TP 11 Due Mon 6 Oct

PROVE OR DISPROVE

A. FOR ANY RELATION R WE HAVE EITHER $R \subseteq R^2$ OR $R^2 \subseteq R$.

B. FOR ANY RELATION R WE HAVE $(R^{-1})^2 = (R^2)^{-1}$

TP 12 Due Wed 8 Oct

PROVE BY INDUCTION, If the relation R is reflexive and transitive, then $R^n = R$ $(n \geq 1)$

DM2

TP 13 due Mon 13 Oct.

~~Proof.~~ A. Given x & y are vertices ($x \neq y$) in a directed graph G with a directed path P from x to y and a directed path Q from y to z. Prove that G has a directed cycle.

B. Given x & y are vertices ($x \neq y$) in a (non-directed) graph G and P & Q are two different paths from x to y. Prove that G has a (non-directed) cycle.

TP 14 due Wed 15 Oct. [3.4 9, 10 by induction]

A. Given $n = d^k$ $\quad k \geq 0 \quad a_1 = c$

and for $k \geq 1 \quad a_n = a_{n/d} + c$

Prove for $n = d^k$, $k \geq 0$

$$a_n = c(\log_d n + 1)$$

B Given $n = d^k$ $\quad k \geq 0 \quad a_1 = e$

and for $k \geq 1 \quad a_n = c\,a_{n/d} + e \qquad (c \neq 1)$

Prove for $n = d^k$, $k \geq 0$

$$a_n = e(c\, n^{\log_d c} - 1)/c - 1$$

DM2

TP 15 due mon 20 OCT

~~Prove or Disprove;~~

Prove or Disprove

Given G is connected.

A. x is a cut node of G $\Rightarrow$ ~~there is~~ for all vertices y and z in G with $y \neq z$, there is a simple path P from y to z which traverses x.

B. For all vertices y and z in G with $x \neq y, y \neq z, z \neq x$, there is a simple path P from y to z which traverses x $\Rightarrow$ x is a cut node of G

<u>Definition</u>: A vertex x of a graph G is said to be a <u>cut</u>-<u>node</u>, if the graph H is disconnect where H is G with x and x's incident edges removed

TP 16 due wed 22 oct.

Prove or Disprove

A. If P is a simple path of <u>maximal</u> length from x to y in G and Q is a portion of P which goes from w to z, then Q is a simple path of <u>maximal</u> length from w to z in G.

B. If P is a simple path of <u>minimal</u> length from x to y in G and Q is a portion of P which goes from w to z, then Q is a simple path of <u>minimal</u> length from w to z in G.

DM2

TP 17 due Mon Oct 27

[Helpful hint each even number can be written $2^k p$ where $k \geq 1$ and p is odd]

Given for n odd ≥ 1, $a_n = n \log_2 n$ and for n even ≥ 2, $a_n = 2a_{n/2} + n$.

Prove by induction (on k in helpful hint) for n even ≥ 2 $a_n = n \log_2 n$

TP 18 due Wed Oct 29

Prove or disprove!

A. A maximal directed path in a digraph G must be a directed path of maximal length in G

B. A minimal directed path in a digraph G must be a directed path of ~~maximal~~ minimal length in G.

TP19 was a "prove or disprove" if the following algorithms were correct solutions to TSP <traveling salesman Problem>

A. Greedy take the next shortest step possible

B. Divide & Conquer. Split in half & find "best" way of glueing together.

DM2 TP 20 due Mon 10 Nov

Prove or Disprove

A. If G is a transport network with the property that every edge e in G has non-zero capacity, then G has a non-zero flow.

B. All F-augmenting paths are simple (F is a flow on transport network)

TP 21 due Wed 12 Nov

A. If the digraph G has a closed directed path P which visits each vertex of G, then G is strongly connect.

B. If G is a digraph, let A be the collection of all subsets S of $V(G)$ (vertices of G) so that there is a closed directed path P which visits each vertex in S. Now $A \subseteq$ is poset, prove A has a maximal element T!

C. If the digraph G is strongly connected in B above then $T = V(G)$.

DM 2

TP 22 due Mon 17 Nov.

Given G is a digraph with vertices $\{1, 2, \dots n, n+1\}$ there is a path P in G which goes $1 \to 2 \to 3 \to \dots \to n-1 \to n$ and for each $1 \le i \le n$ there is either an edge i to $n+1$ or an edge $n+1$ to i but <u>not</u> <u>both</u>!

<u>Prove</u>: There is a simple directed path Q in G which "visits" each vertex in G.

TP 23 due Wed 19 Nov

Problem 15 <u>Section 7.4</u>

DM2

TP 24 due Mon 24 Nov.

Given $\begin{bmatrix} a_0 = 6 \\ b_0 = 9 \end{bmatrix}$ and for $n \geq 1$ $\begin{cases} a_n = 2a_{n-1} + 2b_{n-1} \\ b_n = -2a_{n-1} + 7b_{n-1} \end{cases}$

Prove by induction for $n \geq 0$ that

$$\begin{cases} a_n = 2 \cdot 3^n + 4 \cdot 6^n \\ b_n = 3^n + 8 \cdot 6^n \end{cases}$$

TP 25 due Wed 26 Nov

Solve by generating fcns $a_0 = 2$ $b_0 = 3$

and for $n \geq 1$ $\begin{cases} a_n = 19a_{n-1} - 3b_{n-1} \\ b_n = -3a_{n-1} + 11b_{n-1} \end{cases}$

DM2 — the last 2 tp's !

TP 26 due 1 Dec Mon

A round-robin graph with n players is a digraph G on n vertices (one for each player. and for each pair of players (vertices) $1 \le i < j \le n$ ($i \ne j$!) there is either an edge $i \to j$ or an edge $j \to i$ but not both. [$a \to b$ means a "beat" b!]

Prove by induction: A round-robin graph has a simple directed path which contains (visits) all the vertices.

TP 27 due 3 Dec Wed

Prove a digraph G is unilaterally connected if and only if there is a directed path P which visits every vertex of G