Discrete Math II

1–4 worth 10 pts each, 5–8 worth 15 ea, Show all work for

1A. In a network what is $k(P,\bar{P})$? (explain P & $\bar{P}$ as well.)

B. Explain the differences between a tree & a rooted tree.

2A. Draw a venn diagram for each of the three statements to the right

1. Each A is B
2. All C is B
3. ∴ each A is C

B. Is it logically valid or not? why or why not?

3A. Write the permutation to right as a product of disjoint cycles

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 2 & 7 & 8 & 9 & 4 & 3 & 6 & 1 \end{pmatrix}$$

B. Multiply out the cycles $(1,2,3)(3,4,5)(4,5) =$

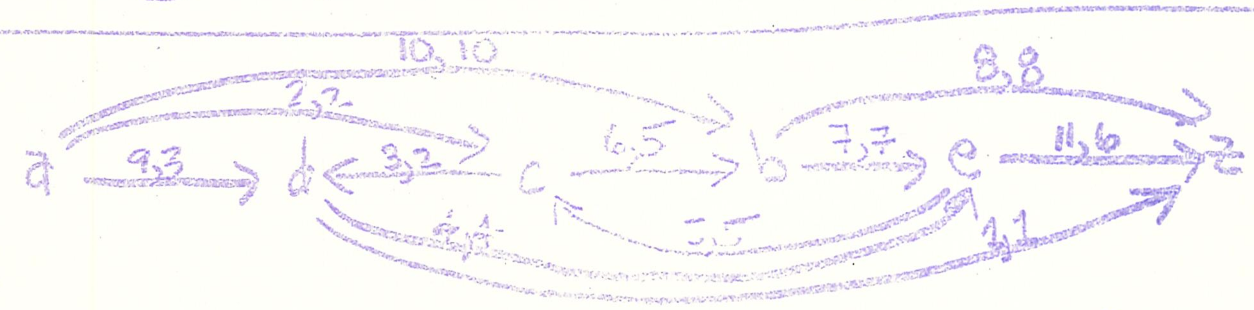


FIG 1.

4. In Fig 1. the numbers above the edges are cap, flow.

A. Use the labeling procedure to label the vertices

B. Show how A. changes the flow on Fig 1.

5. In FIG 1 ignore directions & use only the first number over each edge

AB. In order list the costs of edges chosen to find a minimal spanning tree by

A. Prim's Algorithm

B. Kruskal's Algorithm

C. Label each vertex according to Dijkstra's algorithm to find the shortest path to each vertex to a.

6, Prove a cut edge of a connected graph is on no circu

7, Prove a graph is connected if and only if it has a spanning tree

8. Prove, If a connected graph G has E edges and V vertices and if $k = E - V + 2$, then G has at least k spanning trees.
(Hint induction on k (on the no. of edges (be sure the one you delete is in a circuit)) k is <u>NOT</u> the no. of regions (G could be non-planar))

TEST 4 MAD 3105 by ______________________

SHOW ALL WORK & BE NEAT for CREDIT. 1–4: 10pts ea; 5–8: 15pts

1 A. Define a connected graph:

B. Define a spanning tree for a graph G:

2. Draw a Venn diagram for each statement to right: 1. All trees have paths

2. All connected graphs have paths

3 ∴ All trees are connected graphs.

1. 2. 3.

Is the logic valid or not? Draw a picture which supports your conclusion

3. Draw all trees with 5 edges (Do not include duplicates (i.e isomorphic graphs)) ~~more than once~~) Exactly one of each type.

4. $(x+1)(x-3)^2 = x^3 - 5x^2 + 3x + 9$. For the recurrence relation $a_n - 5a_{n-1} + 3a_{n-2} + 9a_{n-3} = f(n)$ do the following:

AB: Write the general solution to homogeneous equation:

CDE: Write the correct guess for the form of a particular solution when $f(n)$ is the given function

C. $f(n) = 4n^2 - 1$

D. $f(n) = 6 \cdot 3^n$

E. $f(n) = 2n(-1)^n$

5. Solve (find the solution) $a_n + a_{n-1} = 6a_{n-2}$; $a_0 = 5$; $a_1 = 25$.

6. Draw the picture (i.e: graph) required in each part

A. A path with minimal number of edges B. A circuit with minimal no. of edges

CD In C&D Find the rooted tree with 5 edges which has

C. The fewest leaves D. With the smallest height

E. Using the fewest edges possible, find a connected graph which isn't a tree but has a cut edge

7. Prove: An edge in a tree is a cut-edge.

8. The complete graph on n vertices (called K_n) has an edge between any pair of vertices x & y with $x \neq y$. Clearly, K_n has exactly $\binom{n}{2} = \frac{n(n-1)}{2}$ edges by the counting techniques of finite 1. YOUR JOB is to prove K_n has $n(n-1)/2$ edges BY INDUCTION on n (the number of vertices).

