MATH 141 TEST IA BE NEAT; SHOW ALL WORK
ONE SIDE OF EACH PAGE ONLY

1) $A^T = ?$

2) $TR\,A = ?$

WHERE $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 7 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

3) $\vec{x} = (1, 1, 0)$ $\vec{y} = (3, 4, 5)$ what is the dot or scalar product of $\vec{x}$ & $\vec{y}$ (i.e $\vec{x} \cdot \vec{y}$?)

4) Are $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $\begin{bmatrix} 12 & 24 \\ 36 & 46 \end{bmatrix}$ scalar multiples of each other? why?

5) $\det \begin{bmatrix} 0 & 0 & 3 \\ 0 & 2 & 10 \\ 1 & 7 & 9 \end{bmatrix} = ?$

6) $\det \begin{bmatrix} 0 & 7 & 5 & 0 \\ 1 & 3 & 7 & 1 \\ 1 & 3 & 7 & 1 \\ 0 & 7 & 9 & 0 \end{bmatrix} = ?$

7) $B = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & -1 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 3 & 4 \\ 4 & 3 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$ FIND BC and CB

8) $\det \begin{bmatrix} 1 & 3 & 0 & 6 \\ 0 & 1 & 3 & 6 \\ -1 & -2 & 6 & 0 \\ 0 & 2 & 1 & 3 \end{bmatrix} = ?$

9. SOLVE USING CRAMERS RULE:

$$x + y = 3$$
$$y + z = 5$$
$$x + z = 4$$

10) IF D REPRESENTS A COMMUNICATION NETWAY (d_{ij} = no. of ways i can send to j) LET $B = D^2 + D$ What does b_{ij} represent?

MATH 141 FINAL SHOW <u>ALL</u> WORK; BE <u>NEAT</u>
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1. FOR $A = \begin{bmatrix} 1 & 2 & 3 \\ 8 & 9 & 4 \\ 7 & 6 & 5 \end{bmatrix}$ Find $Tr(A)$ and A^T

2. QUICK Determinates

$\det \begin{bmatrix} 2 & 10 \\ -7 & 1 \end{bmatrix} = ?$ $\det \begin{bmatrix} 1 & 7 & 19 \\ 0 & 2 & 5/3 \\ 0 & 0 & -3 \end{bmatrix} = ?$ $\det \begin{bmatrix} 4 & 1 & 4 & 1 \\ 0 & 1 & 0 & 1 \\ 7 & 1 & 7 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} = ?$

3. QUICK RANKS

$\text{Rank} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = ?$ $\text{Rank} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = ?$ $\text{Rank} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = ?$

4. For the system $AX = B$ where A is $n \times n$ matrix:

a) If $\text{rank}[A] < \text{rank}[A \vdots B] = n$ the system is ___ and has how many solutions?

b) If $\text{rank}[A] = \text{rank}[A \vdots B] = n$ the system is ___ and has how many solutions?

c) If $\text{rank}[A] = \text{rank}[A \vdots B] < n$ the system is ___ and has how many solutions?

5. $2\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 6 & 7 \\ 0 & 5 \end{bmatrix} = ?$

6. $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 4 \\ 1 & 0 & 0 & 5 \end{bmatrix}$ $\det A = ?$ $\text{rank} A = ?$

7. $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Find A^{-1} and use it to solve $AX = B$ where $B = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

8. Solve
$$\begin{aligned} x_1 + x_3 &= 4 \\ x_2 + x_3 &= 5 \\ 2x_2 + 4x_4 &= 20 \\ x_1 + 5x_4 &= 21 \end{aligned}$$

9. Find a complete solution to
$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ x_1 + 2x_2 + x_3 \quad &= 6 \\ x_2 + x_3 + 2x_4 &= 6 \end{aligned}$$

10. A factory makes regular, deluxe and super-deluxe tricycles. A regular tricycle uses 1 gismo, 1 beta and 3 whats. A deluxe tricycle uses 2 whats, 1 beta and 3 gismo's. A super-deluxe uses no betas, 2 gismo's and 5 whats. The factory has 26 whats, 10 beta's and 18 gismo's. How many tricycles of each type must be made to completely use the stock of gismo's, beta's and whats?