to receive full credit. Points will be deducted for incorrect work, and unless otherwise stated, no credit will be given for a correct answer without supporting calculations. **No work = no credit!** (unless otherwise stated)
5. You may use appropriate results from class and/or from the textbook as long as you fully and correctly state the result and where it came from.
6. You **do not** need to simplify results, unless otherwise stated.
7. There is scratch paper at the end of the exam; you may also use the backs of pages or get more scratch paper from me.
8. You may need the following trig identities:

$$\begin{aligned}\cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta)\\ &= 2\cos^2(\theta) - 1\\ &= 1 - 2\sin^2(\theta)\\ \sin(2\theta) &= 2\sin(\theta)\cos(\theta)\end{aligned}$$

$$\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$$

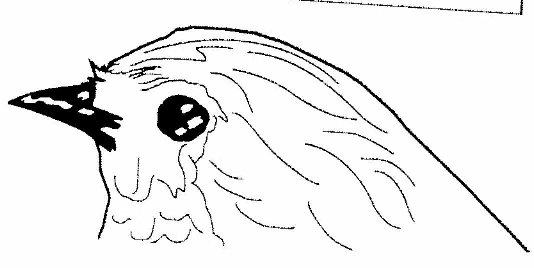
$$\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$$

there is no math on this page

Whatever happens, just remember:

Your worth

is not determined by your performance on this exam!



1. (a) *(5 pts)* Fill in the integration by parts formula:

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

(b) Derive the formula for integration by parts from the product rule for derivatives. *(5 pts)*

SOLUTION: Note that $\frac{d}{dx}\left(f(x)g(x)\right) = f(x)g'(x) + f'(x)g(x)$.

Thus, $f(x)g(x) = \int f(x)g'(x)\,dx + \int f'(x)g(x)\,dx$, and so

$$\int f(x)g'(x) = f(x)g(x) - \int f'(x)g(x)\,dx. \quad ■$$

Part (c) is on the next page

(c) Evaluate $\int e^x \cos(x)\, dx$. *(15 pts)*

SOLUTION:

$u = e^x \qquad v = \sin x$

$u' = e^x \qquad v' = \cos x$

$\Rightarrow \int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx.$

$u = e^x \qquad v = -\cos x$

$u' = e^x \qquad v' = \sin x$

$\Rightarrow \int e^x \cos x \, dx = e^x \sin x - \left[-e^x \cos x - \int -e^x \cos x \, dx \right]$

$\Rightarrow \int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$

$\Rightarrow 2 \int e^x \cos x \, dx = e^x \sin x + e^x \cos x$

$\Rightarrow \boxed{\int e^x \cos x \, dx = \frac{1}{2}\left(e^x \sin x + e^x \cos x\right) + C.}$

2. (a) (*15 pts*) Compute

$$-\int \frac{2-u}{\sqrt{9+u^2}}\,du.$$

You may use the fact that $\int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C.$

SOLUTION: let $u = 3\tan\theta$, $-\pi/2 < \theta < \pi/2$. So $du = 3\sec^2\theta\,d\theta$

$$\Rightarrow -\int \frac{2-u}{\sqrt{9+u^2}}\,du = -\int \frac{2-3\tan\theta}{\sqrt{9+9\tan^2\theta}}\,(3\sec^2\theta\,d\theta)$$

$$= -\int \frac{2-3\tan\theta}{(3\sec\theta)} \cdot 3\sec^2\theta\,d\theta$$

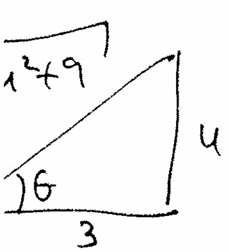
$$= -\int (2-3\tan\theta)\sec\theta\,d\theta = -2\int \sec\theta\,d\theta + 3\int \sec\theta\tan\theta\,d\theta$$

$$\boxed{= -2\ln|\sec\theta + \tan\theta| + 3\sec\theta + C.}$$

Now; substitute:

$$\boxed{= -2\ln\left|\frac{\sqrt{u^2+9}}{3} + \frac{u}{3}\right| + 3\left(\frac{\sqrt{u^2+9}}{3}\right) + C}$$

Part (b) is on the next page



$\Rightarrow$ $\sec\theta = \frac{H}{A} = \frac{\sqrt{u^2+9}}{3}$

$\tan\theta = \frac{u}{3}$

(b) (*5 pts*) Use part (a) to find

$$\int \frac{x}{\sqrt{9+(2-x)^2}}\,dx.$$

Hint: Don't compute both (a) and (b) from scratch; that takes way too much time.