TEST 2 name

1. Find the generating function for the number of integer solutions to $2x_1 + x_2 + x_3 + 8x_4 = r$
so that $0 \le x_1$, $2 \le x_2$, $x_3 \le 5$ and $7 \le x_4 \le 8$

2. Find the probability that in a bridge deal North and South get all the diamonds and hearts

3. Use the binomial theorem to evaluate $\sum_{k=0}^{n} 4^k \binom{n}{k}$

4. $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + \ldots + (n-2)(n-1)n = ?$

5. Write a recurrence relation (with enough intial values) for the number of n-digit ternary numbers (i.e. using only (0, 1 or 2) with no 1 to the right of any 2. DO <u>NOT</u> SOLVE.

6. Find the coefficient to x^{24} in
$\cancel{(x^3+x^4+x^5+x^6+x^7)}$

$$(x^3+x^4+x^5+x^6+x^7)^4(1+x+x^2+\dots)^{12}$$

7. Solve $a_n = 2a_{n-1} + 8a_{n-2}$ $\quad a_0 = 2 \quad a_1 = 17$

8. 8 Find the number of ways to put 16 indentical balls into 5 different boxes with no more than 6 in each box using generating functions.

9. Solve $a_n = 2a_{n-1} + 5^n$ $\quad a_0 = 1$

10. Prove by <u>induction</u> if $a_0 = 2$ and $a_n = 3a_{n-1} + \frac{1}{4} \cdot 4^n$ for $n \geq 1$, then $a_n = 3^n + 4^n$ for $n \geq 0$

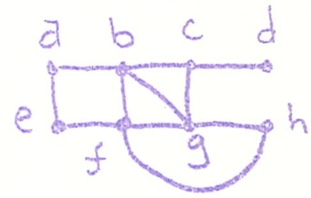
MAD 3105 TEST 2 BY ______________________

be neat, show all work. 1–4 worth 10pts each, 5–8 worth 15 each

1. A Define biconnected component:

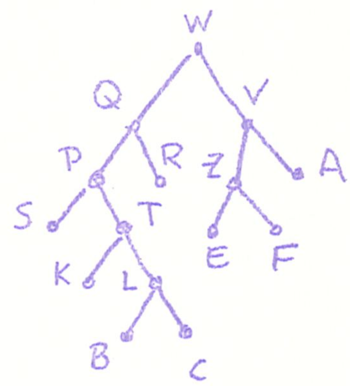
B. Arrange in increasing order $O(2^n), O(n \ln n), O(n), O(3^n), O(n^{3/2})$

2. For the graph on right find (orientated) spanning both by



A. BFS

B and DFS



3. For the tree on right LIST THE VERTICES IN

A Pre-Order:

B Post-Order:

C In-Order:

4. A What (quotient of poly's) function has the generating function

$$\sum_{n=2}^{\infty} 2^n x^n$$

BC. Use partial fractions to rewrite

$$\frac{x^2}{(1+2x+x^2)(1-x)}$$

5. DISPROVE THE FOLLOWING BY GIVING (i.e drawing) A COUNTEREXAMPLE.

A. A connected graph with a cut-edge has an articulation pt.

B. Every graph with 5 edges and 6 vertices is connected.

C. If B & C are different biconnected components of G, then B & C have no points in common (indicate B & C in G)

DE. If G has a circuit C which contains all the edges and vertices of G, then number of edges in G = number of vertices in G. (Indicate the circuit C in G).

6. Prove that a connected graph with at least 3 vertices that has a cut-edge, ~~has~~ then it also has an articulation point.

7. Solve $a_n - 7a_{n-1} + 12a_{n-2} = 12n - 4$; $a_0 = 5$; $a_1 = 8$

8. We have proved that each tree has at least one vertex with degree ≤ 1. Your Induction Problem is to prove by induction that each tree with at least one edge has at least two vertices of degree one.

The classic Finite 2 test 2: a brilliant comeback by ____________

You must show ALL work for full credit
(1–4) worth 10 pts ea. (5–8) worth 15 pts ea.

1. Define A. a tree
   B. a backedge

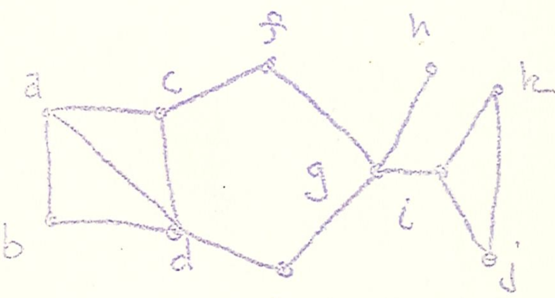
2. A Arrange in increasing order $O(n!)$, $O(n^2)$, $O(4^n)$ & $O($…

   B. Show $O(7n+5) = O(\sqrt{n^2+3})$

3. A. For the list $\{16, 1, 14, 3\}$ show the resulting list after each "pass" (completion of inner do loop) of BUBBLE sort

   B. Show how MERGE sort would sort this same list

4. Let R be the relation on real numbers where xRy if and only if x−y is a non-negative integer
   A. Is R reflexive?
   B. Is R symmetric?
   C. Is R transitive?
   D. Is R anti-symmetric?



