

Graph Theory  
Summer 1991

Do any four problems

1. Define  $\text{rad}(G)$ , the radius of  $G$  and  $\text{diam}(G)$ , the diameter of  $G$ . Prove  $\text{rad}(G) \leq \text{diam}(G) \leq 2 \text{rad}(G)$ . Show both equalities are possible.
2. Prove every planar graph has a vertex of degree 5 or less.
3. Define  $\beta(G)$ , the independence number and  $\chi(G)$ , the chromatic number. Show  $\chi(G) \beta(G) \geq p$  and  $\chi(G) \chi(G^c) \geq p$ . ( $G^c$  is the complement of  $G$ .)
4. Prove that the Ramsey number  $r(3,3) = 6$ . (In particular, show  $r(3,3) < 6$  is false.)
5. Prove every tournament has a Hamiltonian path.
6. For each  $p$ , show there is a unique disconnected graph  $G$  with  $p$  vertices and the most edges [of all such disconnected graphs].