

**Report on the paper**  
**TREE BASIS IN BANACH SPACES**

by  
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The author defines the notion of a *tree basis* of a Banach space  $X$ . Precisely, a sequence  $(x_t)_{t \in \mathcal{D}}$  of non-zero vectors in  $X$  indexed by the dyadic tree  $\mathcal{D}$  is said to be a tree basis provided that it spans the space  $X$  and there exists a constant  $C \geq 1$  such that for every downwards closed subtree  $\mathcal{S}$  of  $\mathcal{D}$  the natural projection  $P_{\mathcal{S}}$  onto  $\overline{\text{span}}\{x_t : t \in \mathcal{S}\}$  has norm at most  $C$ . Among the main results of the author is the following characterization: *a separable Banach space  $X$  has a tree basis if and only if  $X$  is isomorphic to  $Y \oplus U$  where  $Y$  has a Schauder basis and  $U$  has an unconditional basis.*

The paper is nice but I am reluctant to recommend it for publication as it stands for the following reasons.

**(C1)** The paper has many typos/misprints giving the impression that it was rather hastily written. The author should carefully revise the paper and correct all typos/misprints. To this end, he can use an appropriate package of LaTeX.

**(C2)** The author mentions in the introduction (as well as, in the abstract) that the results of the paper were motivated by “well known adaptive approximation algorithms” (notice that the word algorithm in the abstract is misspelled). The author should give appropriate references supporting this claim. Otherwise, these remarks should be deleted since they are clearly not related to the content of the paper.

**(C3)** Recent related work is not mentioned at all. Indeed, there are several applications of basic sequences indexed by the dyadic tree having similar (but weaker) properties with those considered by the author (see, e.g., [1], [2] and [3] below). The author should mention these results and comment on the relation between his work and related material.

Moreover, references [11], [12] and [15] are not cited in the paper (notice that there are several typos in references [11] and [12]). The author should find an appropriate place in main text to mention them.

If the author is willing to revise the paper taking into account the above remarks, then I will be happy to referee it again.

REFERENCES

- [1] S. A. Argyros, P. Dodos and V. Kanellopoulos, *Unconditional families in Banach spaces*, Math. Annalen **341** (2008), 15-38.
- [2] P. Dodos, *Operators whose dual has non-separable range*, J. Funct. Anal. **260** (2011), 1285-1303.
- [3] S. Todorčević, *Introduction to Ramsey Spaces*, Annals Math. Studies, No. **174**, Princeton Univ. Press, 2010.