5. FOR THE GRAPH TO THE RIGHT
   A. Find a spanning tree using
      (i) a breadth-first search and (ii) a depth first search

   B. Using the DFS tree in 5.A.(ii) list the vertices in
      (i) pre-order

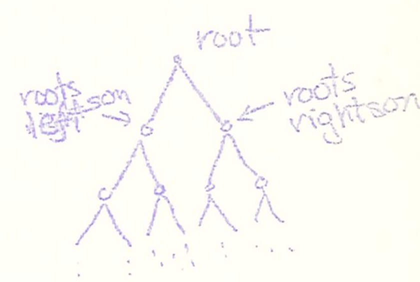
      & (ii) post-order

   C. (i) Find back of each vertex using 5.B.(i)

      (ii) List the articulation pts

6. Prove that each connected planar graph whose circuits all have length ≥ 5 and each vertex has degree ≥ 3 must have at least 30 edges.

7. List all non-isomorphic trees with 7 vertices.



8. Consider the following binary tree like data structure. Each position x has two following positions call rightson & leftson; and x is the parent of these position. Each position except the root has a parent. Consider puting ~~the~~ a list of items into these positions by the following rules: 1 the first item goes into the root
   2: Each latter item can go into any position which is empty but whose parent is non-empty.

Prove by induction that after n "items" have been correctly placed there are exactly n+1 places that the n+1-st "item" could be put.

MAD 3402 (5?05?) Discrete Math II Test 3 by ____________

1–4 worth 10 pts each, 5–8 worth 15 ea. Show all work for full credit

1A. In a network what is $k(P, \bar{P})$? (explain P & $\bar{P}$ as well.)

B. Explain the differences between a tree & a rooted tree.

2A. Draw a venn diagram for each of the three statements to the right

1. Each A is B
2. All C is B
3. ∴ each A is C

B. Is it logically valid or not? why or why not?

3A. Write the permutation to right as a product of disjoint cycles

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 2 & 7 & 8 & 9 & 4 & 3 & 6 & 1 \end{pmatrix}$$

B. Multiply out the cycles $(1,2,3)(3,4,5)(4,5) =$

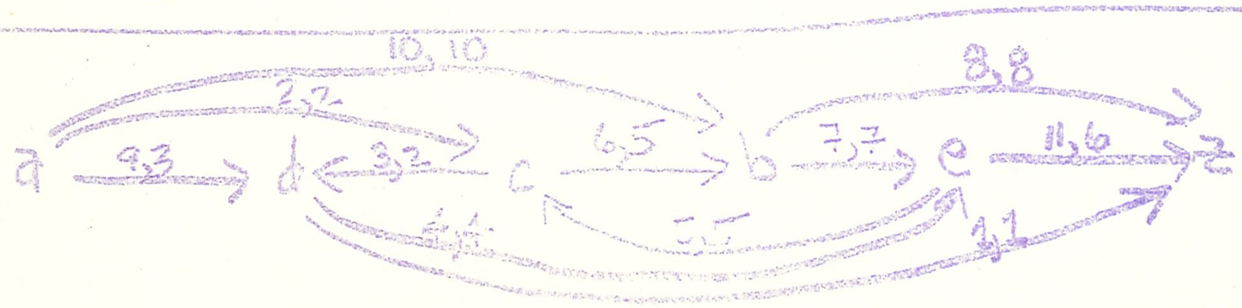


FIG 1.

4. In Fig 1. the numbers above the edges are cap, flow.

A. Use the labeling procedure to label the vertices

B. Show how A. changes the flow on Fig 1.

5. In FIG 1 ignore directions & use only the first number over each edge

AB. In order list the costs of edges chosen to find a minimal spanning tree by

A. Prim's Algorithm

B. Kruskal's Algorithm

C. Label each vertex according to Dijkstra's algorithm to find the shortest path to each vertex to a.

6. Prove a cut edge of a connected graph is on no circuit.

7. Prove a graph is connected if and only if it has a spanning ~~forest~~ tree

8. Prove, If a connected graph G has E edges and V vertices and if $k = E - V + 2$, then G has at least k spanning trees.
(Hint induction on k (on the no. of edges (be sure the one you delete is in a circuit)) k is NOT the no. of regions (G could be non-planar))

Easy Test 3, preformed by ______________________

(1) 15% of all finite students like wine, 60% like orange juice, and 55% like Tea. 35% like any given pair of these beverages and 25% like all three beverages.

