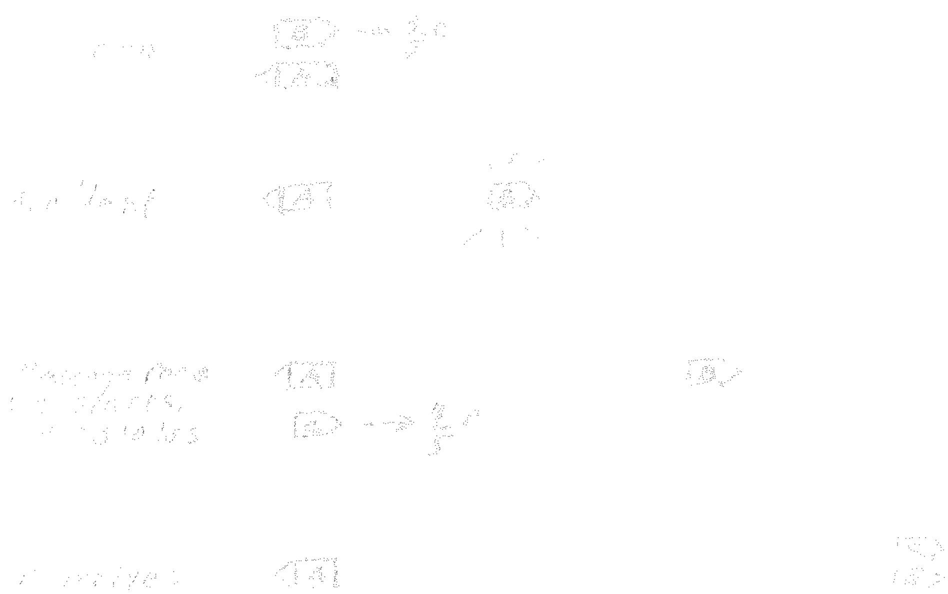


8.

Three space stations, A, B, and C, have synchronized clocks and are stationary with respect to each other. They are 1 lt. year apart, as indicated. At time t = 0 on A's clock, a rocket ship R whizzes by A at 4/5 c. At this same time A launches a ship toward station C with velocity c/2. At time 3/5 years in the ABC reference frame, another ship leaves station B headed for station C with velocity c/2.

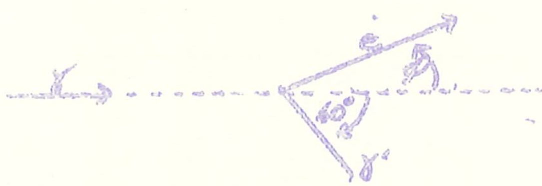
a) What is the distance from A to B according to observers on R?

b) According to R, which station, A or B, launched a ship toward C first? How much earlier, according to R?

c) How long does A's ship require to travel to C according to A's clock?

d) How long does A's ship require to travel to C according to B's clock?

9.



An electron (mass .5 Mev) is at rest in the laboratory. It is "struck" by a photon having energy 1 MEV. The photon moves off with reduced energy, γ' at a 60° angle and the electron moves in the direction φ, as indicated, with momentum p'. Write down the three simultaneous equations from which p', γ', and φ may be calculated. Actual solution of the equations will not be necessary.

10.

A particle with mass Mc^2 = 1300 MEV decomposes into two particles each of mass mc^2 = 250 MEV. Find the momentum, total energy, kinetic energy and speed of each of these particles in the rest frame of the 1300 MEV particle.

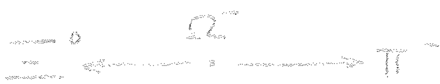
11.

It was shown that the "4-vector," (x, y, z , ict) has a "length squared," $x^2 + y^2 + z^2 - c^2t^2$, which is the same in all inertial reference systems. Another 4-vector of importance in the theory of relativity is the momentum 4-vector of a given particle, (p_x, p_x, p_x, i W/c) which has the "length squared" $p_x^2 + p_y^2 + p_z^2 - W^2c^2$.

a) Show that this "length squared" is the same in all inertial frames.

b) Find its value for a given particle of mass m.

12.

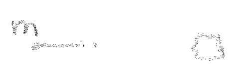


A Ω^- particle at rest in the laboratory decays into a Ξ^0 and a π^-. u_π/c = .92 and u_Ξ/c = [illegible]. The lifetime of a Ξ^0 in its own rest-frame is 1.5 x 10^{-10} sec.

a) As seen in the laboratory, how far will the Ξ^0 move before decaying?

b) How fast is the π meson moving as seen by the Ξ^0?

13.



A particle with mass mc^2 + 250 MEV having velocity relative to the laboratory 12/13 of c, is absorbed by a second particle which is at rest in the laboratory before the collision. After the collision, the system is found to have velocity 3/5 of c relative to the laboratory.

a) What is the mass of the system after the collision?

b) What was the mass of the second particle before it absorbed the first particle?

14. The transformation equations for velocity start at the bottom of page 41 of supplement VI and continue on the top of page 42. Momentum is defined in the middle of page 50, and total energy is defined in page 54. From all this show that the transformation equations for momentum and energy have the same form as those for coordinates and time, and are, in fact:

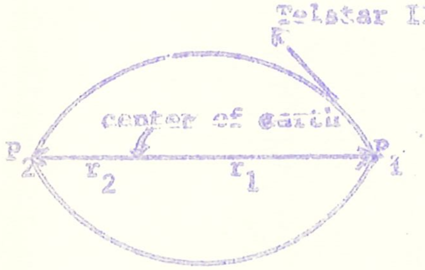
$$p_x' = \frac{p_x - vW/c^2}{\sqrt{1-v^2/c^2}}, \quad p_y' = p_y,$$

15.

$$p_z' = p_z, \quad W' = \frac{W - vp_x}{\sqrt{1-v^2/c^2}}$$

15. A straight wire having mass ρ per unit length, lies along the x axis from $-\infty$ to $+\infty$. Find a formula for the gravitational field strength $\vec{g}$ as a function of r, the perpendicular distance to the wire. Do by direct integration and check by Gauss' law.

16.



The press reports the following relevant data about the orbit of Telstar II:

r_1 = maximum distance of satellite from center of earth = 17.2×10^6 meters

r_2 = minimum distance of satellite from center of earth = 7.34×10^6 meters.

Let v_1 and v_2 be the speeds of the satellite at points P_1 and P_2. Write down two equations which, when solved simultaneously, will determine v_1 and v_2 in terms of the known r_1 and r_2. These equations may also involve

$$G = 6.67 \times 10^{-11} \text{ nt-m}^2/\text{kg}^2 \text{ and } M = \text{mass of earth} = 5.98 \times 10^{24} \text{ kg}.$$

It will be unnecessary, because of time limitations, to actually carry through the calculations. The two equations, properly labeled or boxed, are sufficient answers.

17. Two particles are constrained to move in a plane under the action of no external forces. Particle 1 has a mass m and is initially located on the y-axis at y = 5 cm. Particle 2 has a mass of 3m and is initially located on the x-axis at x = 4 cm. Particle 1 has an initial downward velocity directly toward the origin of 4 cm/sec, and particle 2 is initially at rest. The particles repel each other with forces depending on the distance between them.

a) Calculate the initial coordinates of the center of mass of the two particles.

b) What is the initial velocity of the center of mass?

c) What are the coordinates of the center of mass at t = 20 sec?

d) If particle 1 crosses the x-axis at x = -3 cm when t = 2 sec, what are the coordinates of particle 2 at that time?

18.

The problem is to launch a projectile from the surface of the earth to the moon at such a speed, V_o, that it arrives at the surface of the moon with the least possible speed, V_f. To simplify matters, you may assume that the moon and earth are substantially at rest, that the projectile path is along the straight line connecting the centers of the earth and moon, and that there is no friction. Compute both V_o and V_f. The data in MKS units is:

$$M_1 = 5.98 \times 10^{24}, \qquad M_2 = 7.34 \times 10^{22},$$

$$D = 3.84 \times 10^{8}, \; a_1 = 6.37 \times 10^{6}, \; a_2 = 1.74 \times 10^{6}$$

19. Consider two planets of equal masses M and radii R, a distance D apart (measured between centers). Prove that if a projectile is fired from the surface of one planet to the other along the line of centers, the projectile will have its minimum velocity at a point halfway between the planets.

20.

Figure 1.

The object indicated is a very long cylinder of radius b and uniform density ρ. A cross-section looks like Figure 2. Find the gravitational field strength $\vec{g}$ at points $r > b$ and at points $r < b$. Give a qualitative plot of $|\vec{g}|$. Neglect "end effects".

Note: We strongly recommend the use of Gauss' law.

Figure 2.

21.

The figure shows concentric spheres of radii R_1 and R_2. There is no material within R_1, but the space between R_1 and R_2 is filled with material of constant density ρ. Find the gravitational field strength $\vec{g}$ as a function of r, the distance, from r = 0 to r = ∞. Plot your result qualitatively.

$m = \frac{\pi 4}{3}(r^2 - R_1^2)\rho$

$E =$

$|\vec{g}| = 0 \quad 0 \le r \le R_1$

$|\vec{g}| = \frac{\rho G \frac{\pi 4}{3}(r^3 - R_1^3)}{r^2} \quad R_1 \le r \le R_2$

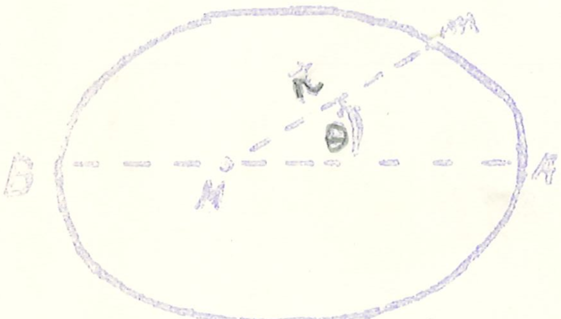
$|\vec{g}| = \frac{\rho G \frac{4}{3}\pi (R_2^3 - R_1^3)}{r^2} \quad R_2 \le r$

22. A space station is to be placed in a circular orbit at an altitude of 6 earth radii. (This means 7 earth radii from the center of the earth.) It is to be done with a 3-stage rocket. The first two stages place the vehicle in a minimum altitude circular orbit. (For practical purposes, the radius of this orbit is simply the radius of the earth.) By a controlled series of firings, the third stage puts the station into the required circular orbit.

a) Let v_1 be the speed in the minimal orbit and v_2 be the speed in the final orbit and calculate the ratio v_2/v_1.

b) Let b be the radius of the earth, M its mass and m the mass of the space station. Calculate the work done on the space station by the third-stage rocket. (In terms of b, M, m, and G.)

23.



In the picture, M is the mass of Jupiter and m is the mass of one of its satellites. M >> m, so the motion of Jupiter may be neglected. Use the indicated polar coordinates.

a. Write the law of conservation of angular momentum for the system.

b. Assume that the distance $\overline{MA}$ = b, and that the distance $\overline{MB}$ = b/2. If $\dot{\theta} = \omega_0$ when m is at A, find $\dot{\theta}$ when m is at B.

c. Write the law of conservation of energy for the system. (In general, not under the special assumption of part b.)

d. Reverting again to the special assumptions of part b, find the total energy of the system in terms of m, b, and ω_0. (No G or M)

24. Two spheres of masses 4000 gm and 200 gm and radii 10 cm and 4.0 cm respectively are fixed in space with their centers 100 cm apart. A body is placed on the line joining their centers 30 cm from the center of the smaller sphere.

$$\gamma = (6.66)\ (10^{-8})\ \text{cm.}^{3}\ \text{gm.}^{-1}\ \text{sec.}^{-2}$$

a) If the only forces acting on this body are the gravitational attractions of the two spheres, what will be the initial acceleration of the body when it is released?

b) What will be the velocity of the body when it reaches the surface of one of the spheres?

25. An H. M. C. rocket which has used up all its fuel is falling freely toward the newly discovered planet Muddo, which turns out to be a homogeneous thin-walled spherical shell with a mass of 2.0×10^{19} kgm. and a radius of 4.0×10^{6} m. The rocket is moving with a speed of 10 m/sec when it is 5.0×10^{6} m. from the center of Muddo. If it happens to hit a hole in the planet and passes into its interior without touching anything, what will be its speed when it is 2.0×10^{6} m. from the center of Muddo?

The diagram is a cross section of a very large, [illegible] spherically symmetric dust cloud [illegible] in space. Take the density as a constant, ρ, and its radius as R. m is the mass of a particle [illegible] which is [illegible] inside the cloud. [illegible] ρ is [illegible] through great distances without significant [illegible] with individual dust particles. [illegible] the field created by the cloud [illegible] in [illegible] with the cloud.

[illegible] inside the cloud. $\vec{g} = -\frac{\frac{4}{3}\pi r^3 \rho G}{r^2}\hat{r}$ $\quad r \leq R$

[illegible] potential energy function for particle [illegible] [illegible]

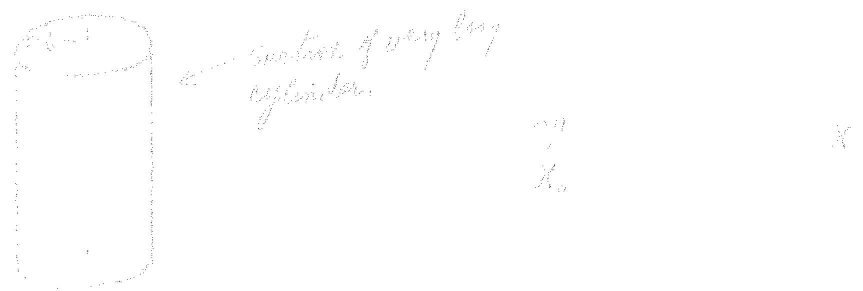
[illegible] at one point [illegible] [illegible]

$U(R) = -\frac{G\frac{4}{3}\pi R^3 \rho m}{R}$

[illegible] to be approximately [illegible]

$\rho = [illegible]\ kg/m^3, \quad R = 10^{[illegible]}$ meters

[illegible] per cubic centimeter and a cloud diameter [illegible] light years. If this is the case, find the [illegible] through the center of the cloud [illegible] for a complete [illegible].

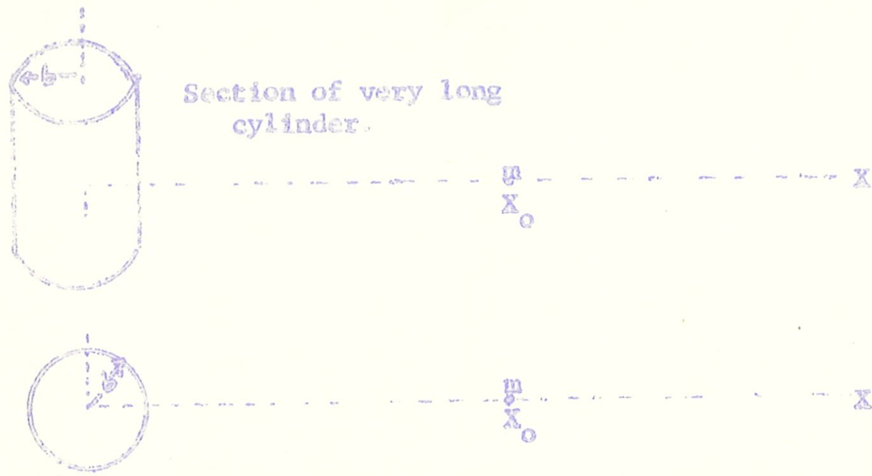


[illegible] cross section of a very long uniform cylinder. [illegible] density [illegible] mass per unit of length. A particle of mass m is released [illegible] from the axis of the cylinder. Assuming that the only force [illegible] is the gravitational pull of the cylinder, find [illegible] of x and the parameters [illegible] and [illegible].

