TRUE OR FALSE:

1. $\{(x,y) \mid x^2+y^2=1\}$ is a function $I \to I$ (where $I = \{x \mid 0 \le x \le 1\}$)
2. $P \Rightarrow Q$ is logically the same as Q or (not P)
3. There is a set of all sets
4. If X is a set then $\bigcup \mathcal{P}(X) = X$
5. If R is the relation "is a horse of a different color" on the set of all horses, then R is symmetric but not reflexive nor transitive.
6. The $\mathcal{P}(X)$ is partially ordered by $\subseteq$.
7. Suppose X is totally ordered by $\le$ and for some $S \subseteq X$ both S and $X \setminus S$ are well ordered by $\le$; then X is well ordered by $\le$.
8. If $\sim$ is an equivalence relation on X then the set of equivalence classes of $\sim$ is a partition of X.
9. $\emptyset$ is subset of every set.
10. The set $\{\{\emptyset, \{\emptyset\}\}, \{\emptyset\}\}$ has 3 elements
11. If all words are finite sequences of letters Then alphabetal order well-orders all words.
12. Every infinite partially ordered set has an unbounded subset.
13. If f is a function: $X \to X$ and f is symmetric and transitive then it is reflexive.
14. If it is true "$\forall x \in X, x \in \mathbb{N}$"; then "$\exists$ integer in X" is true.
15. Same as 7 only just assume X is partially ordered
16. $A \Rightarrow B$, $B \Rightarrow C$ and (not A) $\Rightarrow$ (not C) then A, B and C are equivalent
17. Same as 11 only all words are infinite sequences of letters
18. As sets of ordered pairs on X: (the relation $=$) $=$ (the function 1_X)

19. Every minimal element is also a minimun element

20. Finite totally ordered sets are well ordered.

FILL IN THE BLANKS: (if none say none)
FOR THE P.O. SET GIVEN BY FIGURE ONE

21. the minimal elements are ________

22. the maximal elements are ________

23. the upper bounds to $\{f, c\}$ are ________

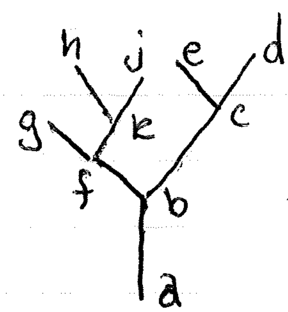

FIGURE ONE

24 the upper bounds to $\{f, k\}$ are ________

25 the greatest lower bounds of $\{g, h, j, e, d\}$ are ____

PROVE:

A. LET $A, B \subseteq X$ show $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$.

B. DEFINE the ordered triple (a,b,c) to be $((a,b),c)$
PROVE: $(a, b, c) = (x, y, z)$ iff $a = x$, $b = y$ and $c = z$.

MATH 505 FINAL

DEFINE

1. Maximal element
2. Maximum element
3. Least upper bound
4. chain
5. order - ideal
6. initial Segment
7. Ordinal
8. cardinal

STATE:

9. Axiom of Choice
10. Zorn's Lemma

PROVE:

11. If S is an ordinal and we define S_n $n = 1, 2, \ldots$ by $S_1 = S$ and $S_{n+1} = \{S_n\} \cup S_n$. Then $T = \bigcup_{n=1}^{\infty} S_n$ is an ordinal, in fact T is a limit ordinal which is smaller than any other limit ordinal bigger than S

12. Let X be a p.o. set such that every $A \subseteq X$ with A well ordered has an upper bound in X; show that every $C \subseteq X$ with C totally ordered has an upper bound.

\#~~12~~ 13 :

Axiom of Regularity : If X is a set then $X \notin X$

Definition:

A set X is transitive if $y \in X$ and $z \in y$ then $z \in X$.

A. Show $\emptyset$ is transitive

B. Show X is transitive iff $\in$ (as a relation on X) is a strict partial order.

C. Show that every ordinal is transitive.

D. Give an example of a transitive set that is not an ordinal.

$$\{ \emptyset, \{\emptyset\}, \{\{\emptyset\}\} \}$$