A. ~~what~~ percent like wine only? ________

B. What pe~~z~~cent like exactly two of the three beverages? ________

(2) The roots of characteristic ~~equa~~ polynomial to

$$a_n + ba_{n-1} + ca_{n-2} + da_{n-3} + ea_{n-4} = f(n)$$

(with $f(n) = 0$) are 2, 2, 1, 5

A. Write the general solution to the problem with $f(n) = 0$

B. What is the form of the "undetermine coefficients" guess for a particular solution if $f(n) = 3 \cdot 4^n$? Do not solve for the coefficients

C. Same as B if $f(n) = n^2 + 7$ ________

D. same as B if $f(n) = \frac{1}{2} \cdot 5^n$ ________

E. same as B if $f(n) = 7 \cdot 2^n$ ________

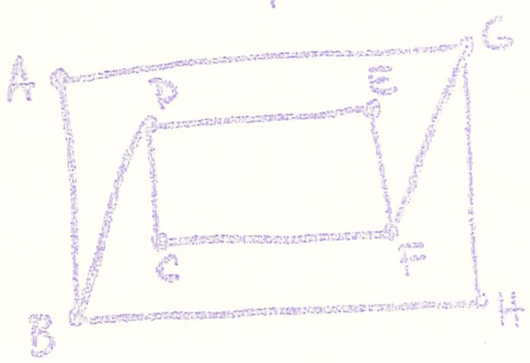
(3) How many 12-card hands from a deck of 52 have at least one card in each suit?

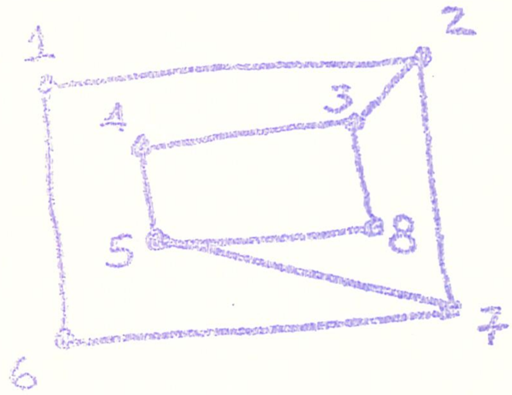
(4) Find the generating function with $a_r = (r+1)r(r-1)$

Pg 2

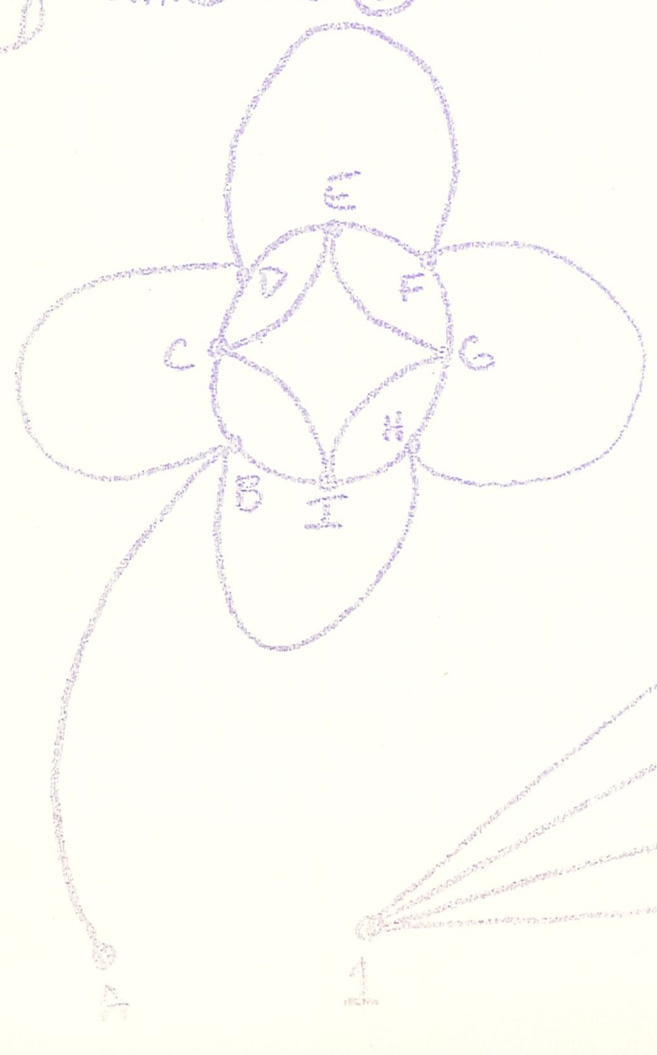
(5) Solve $a_n = 8a_{n-1} - 16a_{n-2}$ $n \geq 2$ $a_0 = 2$ $a_1 = -4$

