graph theory

(mat 5932)

spring 1987

instructor: Bellenot

office 218 LOV; office hours MWF 12:30-1:20 & by appointment.

Text: Bondy & Murty, Graph Theory and Applications.
Coverage: as much as time permits, more or less in the text's order.

Tests: There is only one "in class & closed book" test. It is tentatively scheduled for 11 Mar. and will be worth 20% of your grade. The remaining 80% of your grade is based on "homework".

Grades: The relaxed 87.5%, 75%, 62.5%, 50% cut-offs.

Homework:

1. It must be your **OWN** work!
2. Graded on a 0-10 basis on your reasoning, your ability to express your reasoning, neatness and your English.
3. Failure to follow the rules below cost a point a piece:
   - A. Must be on 8-1/2 by 11 paper.
   - B. Must be written in ink (or typed).
   - C. Must use only one side of each page.
   - D. If they require more than one page, then the pages must be stapled or paper-clipped together.
4. Late work isn't accepted.
5. Your homework average is the average of your best 4/5-ths.

Due friday 9 Jan: 1.2.3
Due monday 12 Jan: 1.5.4
Due wednesday 14 Jan:1.6.5
Due friday 16 Jan: 1.7.2

Graph Theory Test Spring 1987

Problems 2 & 3 are worth 20 points, all others worth 15 points.

1. Let $G = K_{n,m}$ where $1 \le n \le m$. Find the constants: $\omega, \nu, \varepsilon, \delta, \Delta, \kappa, \kappa'$. For what values of m and n does G have an Euler cycle? For what values of m and n does G have a Hamilton cycle?

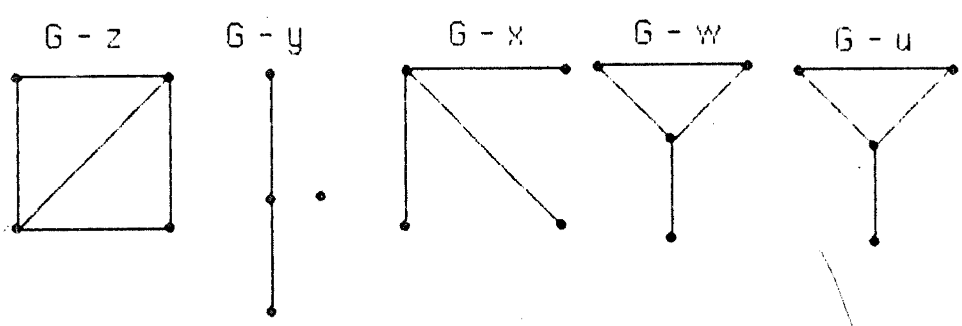
2. Give examples: A connected 3-regular simple graph J with at least two cut edges. An edge cut of K which isn't a bond. A graph G which is isomorphic to its complement G^c. A simple connected graph H whose diameter and radius are equal.

3. Give counter-examples: If v is a cut vertex of a graph H, then {v} is a vertex cut of H. A vertex v in a cycle of a simple graph J isn't a cut vertex of J. A graph G with two distinct walks from x to y has a cycle. A matching M of K, which isn't properly contained in any other matching of K is a maximal matching of K.

4 Prove: T is a tree, if and only if, T is acyclic and for any edge e not in T, T + e has a cycle.

5. Prove: If G is k-regular, G has 2k vertices and the length of any cycle in G is ≥ 4, then G is isomorphic to $K_{k,k}$.

6. The graph G has five vertices z, y, x, w and u. Below are drawings of the five subgraphs of the form G – v for v a vertex of G. Your job is to reconstruct G and label the vertices in G and the subgraphs below.



Graph Theory
Final Problem Set
Spring 1987
Due: 28 April 1987 @ high noon

The Reconstruction Problem

Given the graphs G_i for $i = 1$ to n, construct a graph G with vertices v_i for $i = 1$ to n, so that G_i is isomorphic to $G - v_i$. The reconstruction problem asks if G is uniquely determined (when $n > 2$). This problem is unsolved in general but many special cases are known. (For example the last problem on the test was a special case of this problem.)

1. A. Give two non-isomorphic simple graphs with $n = 2$ and and the same subgraphs G_i.

B. Show how to compute $\varepsilon(G)$ from the $\varepsilon(G_i)$'s.

C. Show how to compute the degrees of the vertices v_i in G.

D. Show how to compute $\omega(G)$ from the $\omega(G_i)$'s.

2. A property P is said to be *recognizable* if for each graph G with $n > 2$ vertices, it is possible to determine if G has property P just from the subgraphs G_i. Show the following properties are recognizable:

A. G is k-regular.

B. G has a isolated vertex.

C. G is simple.

D. G is connected.

3. Show these are more recognizable properties:

E. G is a tree.

F. G is a block.

G. G is eulerian

H. G is bipartite

4. Prove: If G is disconnected and G has at least 3 vertices, then G can be reconstructed.

5. Prove: If the simple graph G is eulerian with at least 3 vertices, then G can be reconstructed.

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