[illegible] small enough that the length of the cylinder [illegible]

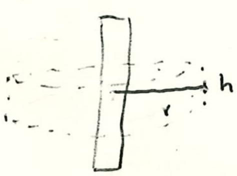
[illegible]

[illegible]



The two views shown are of a very long uniform cylinder. Its density is η, in kilograms per unit of length. A particle of mass m is released at a point distant X_o from the axis of the cylinder. Assuming that the only force acting on the particle is the gravitational pull of the cylinder, find its velocity as a function of X and the parameters η, X_o and G.

Notes:
1. "Very long" means here that the length of the cylinder is many times X_o
2. "Released" means that, at $X = X_o$ $\dot{X} = 0$.
3. The cylinder is so much more massive than the particle that its (the cylinder's) motion may be neglected.



$$\int_S \vec{g} \cdot d\vec{s} = -4\pi G m$$

only curved adds to $\vec{g} \cdot d\vec{s}$ ($d\vec{s} \perp \vec{g}$ otherwise)

$$= \int_{curv} -|g||ds| = -4\pi G m$$

$$-g \int_{curv} ds = -4\pi G m$$

$$\int ds = 2\pi r h \qquad m = \eta h$$

$$-2\pi r h g = -4\pi G m = -4\pi G \eta h$$

$$g = \frac{2G\eta}{r}$$

$$\vec{g} = -\hat{r}\,\frac{2G\eta}{r}$$

29. The following equations describe two different velocity fields:

i. $\vec{V}_1 = \lambda\{i(x+y) + j(-x+y) + k(-2z)\}$ div = [illegible]

ii. $\vec{V}_2 = \lambda\{i(2y) + j(2x+3z) + k(3y)\}$ $(2xy + 3yz)\lambda$ div = 0

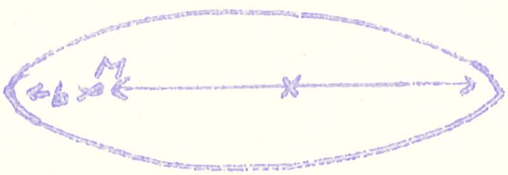
In both cases, λ is a constant with dimensions t^{-1}.

a) One of these fields is irrotational. Which? Show your proof.

b) Find a velocity potential function, φ, suitable for the irrotational field.

c) Calculate the divergence for each of the fields, $\vec{V}_1$ and $\vec{V}_2$.

30.



A comet, having mass m, moves in an elliptical orbit about the sun. Its point of closest approach to the sun is b, and at that point the total mechanical energy of the comet is

$$\frac{-GMm}{1001\ b},$$

where M is the mass of the sun. Find x, the maximum distance of the comet from the sun, in terms of b.

31 A particle of mass m moves under the action of a central force given by

$$\vec{F} = -\hat{r}_1 \, Cmr^3$$

where C is a constant and $\hat{r}_1$ is the usual unit vector.

a) We let the potential energy associated with this force be zero at $r = 0$. What is the potential energy for arbitrary r?

b) For what total energy and angular momentum will the orbit be a circle of radius b? (Answer in terms of C, b, and m.)

c) What is the period of this circular motion?

32. Given that $\vec{V} = \nabla \varphi$ and that $\varphi = x^2 + y^2 + z^2$. Find:

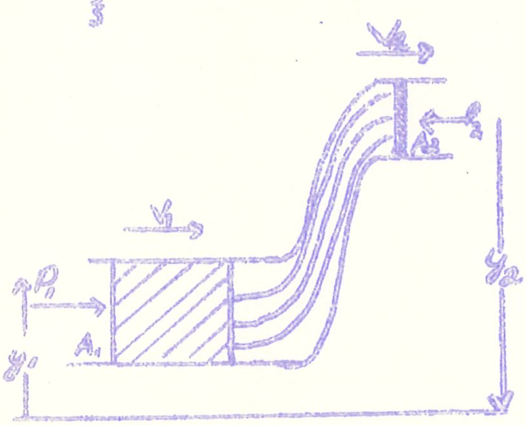
a) $\vec{V}$

b) div $\vec{V}$

c) Curl $\vec{V}$

d. $\int_s \vec{V} \cdot d\vec{s}$ where s is a curve connecting point (1,1,1) to point (2,3,4).

33.



v_1 - speed, p_1 - pressure

v_2 - speed, p_2 - pressure

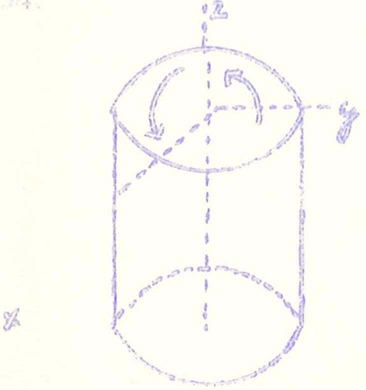
A_1, A_2 = cross-section areas

X_1, X_2 = distances fluid moves in time Δt.

With reference to the above diagram, derive Bernoulli's equation for the case of non-viscous, incompressible flow. The density of the fluid is ρ.

$\text{grad}\, u = \hat{r}\frac{\partial u}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial u}{\partial \theta} + \hat{z}\frac{\partial u}{\partial z}$

34.



The flow of the ideal fluid in the cylindrical container is described by

$$\vec{v} = \vec{\theta}_1 V_o = V_o(-i \sin\theta + j \cos\theta)$$

$$= V_o \frac{-iy + ix}{\sqrt{x^2 + y^2}}$$

where V_o is a constant. This description implies that the linear speed of the fluid is the same at all points. V does not depend on Z. ρ is a constant in space and time, and $\frac{\partial \vec{V}}{\partial t} = 0$.

a) With reference to Euler's equations, calculate $(\vec{V}\cdot\nabla)\vec{V}$

Physically, this is the applied field necessary to produce the flow described. Calculate it in both rectangular and cylindrical co-ordinates and show that they agree. The result, particularly in polar coordinates, should look familiar to you.

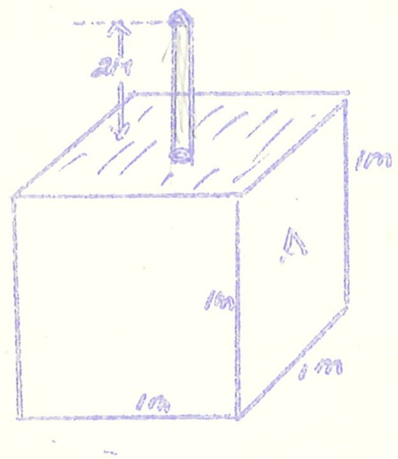
b) Show that the ρ and $\vec{V}$ described are consistent with the equation of continuity by a direct calculation of div $(\rho\vec{V})$.

c) Calculate the curl of the field found in part a, and, on the basis of the calculation, decide if the field is conservative. For this part of the calculation, rectangular coordinates are sufficient.

35. A wooden cylinder of uniform cross-sectional area A is weighted at one end so that it floats vertically in a liquid of density ρ_L. The length of the weighted cylinder is L and its total mass is M. The cylinder is pushed down somewhat from its equilibrium position, though not totally submerged, and then released so that it bobs up and down. Derive an expression for the period of this motion in terms of the given quantities.

$$\frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\theta} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ V_r & rV_\theta & V_z \end{vmatrix}$$

26.



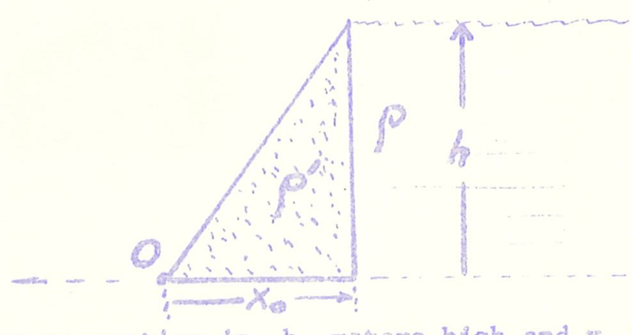
Indicated is a cubical tank of edge 1 meter, which is filled with water. The top of the tank is completely closed, except for a pipe of $10^{-4} m^2$ cross section which exposes the water in the box to atmospheric pressure ($10^5 nt/m^2$).

a) Calculate the total force (due to water and air) against the side of the tank marked A in the picture. The density of water is $10^3 kg/m^3$.

b) The pipe is now filled with water to a height of 2 meters (above the top of the box). Find the amount by which the force against side A has been increased, and

c) Calculate the ratio: $\dfrac{\text{increase of force against A}}{\text{weight of } \underline{\text{added}} \text{ water}}$

27.



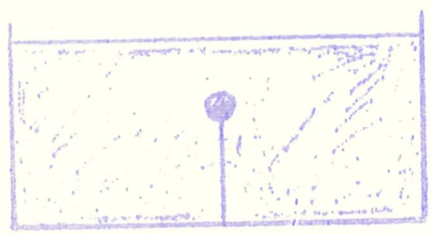
A dam of triangular cross-section is h meters high and x_o meters thick at the base. The density of the concrete is ρ' and that of the water is ρ. (Practically, $\rho' \cong 2.5\rho$).

Find the ratio h/x_o in terms of ρ and ρ' such that the dam will just remain in place when the water comes to the top.

(Assume that the dam is free to turn about an axis perpendicular to the paper at O. There are no other "pins" along the base x_o.)

38. A certain dam is 20 meters high and 30 meters wide. The water in the lake behind it comes just to the top of the dam and is clear for a depth of 15 m. The bottom 5.0 meters of the lake, however, is mud which is watery enough to act as a liquid but has a density of 2.0 gm/cm^3. What is the net force on the dam due to the water and mud?

39.

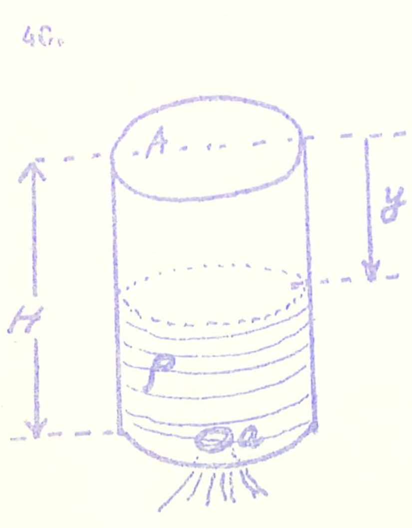


A piece of cork (volume V, density ρ) is attached by a string to the bottom of a tank containing water (density ρ_0). The tank is given a constant acceleration a, to the right. After a time the cork comes to rest relative to the fluid, and the string is at some angle φ with the vertical.

Find φ and the magnitude of the tension in the string in terms of a, ρ, ρ_0, V, and g.

(There is more than enough water to keep the cork submerged.)

40.



At time t = 0 the indicated cylinder, height H and cross-section A, is filled with a liquid of density ρ. At this time a hole, of cross-section a, is made in the bottom of the cylinder. Find the rate, $\dot{y}$, at which the surface of the liquid lowers as a function of y and the specified parameters.

41. A problem commonly found in mathematics books states that the flow of fluid from a small hole in the bottom of a tank is proportional to the amount of fluid left in the tank. Show that this statement cannot be true for incompressible ideal fluids flowing from rectangular tanks.

42. The density of a star may be roughly represented by the function

$\rho = c/r$, where c is a constant.

a. Find the gravitational field, $\vec{g}$, for points $r > R$, where R is the radius of the star.

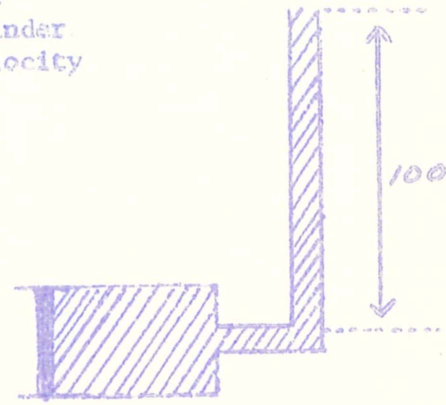
b. Find $\vec{g}$ for poinrs $r < R$.

c. Find the pressure gradient, ∇P, which must exist inside the star to prevent it from collapsing. In this part assume that the stellar material is not in motion; that is, that $\vec{v} = 0$.

d. Find the pressure, P, as a function of r, inside the star. Assume that $P = 0$ at $r = R$.

Note: The gradient operator, in spherical coordinates, simplifies to $\vec{r}_1 \frac{d}{dr}$ in cases where no angular dependence is involved.

43. A piston moving in a cylinder of 10 cm radius forces water out of a pipe of 3.0 cm radius that has its opening 100 cm. above the cylinder. The water is ejected from the pipe at a velocity of 80 cm/sec. Ignore viscosity effects.

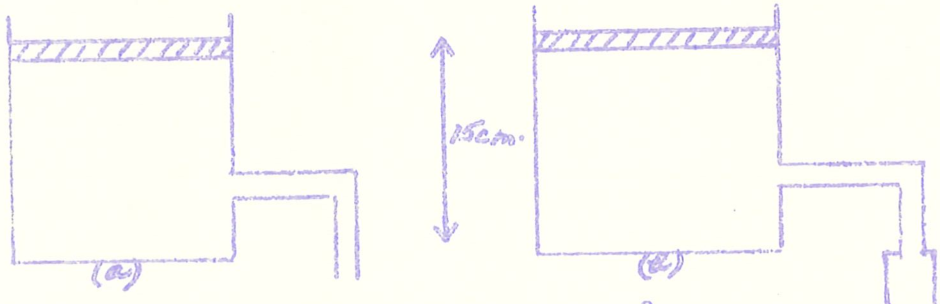


a) What is the velocity of the piston?

b) What is the pressure of the water in the cylinder?

c) What is the power output of the machine which is driving the piston?

44. A cylindrical container has a water tight piston fitted to it that rests on the surface of the liquid. The cylinder contains alcohol ($\rho = 0.81 \frac{gm}{cm^3}$) and has a cross sectional area of 100 cm^2. The piston resting on the surface of the liquid has a mass of 1000 gm.

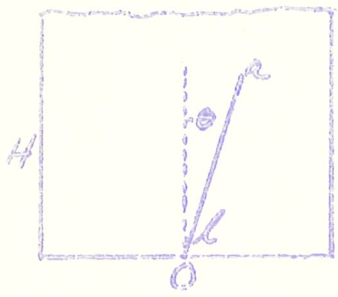


A small tube (cross sectional are = 00.5 cm^2) is attached to the cylinder as shown.

a) What is the rate at which alcohol flows out of the end of the tube when the piston and upper surface of the liquid is 15 cm. higher than the end of the tube open to the air? Ignore viscosity effects.

b) At this time what is the pressure in the horizontal section of the tube?

c) If on the open end of the tube we replace a very short section of the tube with a piece of tube of 2.0 cm^2 in cross sectional area (as shown in diagram (b), will the rate of flow of alcohol be increased by a factor of four? If it will, explain why. If not, why do our equations give the wrong result? Be very brief.

45. 