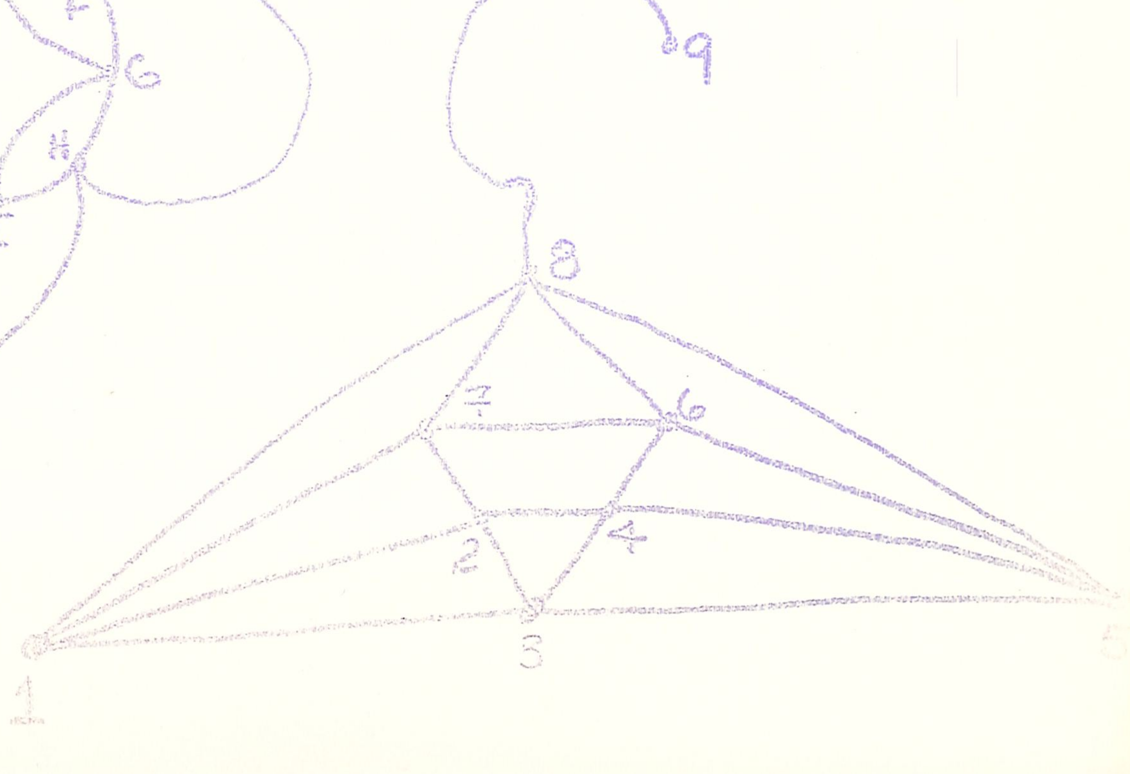
(6) Either show why the graphs are not isomorphic or find an isomorphism.





(7) Same as (6) for:





(8) Given $a_n = \frac{1}{a_{n-1}} + \frac{2}{n}$ f $n \geq 2$ and $a_1 = 2$

Show by induction $a_n = \frac{n+1}{n}$ for $n \geq 1$

(9) How many arrangements of DIGRAPHS have the D before the G or the G before the P or the D before P? (there could be letters between.)

(10) Solve using generating functions $a_n = 4a_{n-1} + 3^{n-1}$ $(n \geq 1)$, $a_0 = 1$

MAD 3105 The Verbose 3rd Test BY ______________

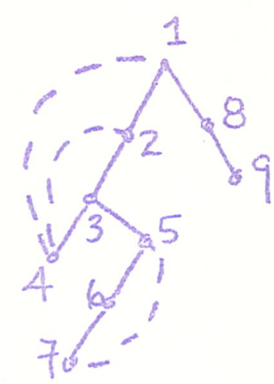
1-4 10pts 5-8 15pts

1 Define

A. Cutset S of a connected graph G

B. $k(P, P^c)$ (a cut in a network)

2 Below are a list of 4 vertices in the graph to the right. Your job is to find "BACK" of each of these and write either "none" or "__ is arty pt" (articulation point) depending on if from the information given (i, back i) allows you to conclude nothing or something. Each part is separate do not use any other info. (fill in the blank of course)



Solid lines: edges in a DFS spanning tree

dashed lines: other edges in the graph

A. Vertex 9

B. Vertex 6

C. Vertex 3

D. Vertex 2

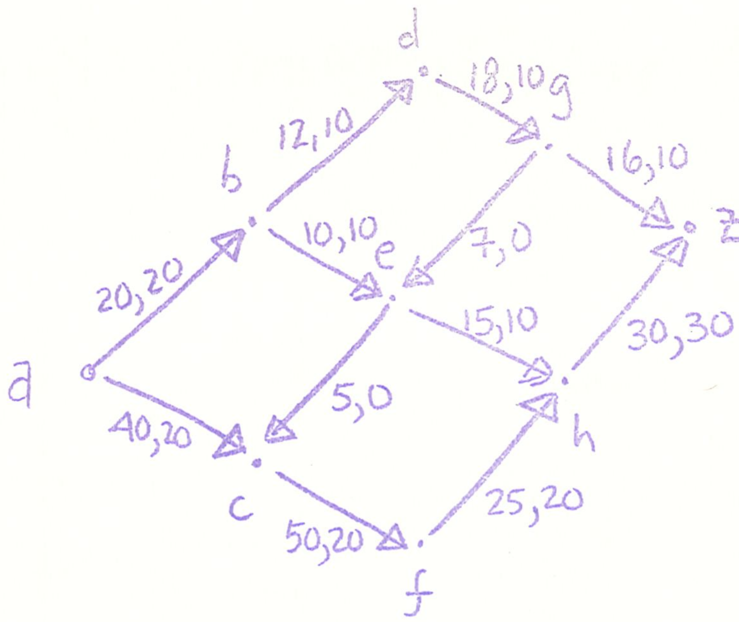
3. Find (do not solve) a recurrence relation (and enough intial conditions) for the number of n-digit quaternary (i.e base4 (i.e. using 0,1,2,3)) with no consecutive even digits (i.e. no 00, 22, 02 or 20).

