MATH 534 FINAL

1. IF $T: X \to X$ is cont linear operator on the B-space X define the spectrum of T and the point spectrum of T.

2. What does it mean for an operator $T: l_2 \to l_2$ to be self-adjoint?

3. Define $X^{\perp}$ where $X \subseteq$ B-space Y and X^{T} where $X \subseteq$ B-space Y^*.

4. State the principle of uniform boundedness.

5. Define an Orthonormal basis for a Hilbert space?

6. Define a Hamel basis for a Vector space?

7. What is a Basis for a Banach space?

8. Define an isometry onto.

9. If $\lambda = (\lambda_n)$ what does $M_\lambda : l_2 \to l_2$ do to the sequence $\xi = (\xi_n)$? [what is $M_\lambda(\xi)$?]

10. Define a Precompact set in the norm space X.

Problems

20. Show $l_2 \times l_2$ is isomorphic to l_2.

11. Show $T: l_1 \to c_0$ given by $T(\{\xi_n\}) = \{\xi_n\}$ is cont.

12. If $A: X \to Y$ Show $(\text{range} A)^{\perp} = \ker(A^*)$

13. Prove if x & $y \in l_2$ then $\|x-y\|^2 + \|x+y\|^2 = 2(\|x\|^2 + \|y\|^2)$

14. If $\xi_n = 1/2^n$ then show that for all $\eta = (\eta_n) \in c_0$ with $\|\eta\|_{c_0} \le 1$ $\left|\sum_{n=1}^{\infty} \xi_n \eta_n\right| < 1$ but $\sup_{\|\eta\|_{c_0} \le 1} \left|\sum_{n=1}^{\infty} \xi_n \eta_n\right| = 1$.

15. Use the closed graph theorem to show that if X & Y are B-space $T: X \to Y$ is linear, 1-1, onto & cont Then T^{-1} is cont.

16. If $P_N : l_2 \to l_2$ $P_N(\{\xi_n\}) = \{\eta_n\}$ where $\eta_n = \begin{cases} \xi_n & n \le N \\ 0 & \text{otherwise} \end{cases}$ and $f \in l_2^*$ show $\lim_{N\to\infty} \|f(I - P_N)\| = 0$

17. Let P_N as in 16. Suppose $A \subseteq l_2$ s.t. $\xi, \eta \in A$ $\Rightarrow \|\xi - \eta\| < \varepsilon/3$ and suppose for some fixed N and fixed $\xi \in A$ $\|\xi - P_N \xi\| < \varepsilon/3$ Show for $\eta \in A$ $\|\eta - P_N \eta\| < \varepsilon$.

18. Show $\exists \varphi \in l_\infty^*$ $\|\varphi\| = 1$ s.t. $\xi = (\xi_n) \in l_\infty$ with $\lim_{n\to\infty} \xi_n$ exists then $\varphi(\xi) = \lim_{n\to\infty} \xi_n$.

19. X, Y B-spaces $\{x_n^*\} \subseteq X^*$ $\{y_n\} \subseteq Y$ $\{\lambda_n\} \in l_1$ $\|x_n\| = \|y_n\| = 1$ show $T: X \to Y$ $T(x) = \sum_{n=1}^{\infty} \lambda_n x_n^*(x) y_n$ is compact.