ENGINEERING 5

Final Home Quiz (No. 7) - January 17, 1967

Due Date: 5:00 P.M., January 26, 1967, (turn in your source deck and plot)
(N.B.: January 26, 1967 is the last day of this semester that the computer laboratory will be open with the laboratory assistants present.)

SOLUTION OF A SET OF ORDINARY DIFFERENTIAL EQUATIONS WITH DIGITAL COMPUTER

STATEMENT OF PHYSICAL PROBLEM:

A Biological Epidemic Problem: In a total population of n individuals there are, at any time t, $y(t)$ infectious carriers of a contagious disease, $x(t)$ members of the population who are susceptible to the disease, and $z(t)$ individuals who are recovered and immune. It is clear that $x(t) + y(t) + z(t) = n$ for all $t \geq 0$. The following system of differential equations, known as Kermack-McKendrick equations, characterize the epidemic.

$$\begin{aligned} \dot{x}(t) &= -\lambda x(t)y(t) \\ \dot{y}(t) &= \lambda x(t)y(t)-\mu y(t) \qquad (1) \\ \dot{z}(t) &= \mu y(t) \end{aligned}$$

If t is the time in days, $\lambda = 0.001$, $\mu = \frac{1}{14}$, $n = 1000$,

$x(0) = 900$, $y(0) = 10$, and $z(0) = 90$.

STATEMENT OF COMPUTER PROBLEM:

Write a NEWTRAN program which will solve the set of differential equation (1) for

a) the number of susceptibles, $x(t)$;

b) the number of infectives, $y(t)$;

c) the number of recovered and immune, $z(t)$; and

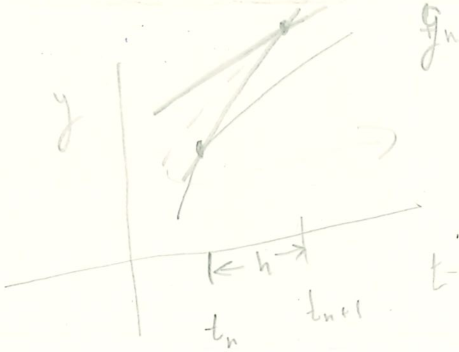
d) the epidemic curve - $\dot{x}(t)$, i.e. the rate at which new disease occur.

Your program must have the following features:

1. It must use iterative form of improved Euler's Method with number of iterations being 3.

2. It must use interval of computation h = 0.1 days but print out the results for every day and up to 15 days.

3. Plot on a single graph paper the variables x(t), y(t), z(t) and $-\dot{x}(t)$ versus time, t.



$y_n + h f(t_n, y_n)$

$\frac{1}{2}\left[f(t_n, y_n) + f(t_{n+1}, \bar{y}_n)\right]$

$$y_{n+1} = y_n + \frac{h}{2}\left[f(t_n, y_n) + f(t_{n+1}, \bar{y}_{n+1})\right]$$

$$\bar{y}_{n+1} = y_n + h f(t_n, y_n)$$

$\dot{y}_n = f(t_n, y_n)$

$\bar{y}_{n+1} = = y_n + h\dot{y}$

$f(t_{n+1}, \bar{y}_{n+1})$

$y_{n+1} = y_n + h\bar{\Phi}$

$\bar{\Phi} = \frac{1}{2}\left[\dot{y}_n + f(t_{n+1}, \bar{y}_{n+1})\right]$

2/17/67
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