4. Disprove by giving a counterexample.

A. If E is an edge $x \to y$ in the connected graph G and E is a cut-edge of G then the only path $x \to y$ in G is E.

B. If V, W are subsets of vertices of the connected graph G so that each vertex of G is in exactly one of V and/or W. Let S be the set of edges in G which join a vertex in V to a vertex in W. Then S is a cut set.

5.



A For the network above label the vertices per our algorithm to increase the flow.

B. Use your labels to show how the flow changes on the graph.

C. Find a minimal spanning tree for graph above (ignore direction, 1st number is the cost of the edge) DRAW the tree below

6. A. $4^{\log_2 n} = n^k$, so $k =$

BCDE. Solve $a_n = 2a_{n/3} + 4$ $a_1 = 5$

7. A. Find the number of arrangements of VERBOSE with either "the B at the begining" or "the V at the end" or "the V before the B".

B. Find the number ways you can roll 101 distinct dice so that at least one of each of the numbers 1 through 6 appear.

8. Prove by induction (on number of vertices) (choose your vertex carefully) that the vertices of a tree can be colored with two colors (or fewer) so that no two vertices of the same color are adjacent.

MAD 3105 FINAL by ______________________

1-8 10 pts ea. 9-16 15 pts ea. Show All Work

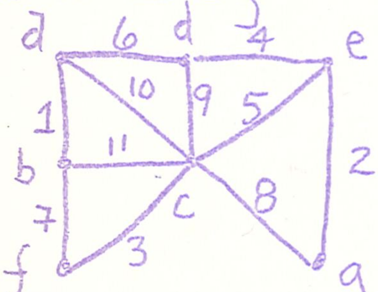
1. A. Define a <u>tree</u>:

B. Define a <u>circuit</u>:

2. A. Arrange in increasing order: $O(2n^3), O(3^n), O(n!), O(10n^2 \log n)$

B. Arrange in increasing order: $2 \cdot 4^3,\ 3^4,\ 4!,\ 10 \cdot 4^2 \log_2 4$

3. For the graph below find the spanning tree (i.e. draw it) obtained by A. BFS ⟶ B. DFS



4. For the graph above list the <u>COSTS</u> in the <u>order</u> picked to find a minimal spanning trees by algorithm given

A. Prim's:

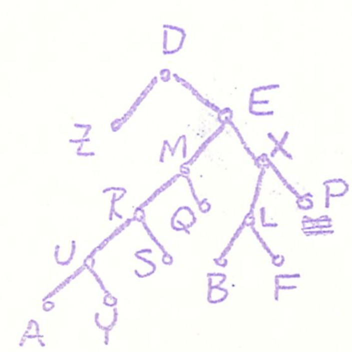
B. Kruskal's:

5. For the tree to right list the vertices in the required order

A Pre Order

B Post Order

C In Order



6. A certain 5-ary tree (i.e. each internal vertex has exactly 5 children) has 1001 Leaves. How many internal vertices does it have?

7. $(x+1)(x-3)^2 = x^3 - 5x^2 + 3x + 9$. For the recurrence relation $a_n - 5a_{n-1} + 3a_{n-2} + 9a_{n-3} = f(n)$ do the following

AB. Write the general solution to the homogenuous equation

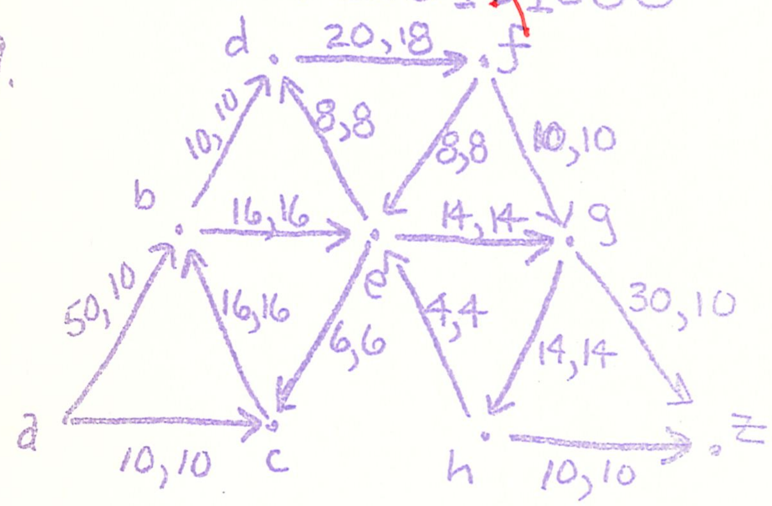
CDE Write the correct guess for the form of a particular solution when $f(n)$ is the given function.

