MAC 1140 — Precalculus Mathematics

Section 10, Spring 1994.

Instructor: Bellenot.

The good doctor's Office: 002-B Love, Office Hours: MWF 9:20–10:00 or by appointment.

Eligibility: A grade of C- or better in MAC 1102 or placement by AMP (groups 1, 2, 3A, 3B, 4, or 5). You must not have passed an equivalent or "higher" math class.

Text: Swokowski and Cole, *Algebra and Trigonometry with Analytic Geometry,* 8^{nd} Edition.

Coverage: Chapters 3, 4, 5 and parts of 9 and 10.

Final: At 10-12 Monday Apr 25, 1994.

Tests: (3) Tentatively at Jan 26, Feb 23(Mar 2?) and Apr 13. No Makeup tests.

Quizes: Every Wednesday (except test days). No Makeup quizes.

Grades: 90% A, 80% B, 70% C, 60% D.

Relative Weights F = 2T and T = P (F is 1/3, each T is 1/6 and P is 1/6).

Homework and Attendance are required. Always bring your FSU id to class.

Calculators: Only certain types of calculators are permitted. No printing, programmable, alpha-numeric or graphic-capability calculators. Only some tests will permit calculators.

Cheating: Don't, it will ruin your grade and it gets reported to the University Judical Officer.

Help Center: 110 MCH; open Sunday-Friday.

MAC 1140 Pre-Calculus Math **Test 2** 23 Feb 1993

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

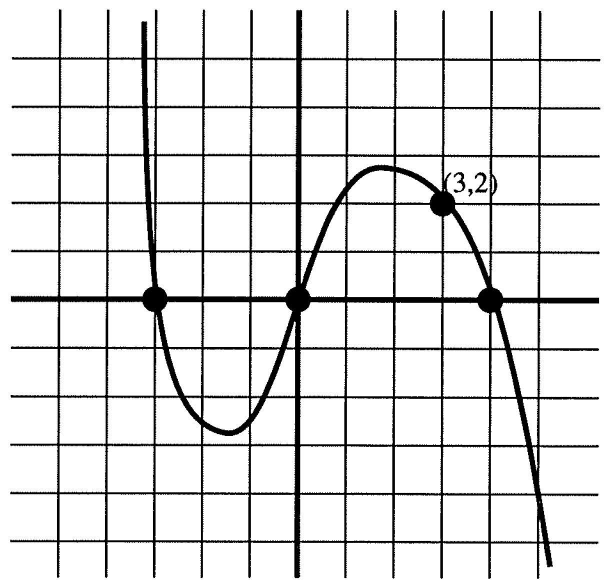
1. Express the statement below as a formula that involves the given variables and a constant of proportionally k, and then determine the value of k from the given conditions. y is directly proportional to the square of x and inversely proportional to the square root of z. If $x = 5$ and $z = 16$, then $y = 10$.

2. Find the zeros of $f(x)$ and state the multiplicity of each zero.
   A. $f(x) = 4x^5 + 12x^4 + 9x^3$.   B. $f(x) = (x^2 + 1)(x - 2)^3(x^2 - 9)(x + 3)$.

3. A polynomial $f(x)$ with real coefficients and degree 5 has zeros at $0, 3i, 4 + i$. Express $f(x)$ as a product of linear and irreducible (over **R**) quadratic polynomials with real coefficients.

4. Find $g(x)$ the inverse function to $f(x) = (3x + 2)/(2x - 5)$. [Your final answer should be $g(x)$, a function of x.]

5. Find the third-degree polynomial function whose graph is shown in the co-ordinate plane below.



6. Find $f \circ g, g \circ f$, and the domains of $f, g, f \circ g$ and $g \circ f$ when $f(x) = \sqrt{3 - x}$ and $g(x) = \sqrt{x + 2}$.

7. Find the quotient and remainder when $f(x)$ is divided by $p(x)$.
   A. $f(x) = -4x^2 + 7x + 3$, $p(x) = x^2$ B. $f(x) = 3x^5 - 4x^3 + x + 5$, $p(x) = x^3 - 2x + 7$

8. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x) = (x + 2)(x - 1)^2(x - 4)$.