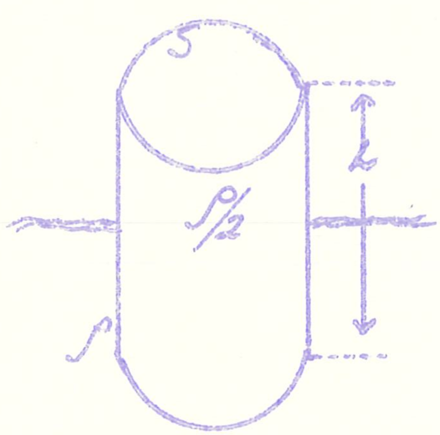
A rod, length ℓ, cross-section area a, and density ρ' is immersed in a liquid of density $\rho > \rho'$ and pivoted at O. Neglecting small effects, find the period of oscillation of the rod for small angles θ.

Notes: H > l. Moment of inertia of a rod with respect to its center of mass is $\frac{1}{12} m\ell^2$.

46 A plane is refueled at a rate of 240 liters per minute using a hose which has an internal cross-sectional area of 80 square centimeters. The hose is connected to a pump 1 meter above the ground. Gasoline with a density of 700 kilograms per cubic meter is delivered to the plane at an elevation 5 meters above the ground through a nozzle which has an internal cross-sectional area of 20 square centimeters. Calculate:

a) The velocity of the fuel in the hose at the 1 meter level near the pump.

b) The pressure in the hose at the same position as specified in part a).

47. 

A solid wooden cylinder, having cross-section area

$S = 20\ cm^2$,
height h = 10 cm, and
density $\rho/2$ grams/cm^3,

floats on a fluid of density ρ grams/cm^3 in the vertical orientation indicated. The cylinder is depressed so that its upper surface coincides with the surface of the fluid, and then released. Neglecting viscosity and surface tension, find the maximum speed of the cylinder during its subsequent motion.

48. A layer of water is floating on top of mercury. A cylindrical block, axis vertical, of metal of density 7.30 gm/cm^3 and length 6.00 cm is floating so that its upper end projects 0.945 cm above the upper surface of the water. When oil to a depth of 1.00 cm is poured on top of the water, the top of the cylinder is level with the upper surface of the oil. Density of mercury is 13.6 gm/cm^3.

a) Find the thickness of the water layer.

b) Find the density of the oil.

49.

The metal bar has cross-section area 1 cm^2, length 10 meters, density 8 $grams/cm^3$, and $Y = 2 \times 10^{+12}$ $dynes/cm^2$. The bar rests on a horizontal frictionless platform. The impulse $F \Delta t$ is delivered. $F = 4 \times 10^9$ dynes and $t = 10^{-4}$ sec.

a) Calculate the velocity of the resulting longitudinal wave pulse. When will the right end of the bar start to move?

b) Neglecting thermal motion, what is the velocity of the particles (or lamina) participating in the wave motion?

c) If 1/200 of the total energy of the wave pulse is eventually transformed into translational kinetic energy of the rod as a whole, how fast will the rod be moving?

50. It is known that deep water waves consist of two components. One component is transverse and the other longitudinal. These components have equal amplitudes, frequencies and velocities, and differ in phase by 90°. Typical data on a great wave might be:

speed $\approx$ 25 meters/sec
wavelength $\approx$ 400 meters
amplitude $\approx$ 20 meters

a) Prove that the "particles" or fluid elements constituting such a wave move in circles.

b) Calculate the speed of a surface particle, assuming the above data.

51.

The graph represents a snapshot of a sine wave traveling in the + x direction from a source of frequency 100 cycles/sec. The y coordinates are in millimeters and the x coordinates are cm.

a) What is the wavelength?

b) What is the amplitude?

c) What is the velocity of the wave?

d) Write the equation describing this wave, putting into it the values you have calculated in a, b and c.

e) What is the maximum velocity of a particle in the string?

52. The differential equation for waves traveling in a certain medium is

$$\frac{\partial^2 y}{\partial t^2} = 25 \frac{\partial^2 y}{\partial x^2}$$

where x and y are measured in meters and t in seconds. As a sinusoidal wave moves through this medium in a <u>negative x direction</u>, the particles of the medium oscillate with a frequency of 5.0 cycles per second and an amplitude of 0.001 meters.

a) Write the general displacement equation for this traveling wave (not the differential equation above), putting in the numerical values appropriate to this particular wave.

b) Prove that your general wave equation (as set up in part a) is a valid solution of the differential equation given above.

c) What is the maximum velocity u of any particle in the medium?

d) At t = 1 second what will be the values of x for which the particle velocity us is a maximum (u_m or $-u_m$)?

e) At t = 1 second what is the phase difference between particles in the medium located at $x_1 = -2.0$ meters and $x_2 = -3.4$ meters respectively.

53. The displacement of the air molecules in an open pipe of length 60.0 centimeters is given as a function of position x (in centimeters) and time t (in seconds) by the following expression:

$$y = y_0 \cos \frac{\pi x}{20} \sin 1600\, \pi t.$$

where x is measured from one of the open ends of the tube.

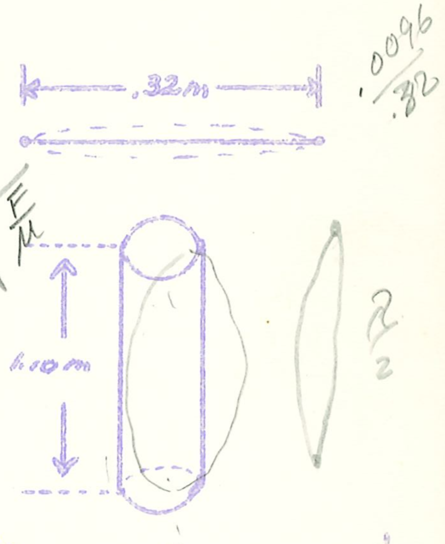
a) What is the wavelength of the wave?

b) What is the speed of the sound wave in this medium?

c) Is this vibration one of the harmonics of the tube? If so, which?

d) How many pressure nodes are present in the tube and where are they located? Use a diagram to illustrate your answer.

54. A tube, 1.10 meters long, is closed at one end. A stretched wire is placed near the open end. The wire is .32 meters long and has a total mass of .0096 kg. It is fixed at both ends and vibrates transversely in its fundamental mode. It sets the air column in the tube into vibration at its fundamental frequency by resonance. The velocity of sound in air is 340 meters/sec. Calculate



a) the frequency of oscillation of the air column.

b) the tension in the wire.

55. A tube closed at both ends is 4 meters in length. (see figure) The tube is filled with hydrogen gas at 0°C temperature and 1 atm-pressure. A small vibrator is located within the tube at point V (therefore making it a displacement anti-node) 3 meters from one end. This vibrator can be made to oscillate at controlled frequencies.

a) What is the lowest applied frequency for which a resonance condition may be achieved in the tube?

b) What is the maximum pressure variation that will be experienced in the tube if the displacement amplitude of the standing wave is 1×10^{-6} meters when resonance is first achieved (for the applied frequency found in part a)?

c) At what positions in the tube will these maximum pressure vibrations be encountered?

d) What is the next applied frequency for which a resonance condition may be achieved in the tube?

56 An analysis of changing electric and magnetic fields in a medium yields the following differential equation for the electric field E.

$$\frac{\partial^2 E}{\partial x^2} = \mu k \frac{\partial^2 E}{\partial t^2} \qquad = \mu k = \frac{1}{v^2} \qquad v = \frac{1}{(\mu k)^{1/2}}$$

where μ is the permeability and k the permitivity of the medium. From this equation, what would you say is the velocity of electromagnetic waves moving through the medium?

57. Two violin strings of the same length and same linear density are adjusted until the first string has a tension of 20 gmf and the second string has a tension of 500 gmf. When the first string vibrates in 12 segments (11 nodes plus one at each fixed end) and the second string vibrates in two segments (one node plus one at each fixed end), the two sounds give rise to 25 beats per second. Calculate the frequencies of the two strings.

58.

The picture represents a string, practically fixed at both ends, vibrating in the manner indicated by the standing wave. The data:

length of string = 2 meters
linear density = .01 kg/meter
tension = 9 newtons
amplitude = 4×10^{-3} meters

a) Calculate the wave velocity, the frequency, the wave number, the angular frequency, and the wavelength.

b) Write down the equation for the standing wave putting in the numerical data.

c) Calculate the maximum particle velocity.

d) Find the maximum positive value of the slope of the string at its midpoint.

e) Write equations for the progressive waves of which the standing wave is composed.

59.

A pair of charges, each +q coulombs, are fixed at y = +s and y = -s. A particle of mass m, bearing charge -q, is free to move back and forth along the x axis. The effects of gravitation, friction and radiation damping are to be neglected.

a) Find $\ddot{x}$ as a function of x and the parameters q, ϵ_0, m, and s.

b) Find the period of the motion for the case where the maximum value of x is much less than s.

c) Using the formula derived in b), compute the period for the case where s = 1 Angstrom, m = electronic mass, + q = proton charge, and -q = electron charge.

60.

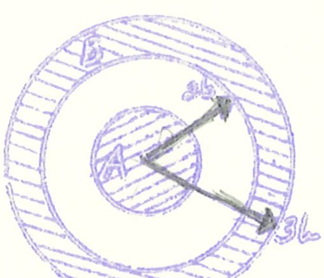
Given 4 spherical, concentric, non-conductor shells having radii b, 2b, 3b, and 4b. These shells have uniformly distributed charges of +q, -2q, +3q, and -2q, respectively. Find the electric field strength, $\vec{E}$, as a function of r. Plot it -- carefully -- from r = 0 to r = ∞.

61.

An electron beam ($q = -1.6 \times 10^{-19}$ coul, $m = 9.1 \times 10^{-31}$ kg) is projected with a velocity of 10^7 meters/sec between the deflecting plates of an oscilloscope between which there is an electric field of 10^4 volts/meter. The plates are 5 cm long and are located 20 cm from the fluorescent screen. What is the deflection of the spot on the screen? (neglect gravity) What is the kinetic energy of the individual electrons upon entering the electric field? What is their kinetic energy after they have passed through the field?

(Note: Relativistic effects are negligible in this problem.)

62.



An insulated metal sphere "A" of radius b and excess charge of +q is concentric with an insulated metal spherical shell "B" of inner radius 2b and outer radius 3b. The spherical shell has an excess charge of -3q.

Determine the electric potential and the electric field intensity vector as a function of distance from the center. Plot these from r = 0 to r = 4b. <u>Describe the charge distribution on both "A" and "B"</u>.

Problem 1:

TRUE OR FALSE (T or F)

a. ______ Bodies in equilibrium are always at rest.

b. T Á 10-lb. force may produce a greater torque about an axis than a 50-lb. force about the same axis.

c. T When the vector sum of the forces acting upon an extended body is zero, the object is necessarily in equilibrium.

d. F Three forces of 3, 4, and 10 lbs. act through a single point. It is possible to arrange their directions so that they will produce equilibrium.

e. F $i \times k . j = 1$.

f. T The vector $\frac{\vec{A}}{|\vec{A}|}$ where $|\vec{A}| \neq 0$ is a unit vector.

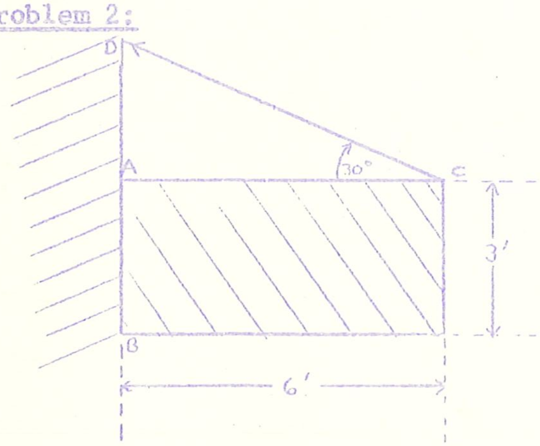
g. F If the scalar (dot) product of two unit vectors is 1, then the vectors are mutually perpendicular.

h. F If the cross product of two non-zero vectors is zero, then the vectors are mutually perpendicular.

i. F $\vec{A} \times \vec{B} + \vec{B} \times \vec{A} = 2\vec{A} \times \vec{B}$

j. ______ $(2i + 3j - 4K) \times (2i - k) = -3i + 6j - 6k$.

Problem 2:

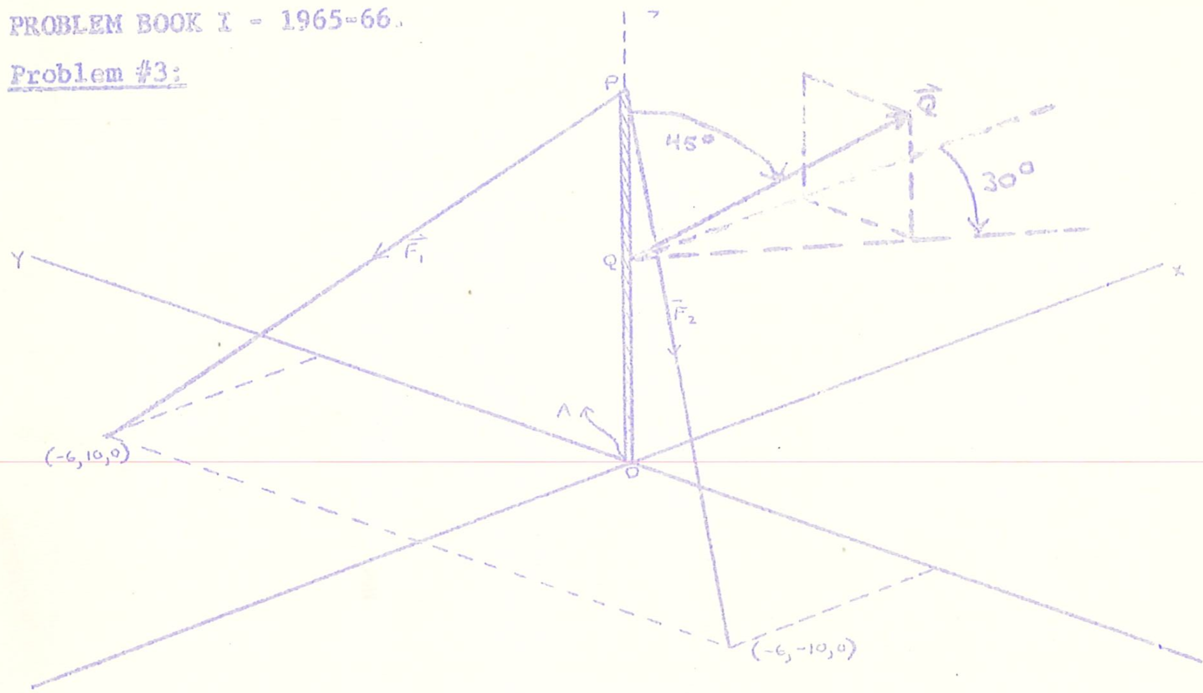


A uniform door, 6' wide and 3' high, weighs 60 lbs. and is hinged at A and B; to relieve the strain on the top hinge, a wire $\overline{CD}$ is connected as indicated. The tension in $\overline{CD}$ is increased until the horizontal component of the force exerted by hinge A is zero. When this is the case, the force exerted by hinge B has no vertical component.

Find the magnitudes of the forces exerted by the wire $\overline{CD}$, the hinge at A and the hinge at B.

PROBLEM BOOK I - 1965-66.