C. $f(n) = 27n^2$ D. $f(n) = \frac{1}{2}\cdot 3^n$ E. $f(n) = 2^n + 4^n$

8. Each binary tree (parents have exactly two kids) can be "coded" as a sequence of zero's & one's as follows. The vertices are checked in pre order, a "1" is written if the vertex is internal and a "0" is written if the vertex is a leaf. DRAW the tree whose "code" is

1110010011~~0~~1000

9.



A. For the network to left use our labeling algorithm to label as many vertices as possible

B. On the graph above show how to increase the flow using your labels in A.

C. Suddenly there is a break in the edge $\vec{ed}$, and flow is stoped along this edge. Without changing the total flow value and changing as few edges as possible, correct this. (i.e. make the flow into = flow out of each vertex without changing the flow out of a)

SHOW the edges & changes HERE

10. Disprove by giving a counterexample

A. If $P, Q : x \to y$ are different paths then P followed by Q backwards is a circuit. (Indicate P, Q)

B. A connected graph with 6 vertices and 7 edges has exactly two circuits (Indicate the circuits)

C. Each graph with 3 bicomponents has 2 articulation points

D. If the connected graph with more than two edges has a unique minimal spanning tree, then the weights on the edges are distinct.

11. A. Find the number of arrangements of ALGORITHM with the A before the L or the L before the G or the G before the A

B. Find the number of 7 card poker hands with at least one card in each suit.

12. Solve by GENERATING FUNCTIONS $a_n = 5a_{n-1} + 2^n, n \geq 1; a_0 = 2$

13. Solve $a_n = 3a_{n/8} + 2n$, $a_1 = 1$

14. <u>Find</u> and <u>Solve</u> a recurrence relation for the number of n-digit ternary (base 3) numbers with no consecutive even digits

15. ~~Prove if~~ G is a connected graph and E is an edge in G
Prove: E is a cut-edge if and only if E is in no circuits of G

16 Prove by induction (on the crossing number) If there is a path x to y in G, then there is a simple path x to y in G.

1&2 How many license plates are there with

A. Three letters followed by three digits? ______

B. With either two, three, four or five letters? ______

C. With 8 digits exactly five of them zero's?

D. With 6 letters exactly two of them vowels?

3&4 How many arrangements of MERRY CHRISTMASS are

A. Altogether? ______

B. with the Y trapped between the M's? (no letters between)

C. with the Y next to the C?

D. with no consecutive vowels?

5&6 How many ways are there pick 20 electronic games from one of Santa's 50 kinds with

A. No repetition? ______

B. Unlimited repetition? ______

C&D Use Inclusion Exclusion to find the number of ways if you can pick up to five games of the same kind.

7&8 What is the probability that a 5-card hand (from a deck of 52) has

A. All face cards (A, K, Q, J, 10 are face cards)? ______

B. At least one red card (heart & diamonds are red)? ______

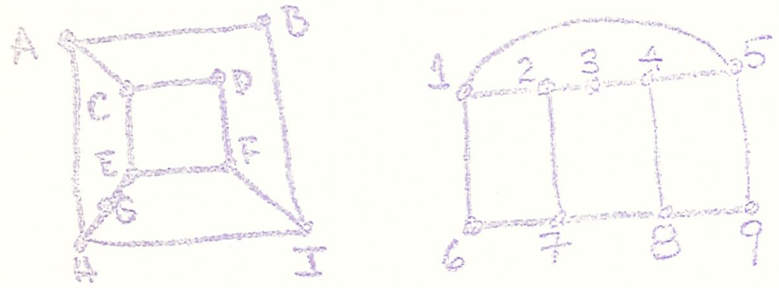
C. Exactly one pair (no three or four of a kind)?

D. At least one pair?

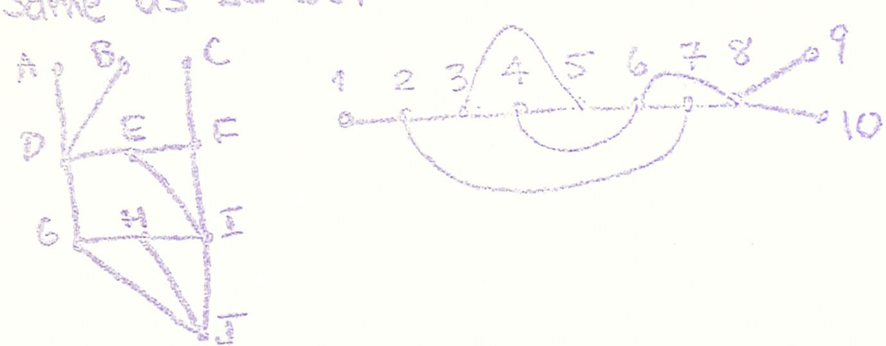
9. $\sum_{k=0}^{100} \binom{100}{k} (-6)^k = ?$

10. Write (but do not solve) a generating function and tell which coefficient we need to find the number of non-negative integer solutions to $x_1 + x_2 + x_3 + x_4 + 2x_5 = 27$ with $0 \le x_1, x_2$, [illegible] $3 \le x_3 \le 7$ and $1 \le x_4 \le 3$.

