MAA 5506 — Functional Analysis

Section 1, Spring 1995.

Instructor: Bellenot.

The good doctor's Office: 002-B Love, Office Hours: MWF 10:20–11:00 or by appointment. Email addressed bellenot@cs.fsu.edu, bellenot@math.fsu.edu, or even bellenot@fsu.edu will get to the good doctor ('bellenot' is enough of an email address on math or cs machines).

Eligibility: Advance Calculus II

Text: DeVito *Functional Analysis* . Available at Target.

Coverage: Chapters 1 – 6? (as time permits).

Grades: The easy going 85% A, 70% B, 55% C, 40% D.

Homework: Your grade will be determined by homework problems. Some problems (most, but not all) will be graded on a 10 point scale. Only the top 90% of your graded homework is used to compute your homework average. Generally three homework problems (often proofs) will be assigned each Monday and due the following Monday.

Homework Rules

- Must be your **OWN** work.
- Must be neat and written in clear English.
- Must be on time – late homework is **NOT** accepted.
- Must be on 8.5 by 11 paper.
- Must be written in ink.
- Must use only one side of each page.
- Multiple pages must be stapled together.

MAA 5506 — Functional Analysis — Problems

$[x_n]$ stands for the closed linear span of $(x_n)_{n=1}^{\infty}$.

Problem A. Use the gliding hump in ℓ_1 to show any infinite dimesional subspace M of ℓ_1 has a sequence (x_n) so that $T : \ell_1 \rightarrow [x_n]$ given by $T((\alpha_n)) = \sum \alpha_n x_n$ is an isomorphism. Furthermore, show there is another sequence (y_n^*) in ℓ_1^* so that $P : \ell_1 \rightarrow \ell_1$ given by $P(x) = \sum y_n^*(x) x_n$ is a continuous projection.

Problem B. Show each separable norm space is isometric to a subspace of ℓ_∞. Hint: Let f_n be norm one functionals on X which norm a separable dense subset and map $x \in X$ to the sequence $(f_n(x)) \in \ell_\infty$.

Problem C. Suppose $T : X \rightarrow Y$ is a continuous map between norm spaces and $T^* : Y^* \rightarrow X^*$ is dual mapping given by $T^* f = f \circ T$.

1. Show $||T^*|| = ||T||$.
2. Show T onto implies T^* is 1-1.
3. Show T an isomorphism into implies T^* is onto.
4. Show if T is a subspace injection, then T^* is a quotient map.
5. Show if T is a quotient map, then T^* is an isometry into.

Problem D. Let $Q_0 : X \rightarrow X^{**}$ be the canonical injection and let $Q_2 : X^{**} \rightarrow X^{****}$ be the canonical injection. Show $Q_2 \neq Q_0^{**}$.