9. For the functions below **JUST** find the equations of any hortizontal, vertical or oblique asymptotes, do **NOT** graph the functions.

$$A. \quad f(x) = \frac{x^2 - x - 6}{x + 1} \qquad B. \quad g(x) = \frac{x + 3}{x^2 - 1} \qquad C. \quad h(x) = \frac{4x^2 - 4x + 1}{x^2 - x - 2}$$

10. Two rational functions $f(x)$ and $g(x)$ are found to have the same asymptotes and zeros. Both have a horizontal asymptote of $y = 0$, vertical asymptotes $x = -1$ and $x = 2$, and the functions are zero only at $x = 0$. On separate co-ordinate planes, draw the graphs of $f(x)$ and $g(x)$ given the additional information below.
    A. $f(x) > 0$ for $(-\infty, -1) \cup (2, \infty)$ and $f(x) < 0$ for $(-1, 0) \cup (0, 2)$.
    B. $g(x) > 0$ for $(-1, 0) \cup (2, \infty)$ and $g(x) < 0$ for $(-\infty, -1) \cup (0, 2)$.

MAC 1140 Pre-Calculus Math **Test 3** 15 Apr 1994

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. Express the logarithm below in terms of logarithms of x, y and/or z.

$$\log \frac{\sqrt{y}}{x^4 z^{1/3}}$$

2. Solve the equations.
   A. $7^{x+6} = 7^{3x-4}$ B. $2^{-100x} = (0.5)^{x-4}$

3. Find the zeros of $f(x) = -x^2e^{-x} + 2xe^{-x}$.

4. Solve for t using logarithms with base a.
   A. $2a^{t/3} = 5$ B. $A = Ba^{Ct} + D$

5. Use the binomial theorem to expand and simplify $(x^2 - 2y)^7$

6. Find the rational number represented by the repeating decimal.
   A. $147.147147\ldots$ B. $5.144444\ldots$

7. Solve the equations.
   A. $\log(\log x) = 2$ B. $\log(x^2) = (\log x)^2$

8. Solve using matrices.

$$\begin{aligned} x - 2y + 2z &= 0 \\ 3y + z &= 17 \\ 5x + 2y - z &= -7 \end{aligned}$$

9. A silversmith has two alloys, one containing 35% silver and the other 60% silver. How much of each should be melted and conbined to obtain 100 grams of an alloy containing 50% silver?

10. Prove (by induction) that the statement below is true for every positive integer n.

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

Solving Numerical Problems Using Calculators

The correct use of calculators can be summed up by the statement *Use the calculator only once, at the end of the problem.* The most common calculator error is saving and re-entering an intermediate result. The second most common error is using too many or too few digits in final answer.

Consider $A(t) = A_0 e^{-0.0245t}$ where you are given $A_0 = 35$ and $t = 2$ and the problem is to find $A(2)$.

The answer is of course $35 * e^{-0.0245*2}$. A calculator key sequence which computes this is 3 5 * e y^x (. 0 2 4 5 +/- * 2) =. The calculator displays an answer of 33.32634. (It actually has stored more digits, you can find the rest by subtracting 33.32634, which yields -4.60552e-07, so the calculator actually thinks the result is 33.326339539448.) What is THE correct answer? Is it 30?, 33?, 33.3? 33.33? 33.326? 33.3263? 33.32634? 33.326340? 33.3263395? 33.32633954? 33.3263395545? 33.326339539448? or some other number?

The correct answer depends on the given numbers, here 35, 2, e and -0.0245. If these numbers are infinity precise, (e certainly is infinity precise and 2 and 35 could be but -0.0245 is likely not), then none of the answers above are correct. Indeed, the simplest way to express this number would be $35e^{-0.45}$, otherwise it would take an infinite number of digits to get the right answer. Thus the correct answer is only near 30 (or 33 or 33.3 or ...)

What is the difference in the 30, 33, 33.3? If you say the answer is 33.3 what does that mean? Well, it means the answer is near 33.3 and it implies the answer is closer to 33.3 than 33.4 or 33.2. The answer is within 0.05 of 33.3. An answer of 33 means it is between 32.5 and 33.5, (an answer of 33.0 means it is between 32.95 and 33.05)

The correct number of digits to keep in the answer is the smallest number of digits in any of the given numbers. Here e is an infinitely precise number but 35, 2, and -0.0245 are not. The number -0.0245 has only three significant digits (it is -2.45×10^{-2}), 35 has two significant digits (it would be written 35.0 if it had three significant digits) and 2 has only one significant digit (it would be written 2.0 or 2.00 if it had more digits). The smallest number of digits is one, so the correct answer should have only one significant digit, namely 3×10^1