Bellenot
11 Aug 1976.

Elementary lemma If a & b are real numbers and

$$(a+b)^2 \geq a^2+b^2 \qquad (1)$$

then $a^2+b^2 \geq \frac{1}{2}(a+b)^2$. $\cancel{(2)}$

Proof: We may assume a & $b > 0$ since by (1) they have the same sign.

By calculus the problem is to show

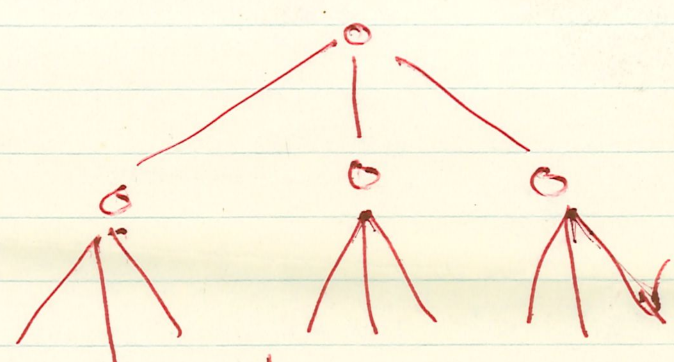
$$\frac{(a+b)^2}{a^2+b^2} = \frac{a^2+b^2}{a^2+b^2} + \frac{2ab}{a^2+b^2} = 1 + \frac{2ab}{a^2+b^2}$$

has max value 2 in the 1st quadrant.

But polar coordinates yield $\frac{2ab}{a^2+b^2} = \frac{2r\cos\theta\, r\sin\theta}{r^2}$

$= \sin 2\theta$ which has max value 1 at $\theta = \frac{\pi}{4}$ (i.e. $a=b$.)

The James Tri-Tree space: JTT = completion X



etc

X sequences of finite support with norm

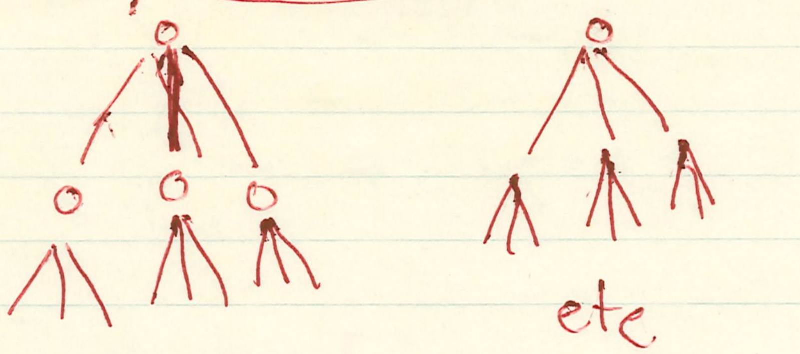
$$\|x\| = \sup \left(\sum_{i=1}^{n} (f_{\alpha_i}(x))^2 \right)^{\frac{1}{2}}$$

sup taken over all disjoint segments $\alpha_1, \ldots \alpha_n$

($f_{\alpha_i}(x)$ = sum of x coordinates on α_i seg.)

JTTT is isomorphic to the following which is isometric to its own square infinitely often