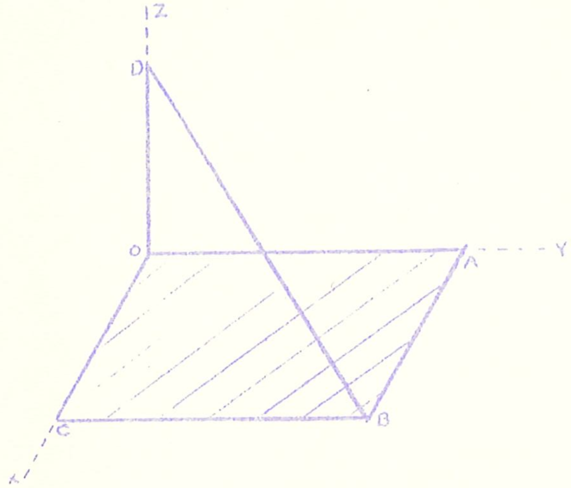
Problem #3:



OP is a 15 ft. pole of negligible weight held in a vertical position by the forces $\vec{F}_1$, $\vec{F}_2$, $\vec{Q}$ and $\vec{A}$. $\vec{F}_1$ and $\vec{F}_2$ are tensions supplied by cables attached at P and having the indicated directions. The force $\vec{Q}$ acts at Q, the midpoint of the pole, and has a magnitude of 800 lbs. The horizontal component of Q makes an angle of 30° with a line drawn through the point Q parallel to the X axis, and $\vec{Q}$ itself is 45° above its horizontal component. Nothing is known about $\vec{A}$ except that it is applied at the origin of coordinates.

Find $|\vec{F}_1|$, $|\vec{F}_2|$, and $\vec{A}$. Write your answer for $\vec{A}$ in the i, j, k form.

Problem 4:



The uniform square plate OABC is held in a horizontal position by hinges at O and A and by wire BD. OA = OD = 2 meters and the weight of the plate is 1000 newtons. It is known that the y-component of the force exerted on the plate by the hinge at O is equal to zero. Find the tension in the wire and the forces exerted by the hinges.

Problem 5:

Given two vectors, $\vec{A} = 5i + 6j - k$, and $\vec{B} = 2i + 2j + 4k$, perform the indicated operations below and express your answers in terms of unit vectors where applicable:

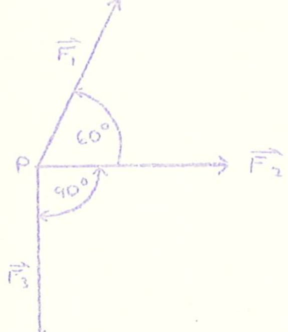
a. $\vec{A} + \vec{B} =$

b. $\vec{A} - \vec{B} =$

c. $\vec{A} \cdot \vec{B} =$

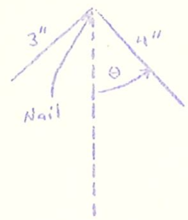
d. $\vec{A} \times \vec{B} =$

Problem 6:



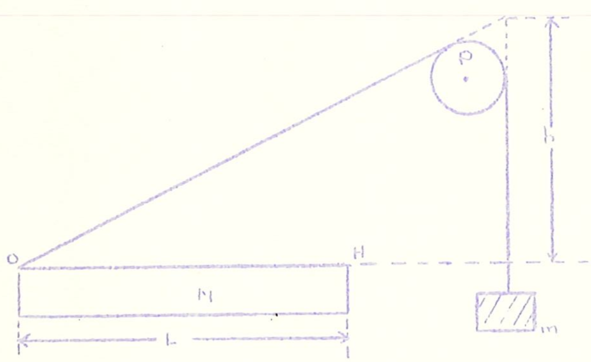
Given 3 coplanar forces, $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ acting on the point P as shown. What are the magnitude and direction of the resultant vector $\vec{F} = \Sigma\vec{F}_1 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ if $|\vec{F}_1| = 10$ newtons $= |\vec{F}_2|$ and $|\vec{F}_2| = 20$ newtons.

Problem 7:



A carpenter's "square" is made of a uniform material, and consists of a 3" arm and a 4" arm forming a right angle. The device is hung over a nail (no friction) as indicated and comes to rest when the 4" arm makes an angle θ with the verticle. Calculate θ.

Problem 8:

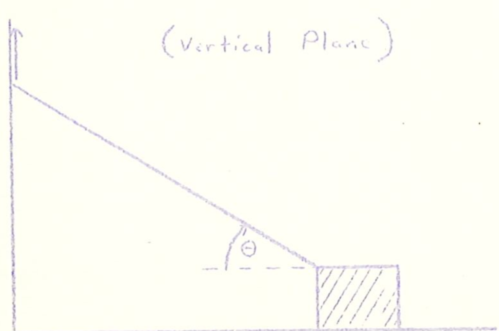


A homogeneous rectangular bar of length L and mass M is suspended horizontally by a string at one end O, and by a frictionless hinge at H (with an axis perpendicular to the plane of the paper) at the other. The string goes over a frictionless, massless pulley at P.

a. Find an expression for the mass, m, which should be attached to the string in order to keep the rod in static equilibrium in terms of h, L, M and g (g is the acceleration due to gravity).

b. Find an expression for the magnitude of the force which the hinge exerts on the rod.

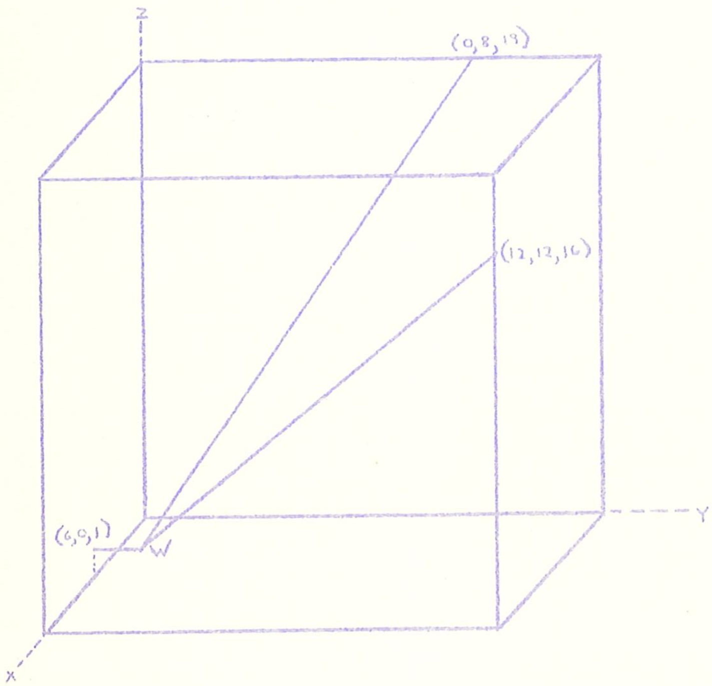
Problem 9:



A block of mass m is pulled along a rough horizontal surface (coefficient of friction μ) at a constant speed by means of a string of fixed length. The free end of the string moves vertically, as indicated, so that the angle θ increases as the block slides toward the wall. Find T, the tension in the string, as a function of θ. (The constants m, g and μ, as well as the variable θ, will appear in your answer.)

Problem 10:

Units: feet.

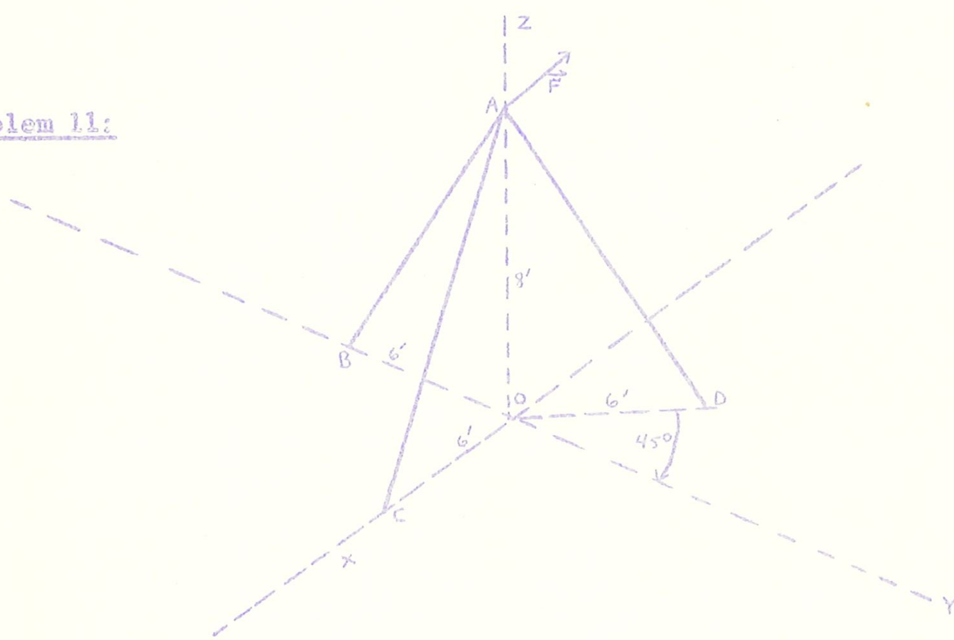


a) A weight of 64 lbs. is suspended in a box as shown. The coordinates of the weight are (6, 2, 1), and the three strings are tied to the box at (6, 0, 1) (0, 8, 18) and (12, 12, 16).

Find the tensions in the strings. The reference system is inertial.

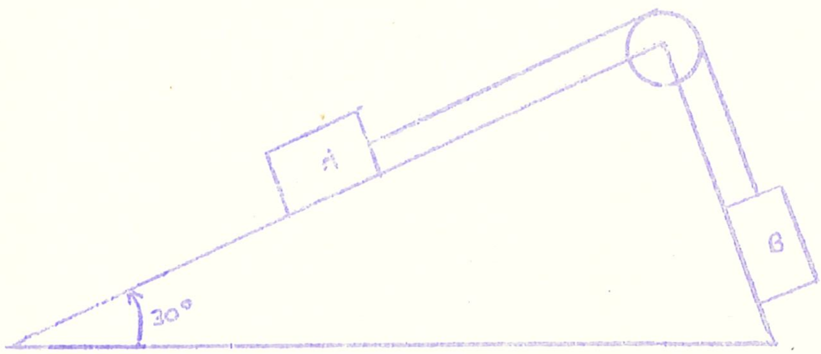
b) If the box were on an elevator which was descending at 4 ft/sec., what would the tensions be?

Problem 11:



AB, AC, AD are the rigid legs of a tripod. They have negligible weight. The ends B, C, D are in ball and socket joints so that the forces exerted at these points are in the directions of the legs. A 1000-lb. force is applied at A, in a direction parallel to the -x axis. Calculate the forces at B, C, and D. (Note: points B, C, D are 6 ft. from O, and point A is on the z axis 8 feet above O. xy is a horizontal plane, and the coordinate system indicated is rectangular.)

Problem 12:



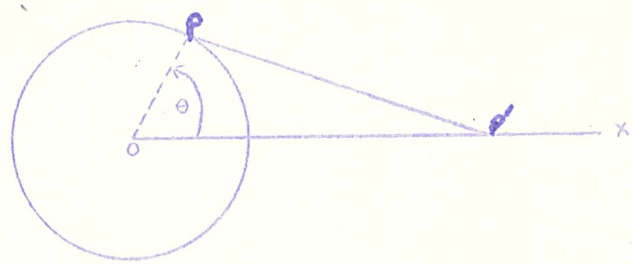
The coefficient of kinetic friction for both blocks is .200. A = 1.02 kg, B = .98 kg. The pulley is the usual massless and frictionless one. Compute the acceleration of A and the tension in the string. (Use g = 9.80 m/sec^2.)

Problem 13:

When a motorboat heads north at 20 mph, the apparent wind is from 30° east of north. When it turns west, the apparent wind is from 60° west of south. Find the true wind velocity.

Note: the wind has no effect on the velocity of the motorboat, and there is no current.

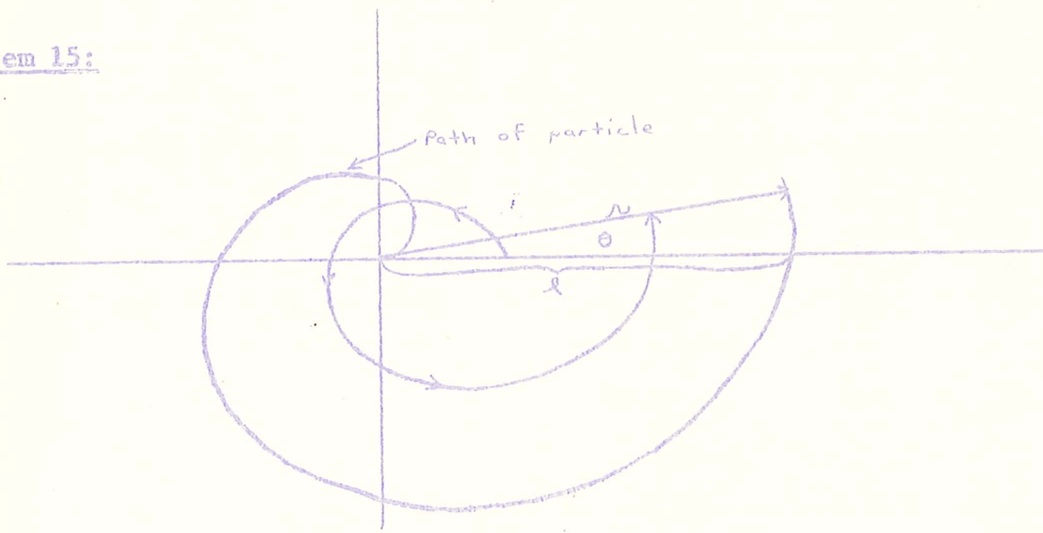
Problem 14:



Point P_1 moves around a circle of 2 ft. radius with a speed of 6 ft/sec. Point P^1 is connected to P by the 4 ft. rod, PP^1, so that P^1 moves along the X axis from X = 6 to X = 2 and back.

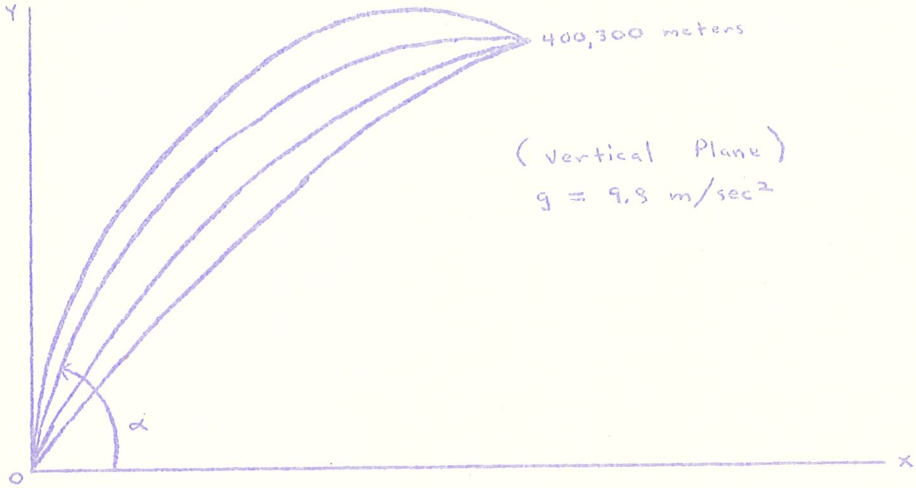
What is the velocity of P^1 when $\theta = \pi/6$? What is the acceleration of P^1 when $\theta = \pi/6$?

