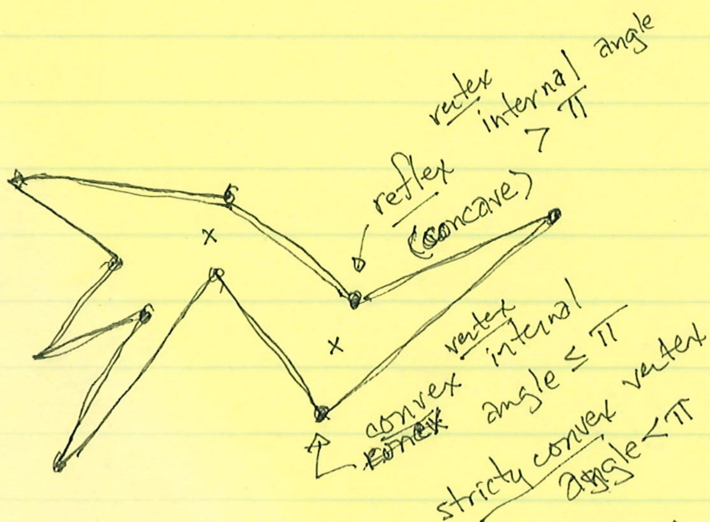
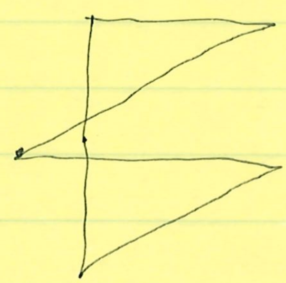
Joseph O'Rourke. "Computation Geometry in C"

KLEE: [1973] How many stationary guards are needed to guard a polygonal art gallery

<u>Thm</u> Chvátal 1975, (Our proof Fisk 1978) $G(n) = \lfloor \frac{n}{3} \rfloor$

<u>definition</u> polygonal P is a region of the plane <u>bounded</u> by a finite collection of line segments homeomorphic to S^1. (i.e. simple closed curve). The lines are ∂P.





x <u>can see</u> y in P or y is visible from x in P

$\Longleftrightarrow$ $\overline{xy} \subset P$ [grazing ∂P is OK]

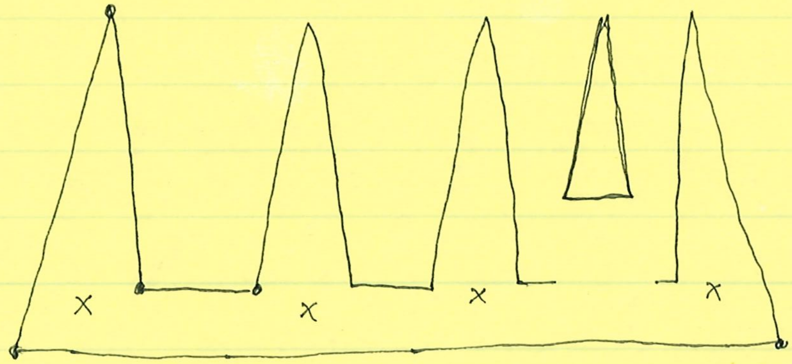
$\{tx + (1-t)y : 0 \leq t \leq 1\}$

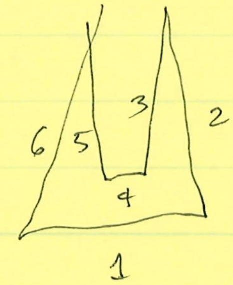
(Guards can see through other guards, guards are points

A set of guards G cover P if every point y is visible from some $\cancel{x} \in G$.

$$G(n) = \max_{P \text{ has } n \text{ sides}} \min \{ |G| : G \text{ covers } P \}.$$

Necessity of $\lfloor n/3 \rfloor$ Chvátal's comb.





$G(n) \geq \lfloor \frac{n}{3} \rfloor$

Triangulation

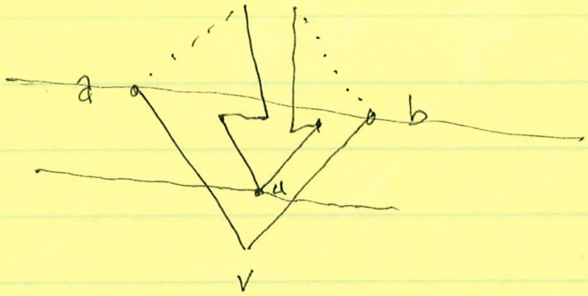
A diagonal of P is a line segment xy between vertices x & y so that $xy \cap \partial P = \{x, y\}$ & $xy \subset P$.

Two diagonals are non-crossing if their intersection is a subset of end points

A triangulation of P is a (maximal) collection of non-crossing diagonals so that P is partitioned into triagles.

Prop. Every polygon can be triangulated.

proof: Strong Induction on number of sides. $n=3$ done.
Inductive step.
rightmost lowest vertex must be strictly convex.



if ab is a diagonal we are done.
otherwise find u, the vertex in Δabv closest to v in direction $\perp$ ab. Now uv is a diagonal.
This divides P into two smaller polygons. ($\checkmark$.)

Consider T a triangulation as a planar graph.
It has a "dual graph." vertex for each Δ, and these vertices are adjacent $\Leftrightarrow$ Δ's share a diagonal.

Prop The dual graph of T is a tree with Δ = max degree ≤ 3.

pf: max degree ≤ 3 is obvious. we need to show connected & acyclic. the same induction will work.

~~Fact~~: Prop T "as a planar graph" can be three colored.

the vertices of P can be colored red, white or blue s.t. vertices adjacent by sides or diagonals have different colors.

pf: A non-trivial tree has a vertex of degree ≤ 1, this is an "ear" of polygon. Induction proceeds by removing of ear.

"pigeon-hole principle"
Some color is used at most $\lfloor n/3 \rfloor$ vertices and this set of guards covers P.

done.

$30°$

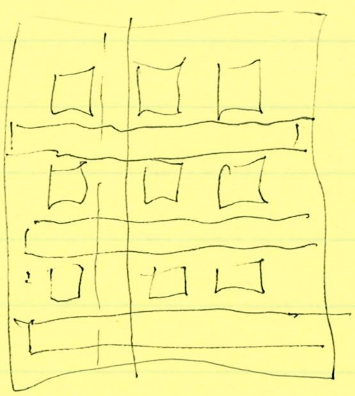
Remarks.

1. This is 2-D. One can't "triangulate" 3-D polyhedron. And there are polyhedrons, where the set of vertices do not cover the polyhedron.
O'Rourke 1987 253-254

* Art Gallery Thm's and Algorithms Oxford Univ Press.
QA 447 .076

* Computational Geometry Annu. Rev. Comp Sci 3, 389-411 1988

$n^{3/2}$



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