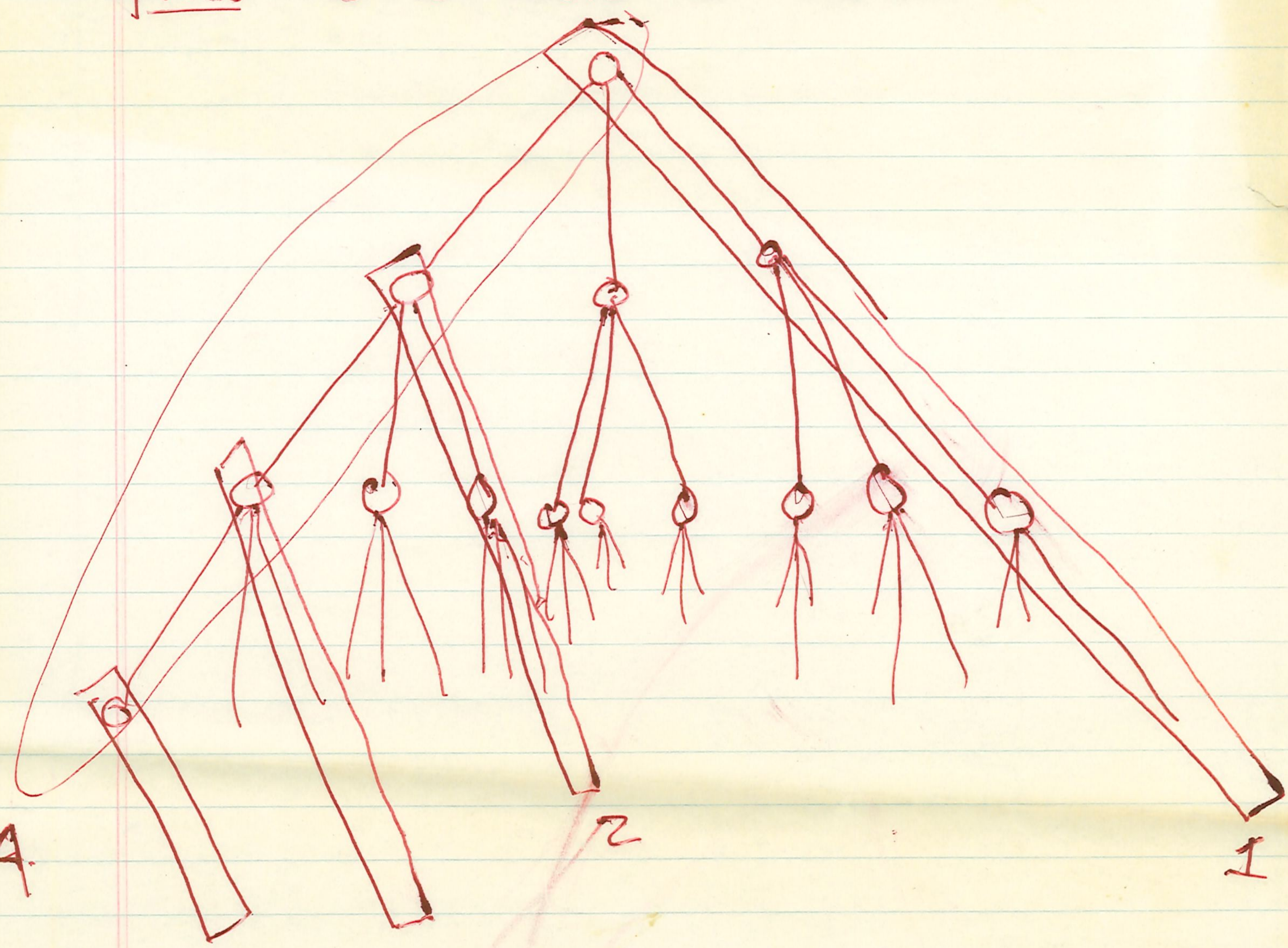
W



etc ... etc

etc

Proof ~~its~~ to move a branch.



A 2 1

4 3

The map π : JTTT $\longrightarrow$ W is decribed as follows

branch 1 goes to limb 2
limb 2 goes to limb 3
limb 3 goes to limb 4
etc

in the obvious way
. otherwised fixed.

The image of $T'(\mathbb{T}'\mathbb{T}')$ is W

Suppose x has finite support in $\mathbb{T}'\mathbb{T}'$

Let $\alpha_1 \ldots \alpha_n$ be disjoint seg s.t.

$$\|x\|^2 = \sum_i (f_{\alpha_i}(x))^2$$

we can divided $\alpha_1 \ldots \alpha_n$ into 3 sets

a = α_i's ~~there~~ that do not intersect the moving sets — leave them alone

b = α_i's that live on the branch A or inside any set Λ move them up a notch — no effect on the norm.

c = α_i's that live off A ~~but~~ or moving set but straddle the line. Slipt in too

$\|T'x\|^2 \geq \sum_i (f_{\beta_i}(x))^2$ where β_i is one of the above.

Thus by the lemma $\|T'x\|^2 \geq \frac{1}{2}\|x\|^2$

$$\|T'x\| \geq \frac{1}{\sqrt{2}}\|x\|$$

$$\|x\| \leq \sqrt{2}\,\|T'x\|$$

$\|T'^{-1}\| \leq \sqrt{2}$ Similarly $\|T'\| \leq \sqrt{2}$.