This is an open book, note and Library exercise, but work is to be done independently. Remember, results about vector spaces are fair game.

To prove $A = B$ as sets, show <u>both</u> $A \subset B$ and $B \subset A$.

To prove $A \subset B$ show <u>either</u> (a) everything in A is in B (in symbols, $x \in A \Rightarrow x \in B$) or (b) everything not in B is not in A (in symbols, $x \notin B \Rightarrow x \notin A$).

<u>Definition.</u> A linear combination $\sum_1^k c_i x_i$, of a finite set of vectors $\{x_1, \cdots, x_k\}$, is an <u>affine</u> combination if $\sum_1^k c_i = 1$.

<u>Definition.</u> A subset P of a vector space is a <u>flat</u> if P contains all the affine combinations of the non-empty finite subsets of P.

<u>Theorem 1.</u> The following are equivalent:

(1) P is a flat.

(2) For each $p \in P$, the set $\{x-p: x \in P\}$ is a vector subspace.

(3) P is the translate of a unique vector subspace. (i.e., there is a vector w and a <u>unique</u> subspace V, such that $P = w + V = \{w+x: x \in V\}$.)

<u>Theorem 2.</u> The flats of $\mathbb{R}^3$ are either:

(1) a single point,

(2) a straight line,

(3) a plane,

or

(4) the whole space.

Theorem 3. A flat P is a subspace, if and only if, the origin belongs to P.

Theorem 4. The intersection of a set of flats (possible infinite) is either empty or a flat.

Theorem 5. Suppose we have an linear system of equations $AX = B$, where A is an $n \times n$ matrix, B is an n column vector and X is the unknown n column vector. Show that the solution space of $AX = B$ (i.e., $\{X: AX = B\}$) is a flat or empty. Furthermore show that this flat is a subspace if and only if B is the origin.