is a semialgebra

B) If μ and ν are finitely additive, then so is $\mu \times \nu$, where $\mu \times \nu\,(E \times F) = \mu(E)\,\nu(F)$.
[Hint: First consider $E = \cup I_n$, $F = \cup J_m$, $E \times F = \cup_{m,n} I_n \times J_m$

C) If $\mathcal{I}$ and $\mathcal{J}$ are both the collections of all intervals of the form $[a,b)$ and $(-\infty,b)$ and if $\mu[a,b) = \nu[a,b) = b-a$, prove that $\mu \times \nu$ is countably subadditive [Hint: Use the fact that a closed bounded rectangle is compact.]

Remark: $\mu \times \nu$ is countably subadditive in the general case, but the proof is <u>much</u> harder.

V) Suppose that χ is a countable infinite set and $\mathcal{R}$ is a σ-algebra of sets of χ. If $\mathcal{R}$ contains all one-element sets, prove that $\mathcal{R} = \mathcal{P}(\chi)$. Find all measures on $(\chi, \mathcal{P}(\chi))$.