SOLUTION: let $u = 2-x \Rightarrow du = -dx$ (so $-du = dx$)

$\hookrightarrow x = 2-u.$

So: $$\int \frac{x}{\sqrt{9+(2-x)^2}}\,dx = \int \frac{2-u}{\sqrt{9+u^2}}(-du)$$

$$= -\int \frac{2-u}{\sqrt{9+u^2}}\,du.$$

By (a), this $= -2\ln|\sec\theta + \tan\theta| + 3\sec\theta + C$

$$= -2\ln\left|\frac{\sqrt{u^2+9}}{3} + \frac{u}{3}\right| + 3\left(\frac{\sqrt{u^2+9}}{3}\right) + C,$$

so substituting $u = 2-x$:

$$\int \frac{x}{\sqrt{9+(2-x)^2}}\,dx = -2\ln\left|\frac{\sqrt{(2-x)^2+9}}{3} + \frac{2-x}{3}\right| + 3\left(\frac{\sqrt{(2-x)^2+9}}{3}\right) + C$$

3. (a) *(5 pts)* Write the partial fraction decomposition of

$$\frac{x^3+x^2+1}{x(x-1)(x^2+x+1)(x^2+1)^3},$$

but **do not** determine the numerical values of the coefficients *or* integrate.

SOLUTION:

$$= \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+x+1} + \frac{Ex+F}{x^2+1} + \frac{Gx+H}{(x^2+1)^2} + \frac{Ix+J}{(x^2+1)^3}.$$

Part (b) is on the next page

Decomp: [redacted] 1
Coeff: 3
Rewrite int: [redacted] 1
Integrate: [redacted] 9
numbers: 1

(b) (*15 pts*) Compute the following definite integral and simplify your answer fully:

$$\int_0^{\sqrt{3}} \frac{18}{(y+3)(y^2+9)}\, dy.$$

SOLUTION:

$$\frac{18}{(y+3)(y^2+9)} = \frac{A}{y+3} + \frac{By+C}{y^2+9} \Rightarrow 18 = A(y^2+9)+(By+C)(y+3$$

$$\Rightarrow 18 = Ay^2+9A+By^2+3By+Cy+$$

$$\Rightarrow 18 = y^2(A+B)+y(3B+C)+9A+$$

So: $A+B=0 \Rightarrow B=-A$

$3B+C=0 \Rightarrow -3A+C=0 \Rightarrow C=3A$

$9A+3C=18 \Rightarrow 9A+3(3A)=18 \Rightarrow 18A=18 \Rightarrow \boxed{A=1}$

So $\boxed{B=-A=-1}$ $\quad C=3A \Rightarrow \boxed{C=3}$

Thus:
$$\int \frac{18}{(y+3)(y^2+9)}\,dy = \int \frac{1}{y+3}\,dy + \int \frac{-y+3}{y^2+9}\,dy$$

$u=y^2+9$
$du=2y\,dy$
$\frac{du}{2}=y\,dy$

$$= \int \frac{1}{y+3}\,dy - \int \frac{y}{y^2+9}\,dy + 3\int \frac{1}{y^2+9}\,dy$$

$$= \ln|y+3| - \frac{1}{2}\ln|y^2+9| + 3\left(\frac{1}{3}\tan^{-1}\left(\frac{y}{3}\right)\right).$$

Hence:

Can cancel

$$\int_0^{\sqrt{3}} \frac{18}{(y+3)(y^2+9)}\,dy = \ln(\sqrt{3}+3) - \frac{1}{2}\ln(3+9) + \underbrace{\boxed{\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)}}_{=\pi/6} - (\ln(3) + \frac{1}{2}\ln(9)) \underbrace{\boxed{-\tan^{-1}(0)}}_{=0}$$

$$\boxed{= \ln(\sqrt{3}+3) - \frac{1}{2}\ln(12) + \frac{\pi}{6} - \ln 3 + \frac{1}{2}\ln 9}$$

Bonus: *(3 pts ea.)* Compute each of the following, and simplify your answer fully:

(a) $\displaystyle\int_0^{\pi} \sin^4(2x)\,dx.$

Solution:

(b) $\displaystyle\int \tan^7(x)\sec^6(x)\,dx$

Solution:

Scratch Paper

Scratch Paper

Scratch Paper