Problem 15:



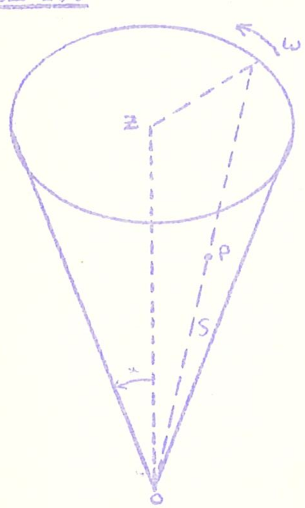
A particle moves along the spiral $\frac{r}{\ell} = \frac{\theta}{2\pi}$ starting at the origin and proceeding at a constant speed v. Find its velocity as as a function of r. (Your answer will include the parameters ℓ and v, the unit vectors $\hat{r}$ and $\hat{\theta}$, but will not contain any reference to θ or t.)

Problem 16:



It is desired to project a particle from the origin so as to strike the point (400,300) (meters). Many parabolic trajectories are possible, depending on the choice of projection angle α, and initial speed v_o. Find the projection angle α such that v_o is a minimum. Neglect air resistance.

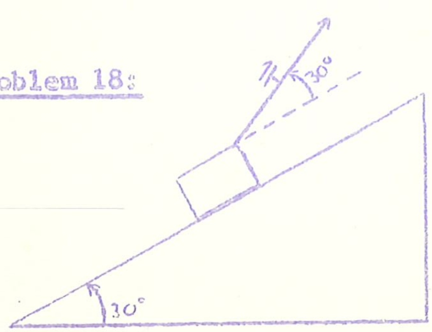
Problem 17:



Indicated is an inverted cone rotating about a vertical axis with constant angular speed ω radians/sec. A particle is at rest* on the inside of the conical surface at P. There is no friction between the particle and the cone. It is given that the cone half angle, α is equal to 30°, $g = 9.8$ m/sec^2, and OP = S = .40 m. Compute ω.

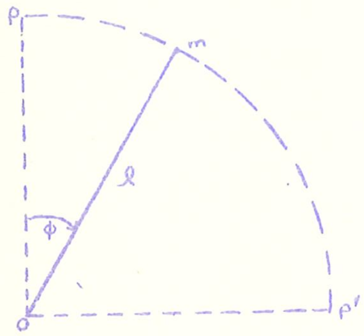
* relative to the cone.

Problem 18:



The block has mass 1 slug. The coefficient of kinetic friction is .200. The magnitude of the applied force $\vec{T}$ is 32.0 lbs. Find the acceleration of the block. (Use $g = 32.0$ ft/sec^2.)

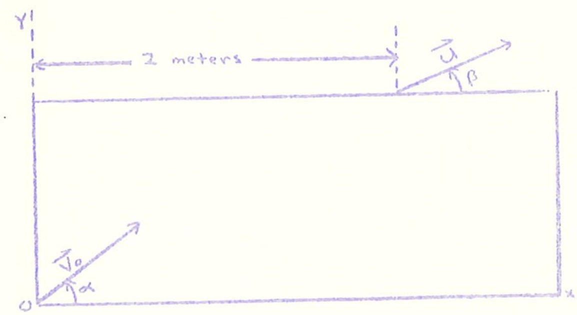
Problem 19:



A rod of length ℓ and negligible mass is hinged at O so that it is free to rotate in the vertical plane between P and P'. A mass m is attached to the upper end of the rod. The rod is held in the vertical position and released.

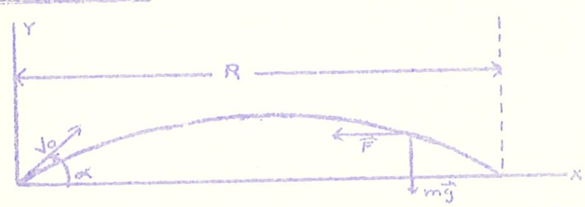
Near the start of the motion, it is evident that the rod exerts a thrust (a force in the $+\vec{r}$ direction) on m. At a certain value of ϕ, this thrust is reduced to zero. Find this value of ϕ, assuming that, when $\phi = 0$, $\dot{\phi} = 0$.

Problem 20:



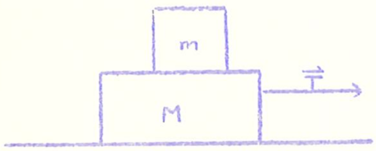
A charged particle is injected into an evacuated box at point O with velocity $\vec{v}_0 = 10^7$ (i cos α + j sin α) in meters/sec where $\alpha = 30^o$. It emerges from the box at the point x = 2 meters, y = 1 meter with the velocity $\vec{U}$ = U (i cos β + j sin β), where $\beta = 10^o$ and U is not known. The mass of the particle is 1.67×10^{-27} kg. Throughout the time it is in the box, the particle is subject to a force having constant but unknown direction and magnitude. The effect of gravity on the particle is negligible. Find the magnitude and direction of the unknown force.

Problem 21:



The initial conditions for the motion of a projectile of mass .50 ounces are given by Vo = 1000 ft/sec, $\alpha = 2.0^o$. The forces to which the projectile is subjected in flight are its own weight and a force of F = -i(.10) lb. Calculate the range. Note: sin 2.0^o = .035, cos 2.0^o = .999, tan 2.0^o = .035.

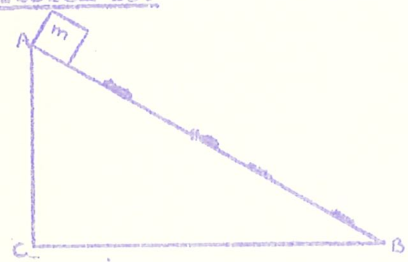
Problem 22:



With reference to the diagram M = 10 kg and m = 1.0 kg. The coefficient of static friction between m and M is .40, and the coefficient of kinetic friction between M and the floor is .20. Find the maximum horizontal force $\vec{T}$ which may be applied to M if m is not to slip.

Note: in this problem it is assumed that M is already in motion with respect to the floor, so that the coefficient of static friction between M and the floor does not enter the problem.

Problem 23:



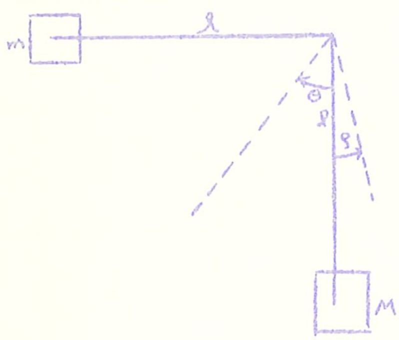
Mass m starts sliding at point A. At point B its speed is 7 meters/sec. AC = 3 meters, BC = 4 meters, AB = 5 meters.

It is known that for some intervals on AB, the coefficient kinetic friction is zero, and that for other regions it is .7.

Calling the sum of the first type of interval X, and the sum of the second type 5-X, calculate the ratio

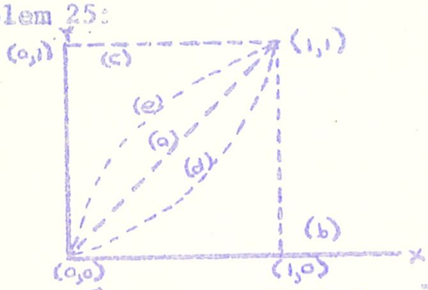
$$\frac{X}{5-X}.$$

Problem 24:



In this picture mass m (4 lbs.) starting at rest in the horizontal position at the end of a light rod of length ℓ, swings downward and collides with mass M (100 lbs.) which was initially at rest. M swings up to the position indicated by $\varphi = 2^{\circ}$, and m rebounds to the position specified by θ. Calculate θ.

Problem 25:



$\vec{f} = k(-iy + jx)$

A force $\vec{f}$ is described by $\vec{f} = k(-iy + jx)$ where $k = 3$ newtons/meter and the units of x and y are meters. A body moves from the origin to the point (1,1) while this force acts on it. The problem is to calculate the work done by the force according to the path taken.

a) the straight line path;

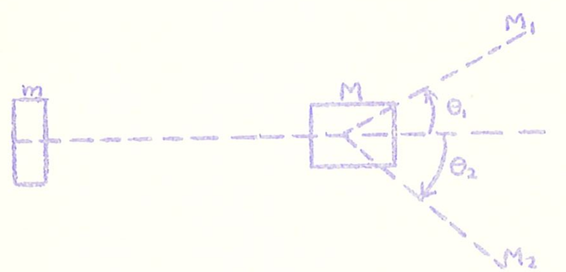
b) along the x axis to 1,0 and thence by a straight line to 1,1;

c) along the y axis to (0,1) and then by a straight line to (1,1);

d) along the parabola $y = x^2$; and

e) along the parabola $y^2 = x$.

Problem 26:



Mass m, with velocity $\vec{U}$ along the X axis, strikes M which is at rest at the origin before the collision. Mass M breaks into two pieces, M_1 and M_2, which go in directions θ_1 and θ_2, while m is left at rest at the origin. Measurement gives the following data:

$m = 1$ gram, $M = 10$ grams, $M_1 = 8$ grams, $\theta_1 = 15^\circ$, $\theta_2 = 30^\circ$.

Calculate the ratio of the final kinetic energy of the system to the kinetic energy it had before the collision occurred.

Problem 27:

A projectile of mass m is fired with velocity $\vec{u}$ at a putty ball of mass M which is initially at rest on a frictionless horizontal table. The projectile passes through the ball and emerges with velocity $\vec{u}/2$. What fraction of the initial kinetic energy of the system was lost or transformed? Assume that $M/m = 4$.

Problem 28:

The picture indicates two boats resting on the surface of a lake, their masses being M_1 and M_2, including passengers, but excluding a ball of mass m held by the passenger in the first boat. This passenger tosses the ball to the passenger in the second boat, who catches it. The horizontal component of the velocity of the ball is iu, and the vertical component is not relevant to this problem. In answering the following questions, you may assume that m, M_1, M_2 and u are known.

a) What is the momentum of the first boat while the ball is in transit?

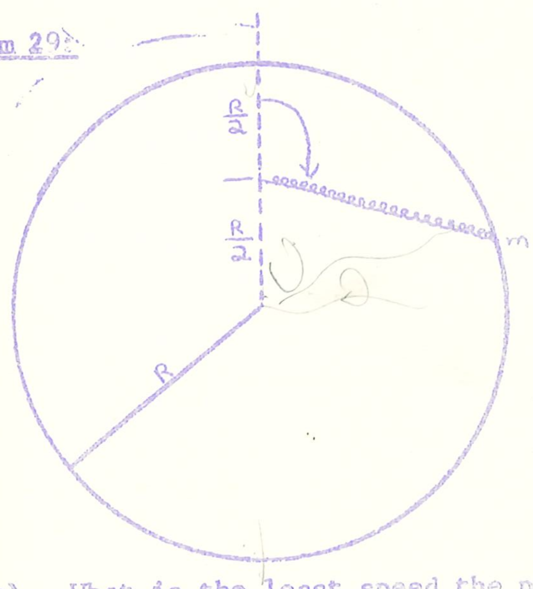
b) What is the momentum of the second boat after the ball is caught?

c) What is the kinetic energy of the system (consisting of both boats and the ball) before the ball is thrown?

d) What is the kinetic energy of the system while the ball is in transit?

e) What is the kinetic energy of the system after the ball is caught?

Problem 29:

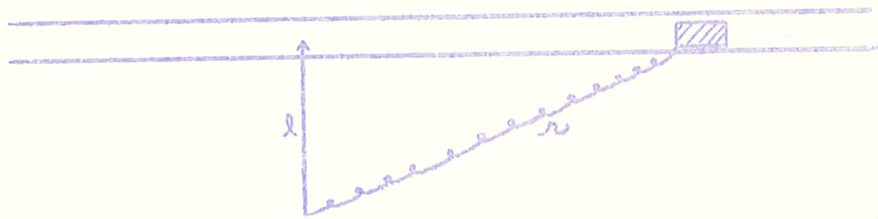


A point mass m can move without friction on a fixed circular track of radius R. The plane of the track is vertical. A weightless spring, having force constant k, is attached to m, the other end being attached to a frictionless pivot which is fixed at a distance $\frac{R}{2}$ above the center of the track. The natural length of the spring is $\frac{R}{2}$, so that it is in its relaxed state when m is at the top of the track.

a) What is the least speed the particle can have at the top of the track and still go all the way around the circle?

b) If the particle has this speed, what force does the track exert on it at the top of the track?

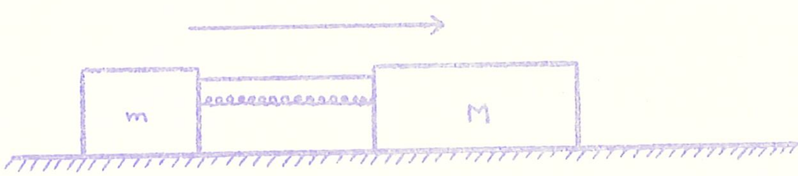
Problem 30:



A car of mass m is constrained to move on a horizontal frictionless track. A spring, constant k and natural length ℓ, is attached to the car, the other end of the spring being fixed at a distance ℓ from the track. Let r be the (variable) distance of the car from this fixed point, as indicated in the diagram.

The car is released from a point $r = r_o$. Find its speed as a function of r and the specified parameters.

Problem 31:



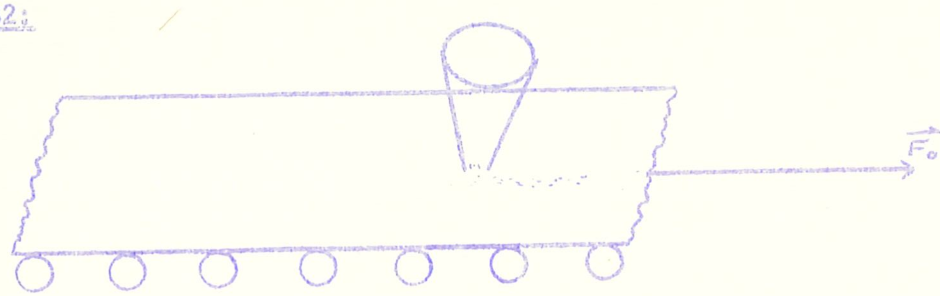
A compressed weightless spring is placed between two masses which are tied together with a string. The spring is attached to M, but not to m. The joined masses are then projected along a straight horizontal frictionless track with speed v_o. Suddenly the string breaks. When the spring is no longer in contact with m, M is found to be traveling in the original direction with speed v. It is given that

v_o = 3.0 meters/sec. M = 10 kg.
v = 4.0 meters/sec. m = 2.0 kg.

a) Determine the final speed and direction of m.

