Graph Theory

MAT 5932 — Instructor: Bellenot
Fall 1990 — Office: 002-B Love
Office Hours: MWF 12:45 - 1:15 or by appointment

Text: Chartrand & Lesniak, Graphs and Digraphs, 2nd ed., Wadsworth & Brooks/Cole, 1986.

Coverage: Parts of most the chapters (as time allows).

Grades: The easy-going 85%, 70%, 55% and 40% cut-offs.

Final: This is "in class" and "closed book" test worth **30%** of your grade. The final will be given Monday 10 December 1990, 10am - 12noon.

Homework: The remaining **70%** of your grade will be determined by homework problems. Some (but perhaps not all) homework problems will be graded on a ten point scale. Only the top 90% of your graded homework is used to compute your homework average. Generally three homework problems will be assigned each Monday and due the following Monday.

Homework Rules:

Must be your **OWN** work!
Must be neat and in clear English.
Must be on time -- late homework is not accepted.
Must be on 8 1/2 by 11 paper.
Must be written in ink.
Must use only one side of each page.
Multiple pages must be stapled together.

Graph Theory Midterm Spring 1989

All problems are worth 10 points.

1. A complete 3-ary tree T has 101 leaves. Find best possible upper and lower bounds on the height of T. Your answers should be simplified integers.

2. A connected graph with no vertices of odd degree has no bridges.

3. Show any two longest paths in a connected graph have a vertex in common.

4. If G is a tree, then G has at least $\Delta(G)$-vertices of degree one.

Suppose C_1 and C_2 are cycles of a graph G. The cycles C_1 and C_2 are said to be *distinct* if $E(C_1) \neq E(C_2)$. The cycles C_1 and C_2 are said to be *edge-disjoint* if $E(C_1) \cap E(C_2) = \emptyset$. The cycles C_1 and C_2 are said to be *unlinked* if $V(C_1) \cap V(C_2)$ has at most one vertex. The cycles C_1 and C_2 are said to be *disjoint* if $V(C_1) \cap V(C_2) = \emptyset$.

5. Give an example of two edge-disjoint cycles which are not unlinked. Give an example of two unlinked cycles which are not disjoint.

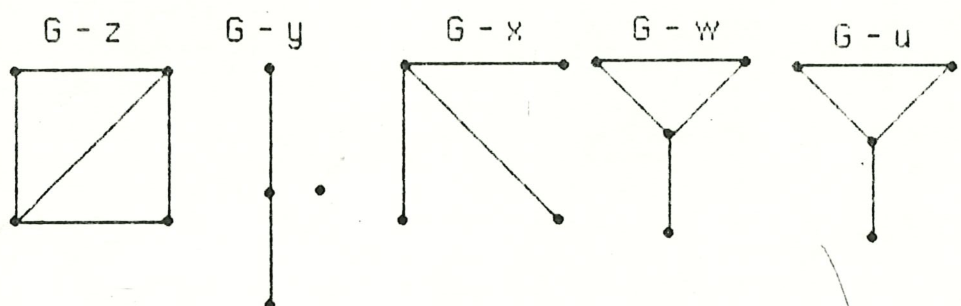
6. If every two cycles in G are edge-disjoint, then every two cycles in G are unlinked.

7. If every two distinct cycles in G are disjoint, G is connected, and G has n distinct cycles, then $q = p - 1 + n$.

Graph Theory Test Spring 1987

Problems 2 & 3 are worth 20 points, all others worth 15 points.

1. Let $G = K_{n,m}$ where $1 \le n \le m$. Find the constants: $\omega, \nu, \varepsilon, \delta, \Delta, \kappa, \kappa'$. For what values of m and n does G have an Euler ~~cycle~~ tour? For what values of m and n does G have a Hamilton cycle?
2. Give examples: A connected 3-regular simple graph J with at least two cut edges. An edge cut of K which isn't a bond. A graph G which is isomorphic to its complement G^c. A simple connected graph H whose diameter and radius are equal.
3. Give counter-examples: If v is a cut vertex of a graph H, then {v} is a vertex cut of H. A vertex v in a cycle of a simple graph J isn't a cut vertex of J. A graph G with two distinct walks from x to y has a cycle. A matching M of K, which isn't properly contained in any other matching of K is a maximal matching of K.
4. Prove: T is a tree, if and only if, T is acyclic and for any edge e not in T, T + e has a cycle.
5. Prove: If G is k-regular, G has 2k vertices and the length of any cycle in G is ≥ 4, then G is isomorphic to $K_{k,k}$.
6. The graph G has five vertices z, y, x, w and u. Below are drawings of the five subgraphs of the form G - v for v a vertex of G. Your job is to reconstruct G and label the vertices in G and the subgraphs below.



Graph Theory
Prelim
Fall 1989

Complete four of the following problems:

1. A graph G is a tree if and only if G is connected and $p = q + 1$.

2. For the graph G, define both $\beta(G)$, the (vertex) independence number, and $\alpha(G)$, the (vertex) covering number. Prove: if G is a graph, then $\alpha(G) + \beta(G) = p$.

3. Every tournament (complete digraph) has a hamiltonian path.

4. A connected graph with no vertices of odd degree has no bridges.

5. The graph $G \otimes H$ has vertex set $\{(g, h): g \text{ in } V(G), h \text{ in } V(H)\}$ and the vertices (g_1, h_1) and (g_2, h_2) are incident iff either $g_1 = g_2$ and h_1h_2 is in $E(H)$ or $h_1 = h_2$ and g_1g_2 is in $E(G)$. The n-cube is inductively defined by Q_1 is K_2 and Q_{n+1} is $Q_n \otimes K_2$. Prove Q_n is n-regular bipartite graph of order 2^n and size $n2^{n-1}$.

Graph Theory Topics

The list of topics below includes more than could be thaught in one semester. The actual choice of topics would depend on the text and the depth of coverage. Almost any one semester course out of any textbook would cover a (large) subset of the topics below.

1. An introduction to the terminology (degree, isomorphism, loops, multigraphs, digraphs, etc.) and the standard examples of graphs (regular, bipartite, complete, cycle, etc.).

2. Connectivity: paths, trails, walks; Dijkstra's Algorithm; cycles and circuits; trees and forests; spanning trees with algorithms; cut-edges, cut-vertices and blocks.

3. Tours: Euler tours with algorithm; The Chinese Postman Problem; Hamiltonian tours; tournaments and Hamiltonian tournaments.

4. Graph colorings: vertex coloring; independence and cliques; edge coloring; The 4 Color Theorem.

5. Planar graphs: Euler's formula; Kuratowski's Theorem; genus and crossing numbers; algorithms.

6. Extremal graph theory: Ramsey's Theorem; random graphs.

7. More on connectivity: Matching and Hall's Theorem; n-connected and n-edge-connected graphs; Menger's Theorem; Networks and the Min-Cut Max-Flow Theorem. Decomposition and factors.

8. Enumeration of graphs: Pólya's Theorem.

Graph Theory

MAT 5??? Instructor: Bellenot
Spring 1989 Office: 218 Love
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