FUN_{ctional} Analysis

MAA 5506 Functional Analysis I
Fall 1989
Instructor: Bellenot
Office: 218 Love
Office Hours: MWF 12:45 - 1:15 & by appointment

Text: Carl L. DeVito, Functional Analysis, Academic Press 1978.

Coverage: The entire text with additions as time allows.

Grades: The easy-going 85%, 70%, 60% and 50% cut-offs.

Midterm: This test is "in class" and "closed book" worth **30%** of your grade. Tentatively scheduled for Wednesday 15 November1989.

Homework: The remaining **70%** of your grade will be determined by homework problems. Some (but perhaps not all) homework problems will be graded on a ten point scale. Only the top 90% of your graded homework is used to compute your homework average.

Homework Rules:

Must be your **OWN** work!
Must be neat and in clear English.
Must be on time -- late homework is not accepted.
Must be on 8 1/2 by 11 paper.
Must be written in ink.
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Multiple pages must be stapled together.

Assignment 1: Read the first two sections by Friday 1 September.
For Wednesday 6 September do Problems 1 & 2 in Section 1 and Problems 1, 2 & 3 in Section 2

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1. Let D : $C^1[0,1]$ -> $C[0,1]$ be the differentiation operator (that is Df = f'). Show that the graph of D is a closed subset of Y = $C[0,1] \times C[0,1]$ (Note Y $\neq C^1[0,1] \times C[0,1]$).

2. Let X be a normed space. If $X^* \supset (f_n)$ has f as a $\sigma(X^*, X)$-cluster point, then $||f|| \leq \liminf ||f_n||$.

3. Let X be a LCS space with dual X^*. For X $\supset$ A, define
$A^\circ = \{x^*$ in X^*: For all x in A, $|x^*(x)| \leq 1\}$. Similarly for $X^* \supset B$, define
$B^\circ = \{x$ in X: For all x^* in B, $|x^*(x)| \leq 1\}$. Show if A is a balanced convex subset of X, then $A^{\circ\circ}$ is the $\sigma(X, X^*)$-closure of A in X.

4. A LCS X [t] (with dual X^*) is said to be *barrelled* if each $\sigma(X, X^*)$-closed balanced convex absorbing set is a t-neighborhood of zero. Show the following version of the Banach-Steinhaus Theorem is true for barrelled spaces: If $X^* \supset (f_n)$ is point-wise bounded on X, then (f_n) is t-equicontinuous on X (that is if D is the ball in the scalar field K, then $\cap f_n^{-1}(D)$ is a neighborhood of zero).

5. Prove that a Banach space is barrelled.

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Functional Analysis Assignment

A. Prove that all continuous linear functionals $F: L_p \to K$ are identically zero for $0 < p < 1$.

B. If X is a norm space and $x_n \to x$ weakly (in $\sigma(X, X^*)$) then there is a sequence y_m, each a convex combination of the x_n, so that y_m converges to x in norm.

C. If X is a norm space with dual X^* and we consider X as a subspace of $X^{*\#}$, then the completion of X is the set of functionals f in $X^{*\#}$ which are $\sigma(X^*, X)$-continuous on $B^* = \{ x^* \text{ in } X^*: ||x^*|| \leq 1\}$.

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FUNctional Analysis

This Fall the good doctor Bellenot is again offering MAA 5506 (better known as Functional Analysis I) MWF 1:25-2:15 104 LOVE (ref# 26533).

The text for the fall semester will be DeVito, Functional Analysis, Academic Press 1978. DeVito's book is a short (about 150 pages) elementary introduction to functional analysis starting with normed spaces.

Topics include Compact linear operators; Open Mapping - Closed Graph Theorems; Hahn Banach Theorem; Banach Steinhaus (Principle of Uniform Boundedness) Theorem; Weak Topologies; Krien Milman Theorem; Eberlein Smulian Theorem; and Applications.

14 November 1988

TO: CurCom
FROM:Bellenot

It seems to be a good time to try to formalize a course which has been taught as a topics class a couple of times. Namely, GRAPH THEORY (MAD 5xyz?) which I taught in Spring 1987 and I will again teach in Spring 1989.

CATALOG STUFF

Prereq's: Graduate standing. Topics: Graphs and digraphs, trees and connectivity, Euler and Hamilton tours, colorings, matchings, planarity, and Ramsey's Theorem. Applications. A proof oriented class which assumes no previous exposure to graph theory, but that the students have some mathematical maturity.

Possible Text: Chartrand and Lesniak "Graphs & Digraphs" 2nd ed.

Thank you for your consideration.