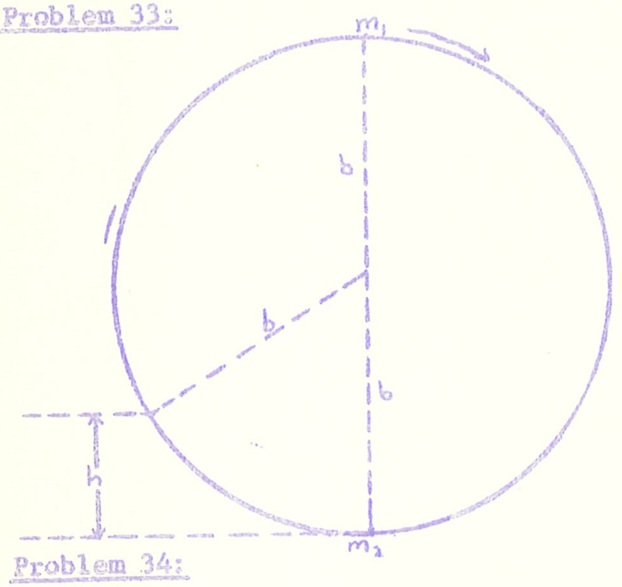
b) Determine the energy U_o that was stored in the spring before the string broke.

Problem 32:



Sand drops from a hopper at a constant rate of λ kg/sec onto a conveyer belt which has mass M. Find a formula for the velocity, v, of the belt. The assumptions are that F_o = const, that, when t = 0, v = 0, and that sand starts to pour when t = 0. Your answer should give v as a function of t and involve the parameters M, λ and F_o.

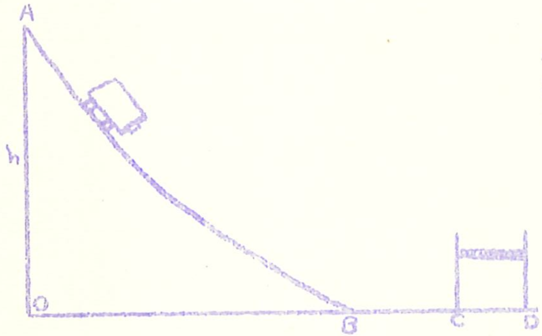
Problem 33:



A particle of mass m_1, initially at rest at the top of a frictionless circular track, slides down the track and makes a perfectly inelastic collision with a particle of mass m_2 which had been at rest at the bottom of the track. The two particles then slide on together and reach a maximum height h, as indicated in the diagram. The radius of the track is b.

Find a formula for h in terms of b, m_1 and m_2.

Problem 34:



A car, starting at A, coasts down the track, AB, arriving at B with speed v. It compresses the indicated spring CD 2 ft. before it is stopped. The data is

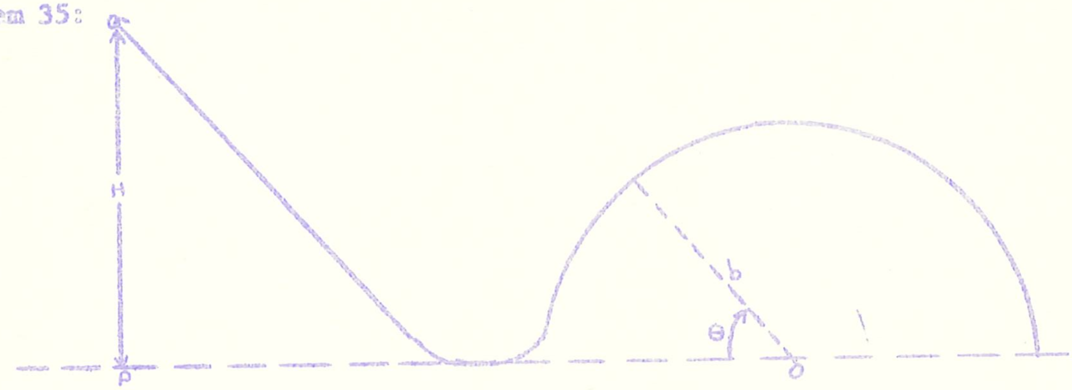
h = OA = 40 feet
mass of car = 10. slugs
g = 32 ft/sec^2
spring constant = 640 lbs/ft
No friction along BCD

Calculate:

a) The energy expended in overcoming friction between points A and B.

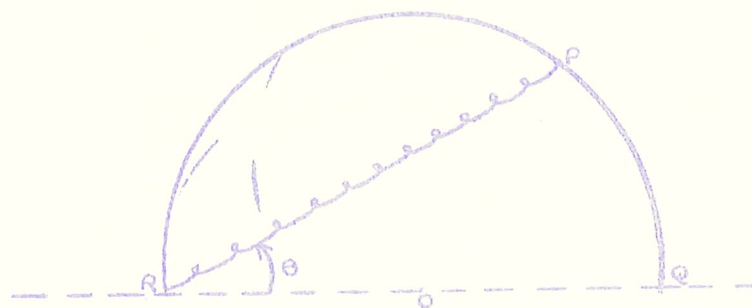
b) The speed of the car when it is at point B.

Problem 35:



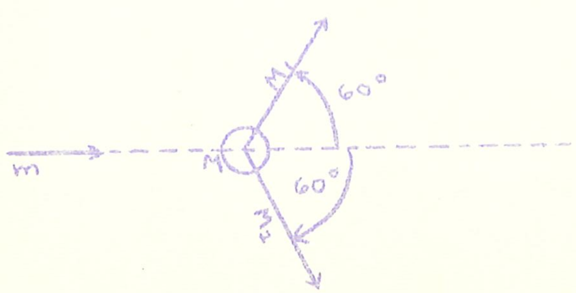
The plane of the indicated frictionless track is vertical, and the latter part of it is a semi-circle of radius b centered at O. A particle of mass m starts sliding at Q with zero initial velocity. At a point indicated by θ on the diagram, the normal force exerted by the track on the particle is reduced to zero. Find θ, given that H/b = 5/4.

Problem 36:



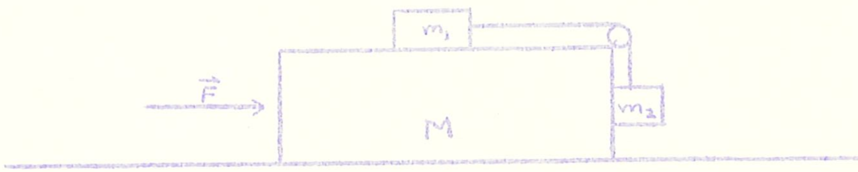
A particle of mass m is constrained to move on a frictionless circular track with center at O and radius b. The plane of the track is horizontal. The particle is attached to point R by a spring, whose natural length is 3b/2 and whose force constant is k. The particle is started at Q with zero speed and allowed to slide around the track under the action of the spring. Find the speed V of the particle as a function of θ and the parameters k, m, and b.

Problem 37:



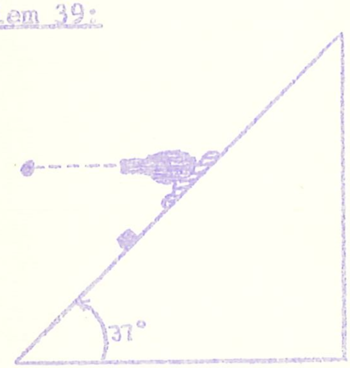
In the collision pictured in the figure, mass m impinges on mass M, which was initially at rest. The combined mass, M + m, breaks into two pieces of unequal masses. M_1 and M_2, which proceed at unequal speeds, in the directions indicated. It is also known that the difference between the initial and final kinetic energy of the system is negligible and that M_1 is greater than M_2. Given that m = 1 gram, M = 7 grams, calculate M_1 and M_2.

Problem 38:



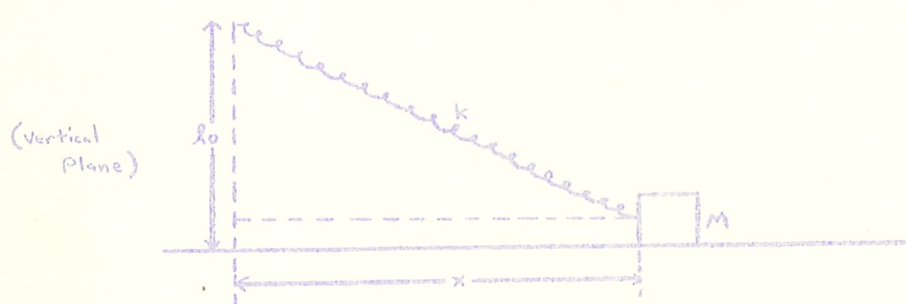
A massless and frictionless pulley is attached to mass M. A massless string is attached to masses m_1 and m_2, as indicated. There is no friction between M and the floor, or between M and m_1, or between M and m_2. A force $\vec{F}$ is applied, as shown, which is such that m_1 and m_2 do not move relative to M. Find F, given m_1, m_2, M and g.

Problem 39:



A cannon rests against a stop on a frictionless inclined plane. After a projectile is fired horizontally, the cannon slides up the plane. The mass of the cannon alone is 100 slugs and that of the projectile is 1.0 slugs. The velocity of the projectile (relative to the earth, not the cannon), is 2000 ft/sec. Find the distance the cannon slides up the plane before starting back to the stop.

Problem 40:

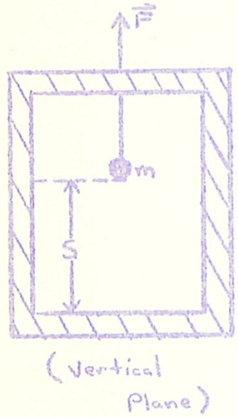


Mass M = 2 kg is on a frictionless horizontal track. The mass is attached to a spring whose natural length is l_o = 1 meter and whose constant k is 8 newtons/meter. The mass is pulled to the point where x = 2 meters and then released.

a) Write a first order differential equation which gives the speed of the mass as a function of x.

b) Find the speed of the mass as it passes the point x = 0.

Problem 41:



A mass m is suspended from the ceiling of an elevator of mass M. (M does not include m. The total mass is m + M.) The elevator is being accelerated upward by a known constant force $\vec{F}$. The distance of m from the floor is S.

a) Find the acceleration of the elevator.

b) What is the tension in the string connecting m to the elevator?

c) If the string suddenly breaks, what is the acceleration of the elevator immediately afterward?

d) How long after the break does it take m to hit the bottom of the elevator?

Problem 42:

True-False. Put a T or an F in the space provided. In grading this question one-half of the number wrong will be subtracted from the number correct. For example, 7 right, 3 wrong, would score 11 out of 20 points.

_____ a) It is impossible for an inertial reference frame to be rotating with respect to a non-inertial frame.

_____ b) The measured acceleration, with respect to a particular inertial frame, of a particle is $\vec{a}$. It will also be $\vec{a}$ in any other inertial frame.

_____ c) The formula for the acceleration of a particle in polar coordinates, namely

$$\vec{a} = \hat{r}\ (\ddot{r} - r\dot{\theta}^2) + \hat{\theta}\ (2\dot{r}\dot{\theta} + r\ddot{\theta})$$

is true only in inertial frames.

_____ d) If one reference frame moves with constant velocity with respect to another, and one of them is inertial, the other one is also inertial.

_____ e) A space ship completes one orbit about the earth each 24 hours. Experiments conducted inside the ship will disclose that $\vec{F} = m\vec{a}$, where $\vec{a}$ is calculated relative to the walls of the ship, is as accurate as it is on the surface of the earth, in spite of the phenomenon of "weightlessness".

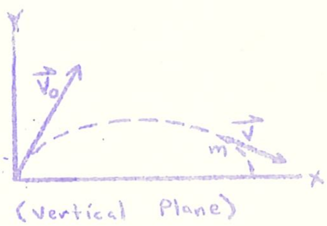
Problem 42, continued:

_____ f) The kinetic energy of a particle, moving along a curve in two dimensions, is always a scalar quantity.

_____ g) It is impossible for the kinetic energy of a particle to be a constant if the particle is accelerated.

_____ h) In the space ship of (e) mass m is attached to a string of length ℓ and revolved in a circle of radius r with angular velocity ω. On the earth, a mass m is attached to a string of length ℓ, placed on a frictionless horizontal platform, and revolved in a circle of radius r. The tension in the string is the same in the two cases.

_____ i)



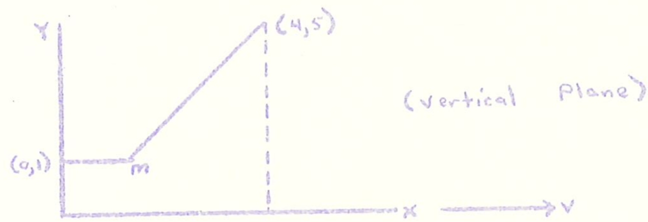
The picture refers to a projectile with mass m, the initial velocity being $\vec{V}_o$. No air friction. Conservation of energy shows that

$$mgy + \frac{1}{2} m \vec{V}\cdot\vec{V} = \frac{1}{2} m \vec{V}_o\cdot\vec{V}_o$$

where $\vec{V}$ is the velocity of the projectile when its altitude is y.

_____ j) With reference to the picture and notation of question i), $i \cdot \vec{V}_o = i \cdot \vec{V}$ throughout the motion. i, of course, is a unit vector in the positive x direction.

Problem 43:



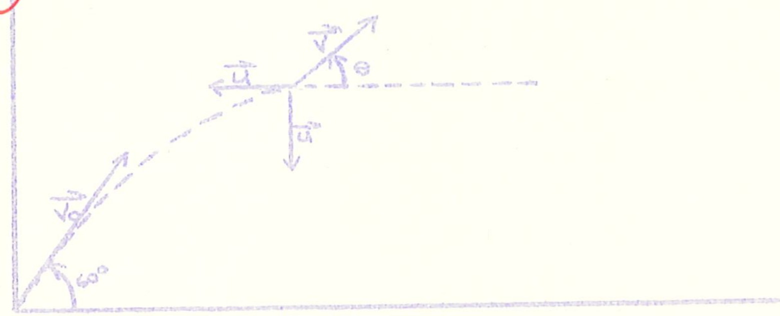
The mass m = 2 kg is held at the point (1,1) by strings anchored at (0,1) and (4,5) as indicated. The whole system is moving horizontally with constant velocity iv.

a) Find the tension in both strings.

b) The whole system is moving horizontally with constant acceleration $\vec{A}$ such that the tension in the string anchored at (0,1) is reduced to zero. Find $\vec{A}$ and the tension in the string anchored at (4,5).

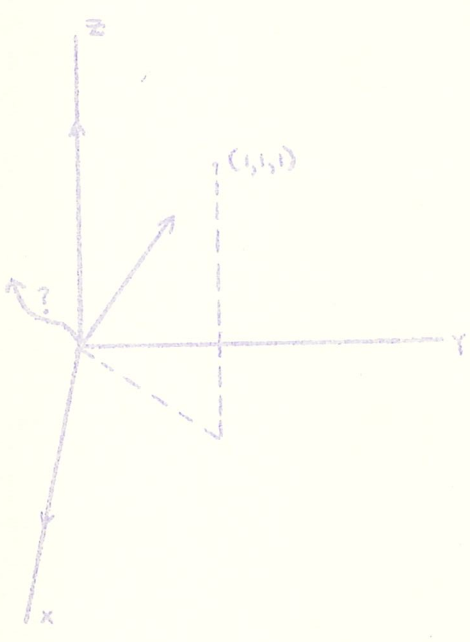
LO-19

Problem 44:



A projectile of mass m has initial speed v_o and a projection angle of 60°. When the projectile reaches its maximum elevation it explodes, dividing into three pieces each with mass m/3. One piece goes horizontally to the left with initial speed $u = 3 v_o$. The second piece goes straight down and has the same initial speed. The third takes off with unknown speed v in unknown direction θ. Find v and θ. v_o is given.

