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Paul T. Bateman
Associate Editor

Department of Mathematics
University of Illinois
Urbana, Illinois 61801

We are happy to inform you that the solution material which you submitted to the Problem Section of the Monthly has been selected for publication. Enclosed is a copy as it will be sent to the printer. Please look it over and see if everything is to your satisfaction. United States law makes it necessary for the editors to request that you sign both copies of the Transfer of Copyright Agreement below. As soon as we receive them we will forward the material to the printer.

Yours sincerely,

Paul T. Bateman

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Steve,

We've arrived!

John

Enclosure
PTB/tb

6577 [1988, 665]. *Proposed by B. Bagchi, G. Misra, and N. S. N. Sastry, Indian Statistical Institute, Calcutta, India.*

(a) Let H be an infinite-dimensional inner-product space. Suppose that finitely many closed balls cover the surface S of the unit ball B in H. Prove that these balls also cover the center of B.

(b) Does the above assertion remain valid if H is any infinite-dimensional, normed linear space?

Combined solution by (independently) the Florida State University Mail Room Problem Group, O. P. Lossers (The Netherlands), and John Henry Steelman, Indiana University of Pennsylvania. The answer to (b), and hence to (a), is yes. If m balls $B_1, \ldots, B_m$ (or any m closed convex sets) cover the surface of the unit sphere of H, the same situation holds for any m-dimensional subspace of H. Inside this subspace, if B_1 fails to cover 0, there is a hyperplane P through 0 that does not intersect B_1. Thus the m-1 balls $B_2 \cap P_1, \ldots, B_m \cap P$ cover $S \cap P$, the unit sphere of an (m-1)-dimensional space. This step may be iterated; if $B_2, \ldots, B_{m-1}$ also fail to cover 0, then B_m will contain the surface of a one-dimensional unit sphere and hence 0 by convexity.

Most solvers stayed in infinite dimensional space and applied some variation of the Hahn-Banach theorem. A few (S. K. Chung (Hong Kong), Jesús Ferrer (Spain), Pei Yuan Wu (Taiwan)) based their solution on results about the weak topology. After his solution, Gerd Herzog (West Germany) remarked that Borsuk proved the following much deeper result (see, e.g., K. Deimling, Nonlinear Functional Analysis, Springer-Verlag, Berlin, 1985, p. 22). Let H be a normed linear space, $n \leq \dim H$, and $B_1, \ldots, B_n$ a cover of S such that $S \cap B_i$ is closed. Then for some x in S both x and $-x$ belong to the same B_j. In response to a query, Tenney Peck showed that in Hilbert space there is no positive r such that if finitely many closed balls cover the surface of the unit ball, then at least one of them contains the ball of radius r centered at the origin.

Solved also by Mahlon Day, K. P. Hart (The Netherlands), H. Hunziker and V. Mascioni (Switzerland), Miguel Lacruz, Reiner Martin (West Germany), Eero Posti (Finland), William H. Ruckle, and Thomas Starbird. Part (a) was solved by A. A. Jagers (The Netherlands) and the proposers.

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