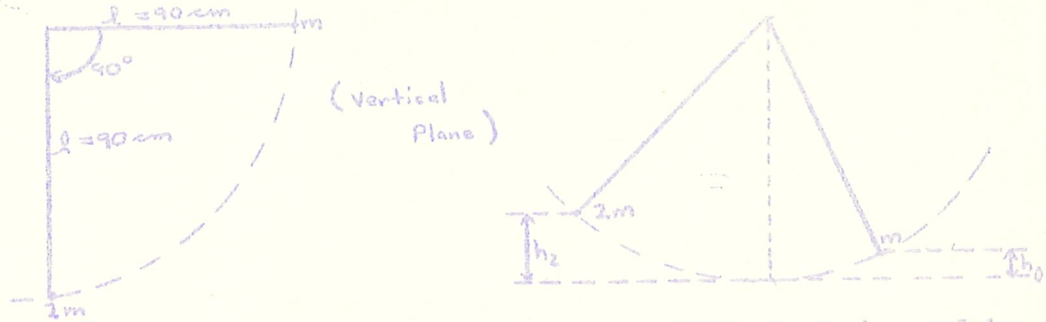
Problem 45:



A bomb, which may be considered at rest in an inertial system in outer space, suddenly explodes. There are a total of 5 fragments. Four of them have masses m/6, where m is the mass of the bomb, and the fifth has mass m/3. The first of the smaller masses remains at rest. The second moves along the x-axis at speed v, the third along the z axis at speed v, and the fourth in the direction (1,1,1) also at speed v.

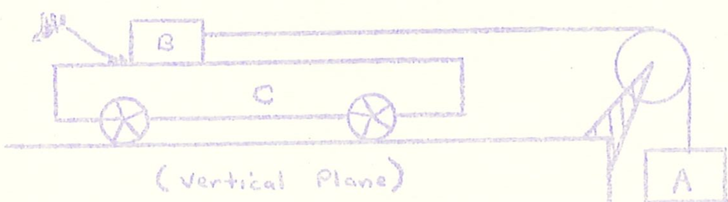
Call the speed of the 5th fragment, which has the mass m/3, u, and calculate the ratio u/v.

Problem #46:



In the first picture, both masses (m and 2m) are attached to strings of length ℓ = 90 cm., and are at rest. When the upper particle is released, it descends and strikes the second particle, making a perfectly elastic collision. The second particle rises to a maximum height h_2, and the first one rebounds and rises to a maximum height h_1. Calculate h_1 and h_2.

Problem 47:

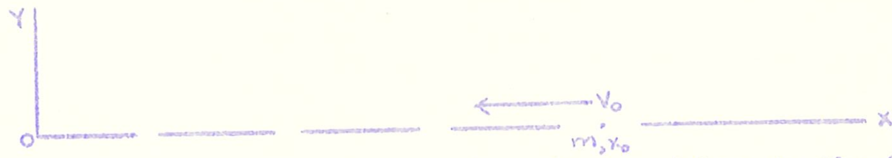


The coefficient of friction, both static and kinetic, between B and C, is μ = .5. There is no friction anywhere else and pulley and the string are massless. The masses A, B, and C, are 1.0, 0.5, and 1.5 kilograms respectively. The system is initially at rest.

a) Find the magnitude of the accelerations of A, B and C. (use g = 9.8 m/sec^2).

b) Determine the total kinetic energy of the system at time t = 1 second. (The system is released at time t = 0.)

Problem 48:



Mass m is constrained to move on the x-axis. It is subject to the force

$$\vec{F} = i\,\frac{c}{x^2}$$

where c is a positive constant. When m is at a distance x_o from the origin, its velocity is $-iv_o$.

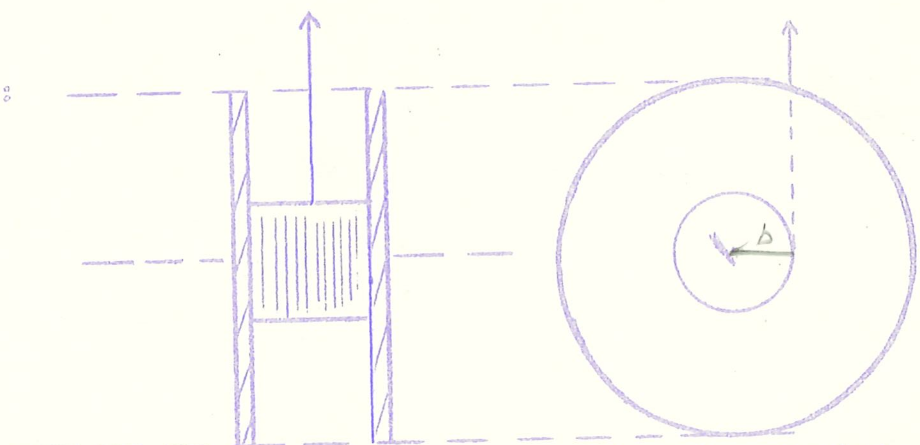
a) Find the potential energy function, U(x), appropriate for this problem. Assume that U is zero for infinite x.

b) Find the speed of the particle as a function of x.

c) Find x_c, the distance of closest approach.

(The answers will involve the parameters c, m, v_o and x_o.)

Problem 49:



$\alpha = \frac{6}{201\,b}$

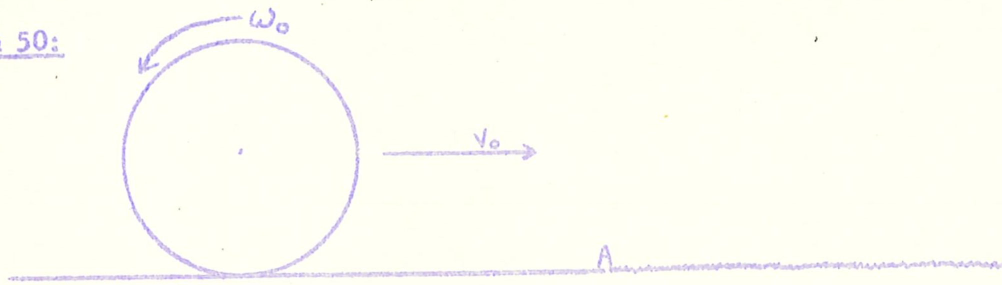
The above are side and end views of a yo-yo; with string attached. The radius of the inner cylinder, about which the string is wound, is b, and the mass of this cylinder is M. The radii of the outer cylinders are 10b and their masses are M each, so that the total mass of the yo-yo is 3M. The free end of the string is held fixed and the yo-yo is released. Find its acceleration.

(The moment of inertia of a cylinder with respect to its axis is $\frac{1}{2}Mb^2$.)

$\frac{1}{2}Mb^2 = \frac{1}{2} I \omega = Mk^2$

$k = b/\sqrt{2}$

Problem 50:

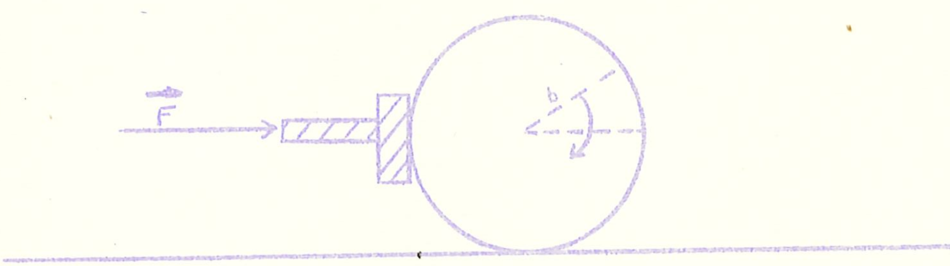


A ring, or hoop, of radius b and mass m, has an angular velocity of ω_o in the direction indicated and a velocity v_o as illustrated. The floor is perfectly smooth as far as point A, after which it is rough, so that after a time the hoop rolls without slipping.

Let K_o and K_f be the initial and final kinetic energy of the hoop, and find a formula $K_o - K_f$, the loss of kinetic energy, in terms of m, b, v_o and ω_o.

(The moment of inertia of a hoop, with respect to an axis perpendicular to the plane of the hoop and through its center of mass is mb^2.)

Problem 51:

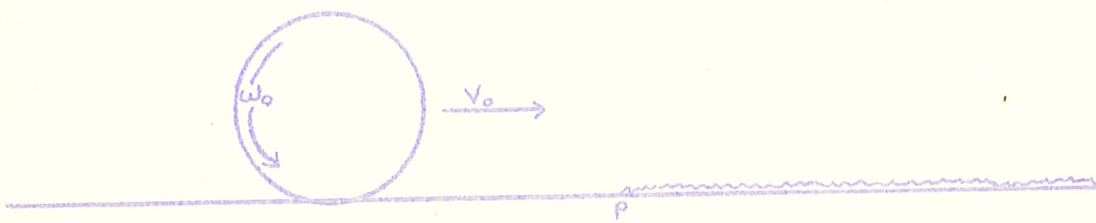


A solid sphere (mass = m, radius b) is rolled along a level floor by means of a piston which exerts a constant horizontal force $\vec{F}$ against the sphere. The coefficient of sliding friction between the sphere and the piston is μ.

a) Calculate the acceleration of the center of mass of the sphere. (The moment of inertia of a solid sphere with respect to axes passing through its center is $2/5\ mb^2$.

b) Calculate the horizontal component of the force exerted by the floor on the sphere.

Problem 52:

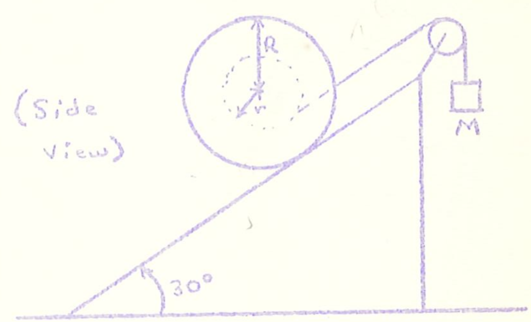


A hoop, radius b, and mass m, is given a "back-spin", $\hat{k}\omega_0$, and launched to the right, the velocity of the center of mass being $\hat{i}V_0$. The floor is smooth at the launching point, but starting at P it becomes rough so that the hoop eventually stops skidding and starts moving to the left. By the time it gets back to P, the hoop is rolling without skidding. What is now the velocity of the center of mass?

It is given that $\omega_0 = 40$ rad/sec, $b = .40$ meters, and $V_0 = 10$ meters/sec, but m is not known.

Problem 53:

A yo-yo shaped object moves on an inclined plane under the action of a string one end of which is wound around the spindle of the object from the lower side (see figure). The other end of the string passes over a pulley of negligible mass and is attached to a weight of mass M. The object is made up of two heavy discs, each of radius R connected by a spindle of radius r. The mass of the spindle is negligible and the total mass of the object is m. Determine the acceleration of the yo-yo shaped object in terms of the acceleration of gravity, g, using the following relations between r and R, m and M:



$$R = 2r \qquad M = 3m$$

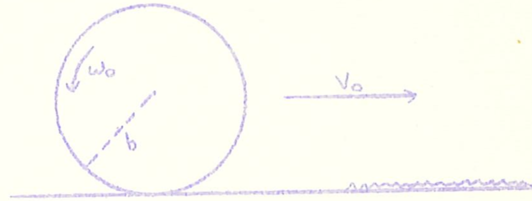
Assume that there is no slipping.

Problem 54:

A sphere (moment of inertia with respect to center of gravity $= \frac{2}{5} mb^2$) is at rest on a perfectly smooth floor. A horizontal impulse, $\vec{F}\Delta t$, is delivered at a point distant h above the floor as indicated. If h is arbitrarily selected, the sphere will partly roll and partly slide. But it is possible to select an h such that the ball will roll without sliding. Find this particular h as a function of b.

Problem 55:

A solid cylinder (moment of inertia with respect to center of mass = $\frac{1}{2} mb^2$) is given a "back-spin" ω_0 and launched with forward speed V_0 on a smooth floor, as indicated. It comes to a rough surface and soon all motion, both translational and rotational, ceases. Find a formula for ω_0 in terms of V_0 and b.

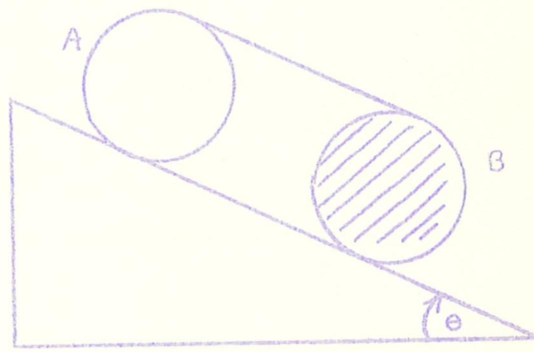


Problem 56:

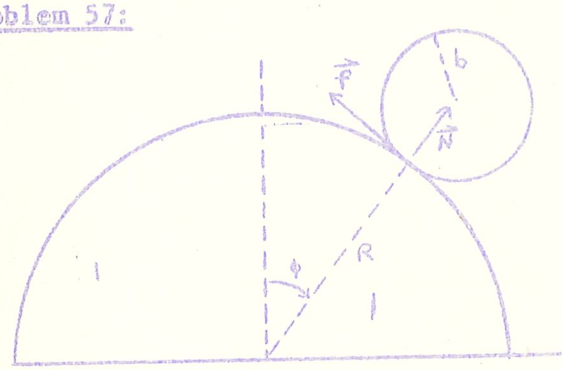
A is a ring of mass m and radius b, while B is a disc of the same mass and radius. A string is wound about A and B, as indicated.

Assuming that no slipping occurs, find a formula for the tension in the string in terms of m, g and θ.

(With respect to center of mass, the moment of inertia of a ring is mb^2 and that of a disc is $\frac{1}{2} mb^2$.)



Problem 57:



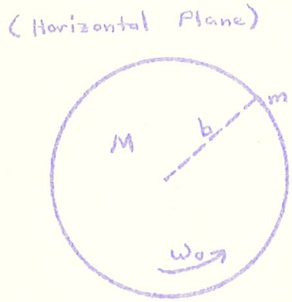
A cylinder of radius b and mass m rolls on the surface of another cylinder, radius R, as indicated. Find

a) the magnitude of $\vec{f}$, and

b) the magnitude of $\vec{N}$,

as functions of φ.
At time t = 0, $\varphi = 0$, and $\dot{\varphi} = 0$.

Problem 58:



A disc is rotating freely about a vertical axis with angular velocity ω_0. A bug, having mass m, is located on the rim of the disc. The bug now crawls inward to the center of the disc. The data are:

M = mass of disc = .100 kg
b = radius of disc = .400 meters
ω_0 = 10 radians/sec
m = mass of bug = .020 kg.

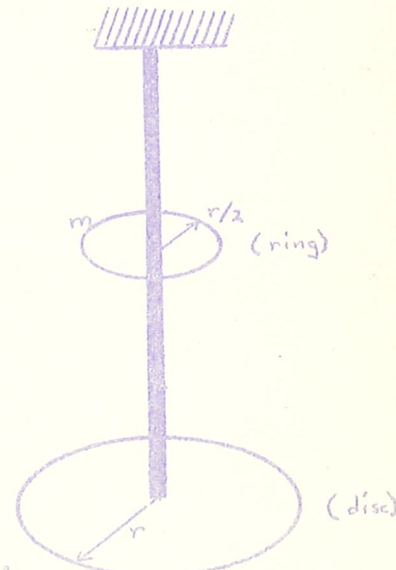
a) Find the angular speed of the disc when the bug has completed his journey.

b) Find the difference between initial and final kinetic energies of the system. (Note: the bug is to be treated as a mass point, so that he has no final kinetic energy.)

c) Explain, with a few well-chosen words, the reason for the difference in energy.

Problem 59:

A uniform horizontal disc of mass m and radius r is suspended at its center by a vertical torsion rod and performs simple harmonic angular oscillations. A thin ring of mass m (same mass as disc) and radius ½r (one-half radius of disc) is dropped onto the disc and almost immediately sticks to the disc. Assume that the ring and disc are concentric and that the ring has no angular motion when dropped.



a) For the case where the ring is dropped onto the disc just when the disc is at its maximum angle of displacement from the equilibrium position, find:

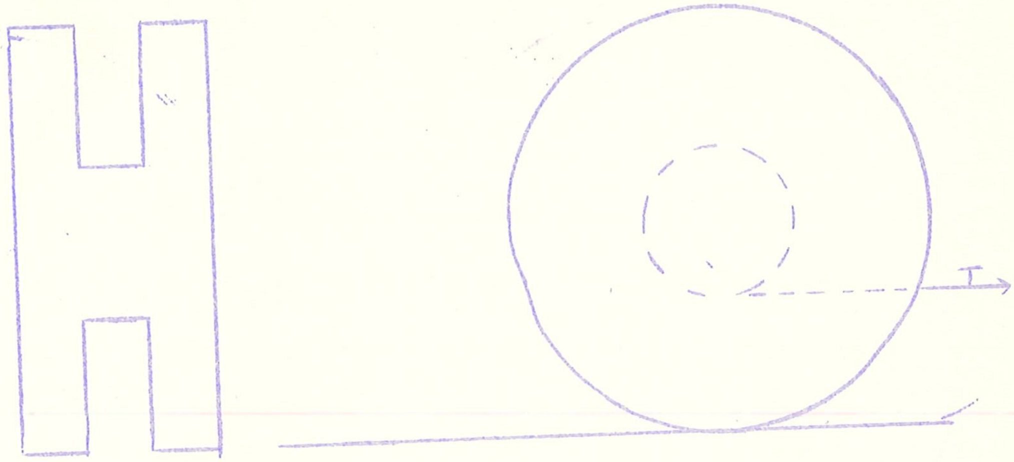
1) the new period, T, of the motion in terms of the original period, T_0;

2) the new angular amplitude, A, of the motion in terms of the original amplitude, A_0;

3) the new total energy, E, of the motion in terms of the original total energy, E_0.

b) For the case where the ring is dropped onto the disc just as the disc revolves through the equilibrium position, find 1), 2) and 3) of a.

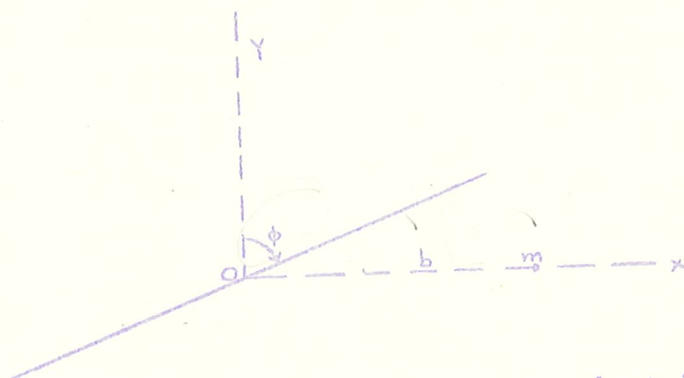
Problem 60:



The yo-yo consists of three concentric solid cylinders as indicated. The radius of the inner cylinder is 1 cm and its mass is 200 grams. The mass of each one of the outer cylinders is 400 grams and their radius is 3 cm.

This yo-yo is being pulled along a horizontal surface, as indicated. It rolls without slipping, and the string is horizontal. The tension in the string is 3000 dynes. Calculate the acceleration of the center of mass of the yo-yo, and the magnitude of the friction force exerted by the horizontal surface on the yo-yo.

Problem 61:



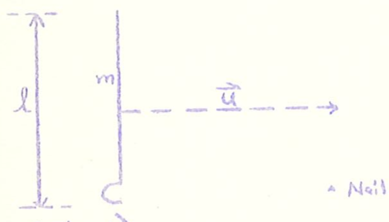
A uniform stick, of length 2b and mass m is rotating about its center on a frictionless horizontal plane with angular speed $\dot{\phi} = \omega$. A particle, also of mass m, is at rest at x = b. The end of the stick strikes the particle, making a perfectly inelastic collision.

a) Find the angular speed of the system after collision and the percentage of the original mechanical energy which was dissipated.

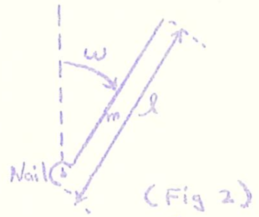
(Note: There is no pin at O, so that the system is free to rotate about any point.)

b) Do the problem in case the collision is perfectly elastic. The questions then are about the velocity, v, of the particle m and the new angular speed of the stick.

Problem 62:



(Fig 1)



(Fig 2)

In Fig. 1 we look down on a uniform rigid rod of length ℓ and mass m sliding on a horizontal frictionless table with velocity $\vec{u}$ and no rotation. This rod has a very small hook (negligible mass and dimensions) which hooks on to a nail in the table, forming a pivot about which the rod will rotate, as indicated in Fig. 2.

Calculate:

a) The angular speed, ω, of the rod.

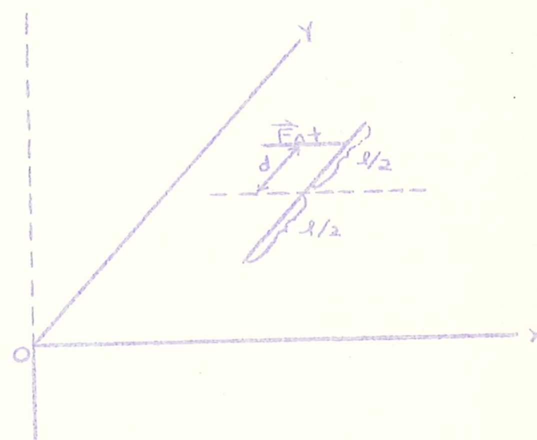
b) The fraction of the initial energy of the rod which is transformed into non-mechanical forms as a result of the collision of the hook with the nail.

Problem 63:

A uniform thin rod of length 10.0 cm lies at rest on a frictionless horizontal plane. It is acted upon by a force $\vec{F}$ for a very short time. The force $\vec{F}$ is in the horizontal plane, perpendicular to the rod, and it acts at a distance d from the center of mass.

It is then found that the translational kinetic energy of the center of mass is equal to one-half of the total kinetic energy.

Calculate the distance d.



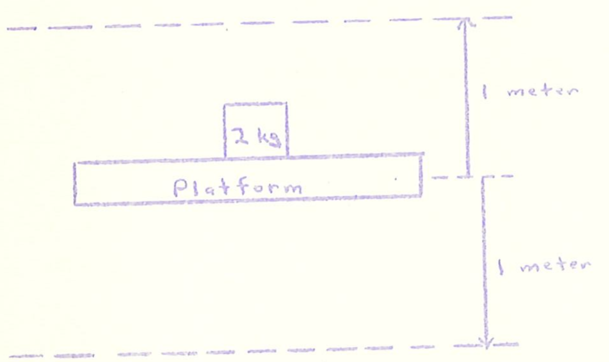
Problem 64:

The equation of motion of the indicated platform is $\ddot{X} + \omega^2 X = 0$ where $\omega = 2\pi f$ and f = frequency. The coefficient of static friction between the platform and the mass m is .2.

a) If the amplitude of the motion is 1 cm. find the greatest frequency such that m does not slip.

b) If the frequency is 2 cycles per second, what is the greatest amplitude for which m does not slip?

Problem 65:



A 2.0 kg. mass is on a horizontal platform which is executing simple harmonic motion in the vertical direction with amplitude 1.0 meters. The frequency of the motion is such that the platform exerts no force on the mass when it is at the top of its cycle, but yet is in contact with it. Find the maximum speed of the platform.

Problem 66:

A 50 gm mass executes simple harmonic motion with a period of 6.0 seconds. It passes through the equilibrium point with a speed of 22 cm/sec.

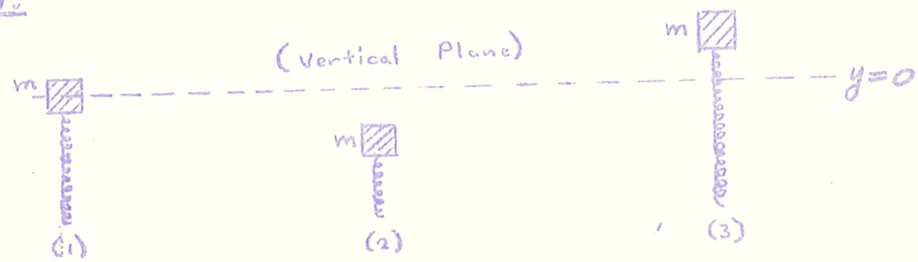
a) What is the amplitude of the motion?

b) If the mass is at its maximum positive displacement when t = 0, where is it located when t = 4.0 sec.?

c) What is the kinetic energy when t = 4.0 sec.?

d) What is the total energy when t = 4.0 sec.?

Problem 67:



In figure (1), mass m, resting on top of a light spring, has compressed the spring until it is 4 cm. shorter than its natural length. In this position the system is in equilibrium. In picture (2), the mass has been pushed down an additional centimeter. In picture (3), the mass has been released, from the position it occupied in figure (2), and is now oscillating about the equilibrium position, indicated by $y = 0$. It is understood that the mass and spring can move only vertically, and that Hooke's law is obeyed.

Find the period of the motion and also the maximum speed attained by mass m.

Problem 68:

A uniform slim rod is 4.0 ft. in length. Find the points on this rod through which a horizontal axis may be passed so that the period of small oscillations about the vertical will be 2.0 sec. (The moment of inertia of a slim rod of mass m about its center is $\frac{1}{12}\, mL^2$ where L is its total length.)

Problem 69:

A block of mass M is sliding east and west on the frictionless platform under the action of the spring, whose constant is k. The amplitude of the motion is A.

a) Just as the block passes through equilibrium and heading east, it is hit by the bullet whose mass is m and whose velocity is u. The bullet imbeds itself in the block. Find a formula for the new amplitude, A', of the ensuing motion.

b) The bullet, instead of hitting the block at the position described in part a), strikes it when it is at the extreme of its westward motion. Find a formula for the new amplitude, A'', of the ensuing motion.

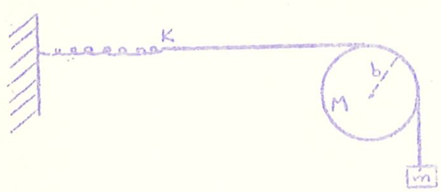
c) In case you find that A'' is not equal to A', explain the difference.

Problem 70:

A uniform rod, mass m and length L, is smoothly hinged at H. It is held in a horizontal position by a spring (constant k) which is attached at a distance s from H. The free end of the rod is pushed up a very short distance (much less than L, thus compressing the spring slightly, and then released. The ensuing motion is indicated in the second picture, where the maximum value of θ is to be taken such that cos θ ≅ 1. Find the period of the motion.

The moment of inertia of the stick with respect to H is $1/3\ mL^2$.

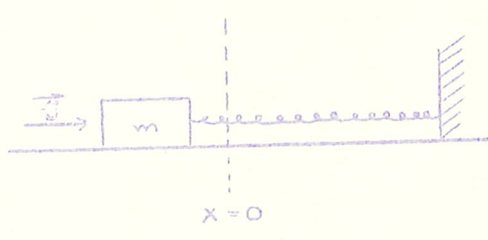
Problem 71:



A spring, constant K, is attached to a cord when runs over a pulley (moment of inertia about center of mass = $\frac{1}{2}Mb^2$) to a mass m, as indicated. Assuming that there is no slipping between the cord and the pulley, find the period of the oscillation of the mass m.

Problem 72:

Mass m is engaged in a simple harmonic motion on the frictionless platform. The motion is described by $X = X_1 \sin \omega t$. When $X = -X_1$ an impulse $\vec{J}$ is delivered horizontally to the oscillator, in consequence of which the amplitude of oscillation is increased to X_2.



Given that m = 100 grams, ω = 10 rad/sec, X_1 = 3 cm., X_2 = 5 cm.

Find $\vec{J}$.

Problem 73:

A body of mass 100 gm hangs from a long spiral spring. When pulled down 10 cm below its equilibrium position and released, it vibrates with a period of 2.0 sec.

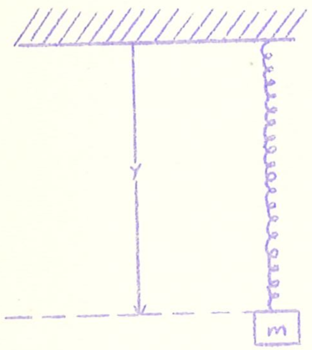
a) What is its velocity as it passes through the equilibrium position?

b) What is its acceleration when it is 5.0 cm above the equilibrium position?

c) When it is moving upward how long a time is required for it to move from a point 5.0 cm below its equilibrium position to a point 5.0 cm above it?

d) How much will the spring shorten if the body is removed?

Problem 74:



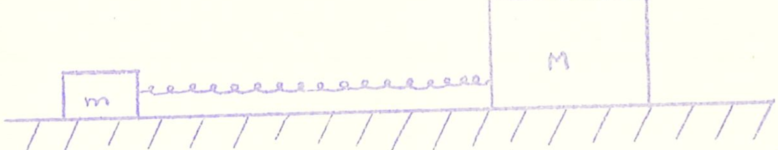
The diagram indicates a massless frictionless spring from which a mass m = .10 kg is suspended. The natural (unstretched) length of the spring is 4.0 meters, and the spring constant is 10 nts/meter.

At time t = 0, y = 4.0 meters, and the velocity is 2.0 m/sec downward.

a) What is the maximum value of y?

b) What is the highest speed that the mass will attain in its motion?

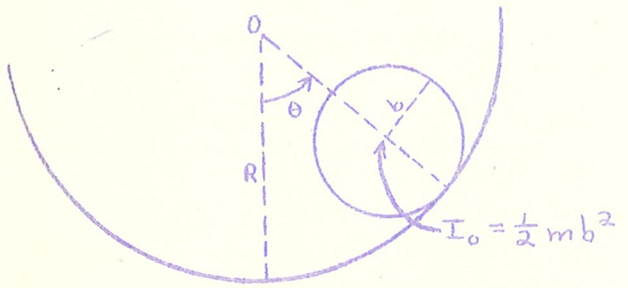
Problem 75:



Two bodies of mass m and M are free to slide without friction along a straight line on a horizontal surface. They are connected by a spring of negligible mass and spring constant k. The spring is compressed from its relaxed length and then the bodies are released in such a way that the center of mass of the system stays at rest.

Derive an expression for the period, T, of the resulting motion.

Problem 76:



The diagram indicates a stationary cylindrical surface of radius R inside of which rolls a smaller cylinder of radius b and mass m.

a) Write the equation of motion of the small cylinder.

b) Prove that, for small angles θ, the motion is simple harmonic, and find the period.