11. Either find the isomorphism or show why the graphs are not isomorphic



12. Same as 11 for



13. How many ways are there to roll 100 distinct dice with at least one of each kind of face showing?

14. Find the coefficient of x^{24} in $(x^3 + x^4 + x^5 + x^6 + x^7)^4 (1 + x + \dots)^{12}$.

15. Given $a_n = 4a_{n-1} - 4a_{n-2}$ $(n \geq 2)$; $a_0 = 3$; $a_1 = 4$ prove by induction $a_n = 3 \cdot 2^n - n2^n$, $n \geq 0$.

16. Find ~~a~~ the generating function with $a_r = 4r^2$

17. The are 4 roads from Success to Scandal and 10 roads from Scandal to Ruin. Also there are 5 roads from Success to Ruin which avoid Scandal.

A. How many roads to Ruin (routes from Success) are there?

B, C, D, E How many ways are there to go from Success to Ruin and back that doesn't repeat any portition of the route there on the way back.

18. Solve $a_n - 5a_{n-1} + 6a_{n-2} = 4^n$ $a_0 = 9$ $a_1 = 36$

19. Solve $a_n = 4a_{n-1} + 6^n \cdot (n \geq 1)$ $a_0 = 2$ by GENERATING FUNCTI

20. Use generating functions to find how many ways there are to put 30 identical balls into 9 distinct boxes so that the first three boxes have at least two balls each and the last three boxes have no more than 10 balls each.

Name ______________________________

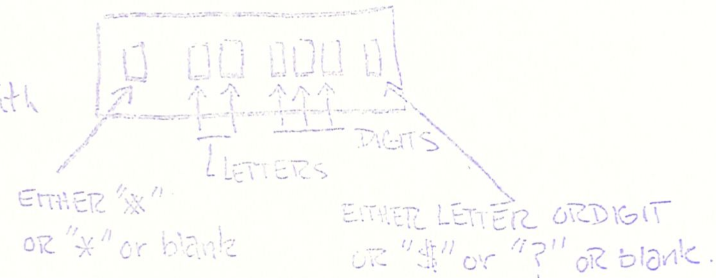
ALL PROBLEMS (OR PARTS OF PROBLEMS) ARE WORTH 5 PTS EXCEPT FOR THE INDUCTION PROBLEM #14 WHICH IS WORTH 20 PTS.

1. How many arrangements are there of the word ARRANGEMENTS ?

2. President Reagan wants to give you nine Jelly Beans of the same color. (It was to be ten, but budget cut backs you know.) On his desk he has a large paper bag full of Jelly Beans in 13 different colors. What is the <u>fewest</u> number of Jelly beans old Ron needs to take out of the bag to <u>always</u> have enough of one color to give to you?

3. A & B are sets. A has 6909 elements, B has 1107 elements and $A \cap B$ has 255 elements. How many elements does $A \cup B$ have?

4. How many license plates are there with



5. What is the probability that a five card poker hand has the Jack of diamonds, the seven and three of spades, the ace of clubs and the six of hearts?

6. How many ways are there to select 23 ice cream cones from 31 flavors?

7. A group of 10 men and 3 women want to form a committee with at least two people which has twice as many men as women. How many ways can they do this?

8. How many 4 card hands (from a deck of 52) have

A. Exactly one card of each suit?

B. Exactly one pair?

C. At least one pair?

9. How many ways are there to put 17 identical pennies into 6 different parking meters so that exactly two of the parking meters get none?

10. How many ways are there to roll 101 distinct dice so that exactly 37 "three"'s are rolled?

11. How many non-negative integer solutions are there to $x_1 + x_2 + x_3 \le 10$?

12. How many arrangements of FINITEMATH are there with

A. No consecutive vowels?

B. With the "F" and "M" next to each other?

13. There are 3 different roads from A to B and 4 different roads from B to C. There is also one road from A to C which by-passes B. How many ways can you go from A to C and back to A so that you do not repeat any road (or portion thereof) on the way back?

14. Prove by Induction $\sum_{i=1}^{n}(2i-1) = n^2$
(20 pts!)

1–8 are 10 pts each. 9–16 are 15 pts each. Show ALL work for credit

1. Define A: a path in a graph

B: a 7-coloring of a graph

2. Define A. An Euler circuit

B. a flow on a network

3A Write $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 9 & 6 & 2 & 5 & 3 & 1 & 4 & 8 \end{pmatrix}$ as a product of disjoint cycles

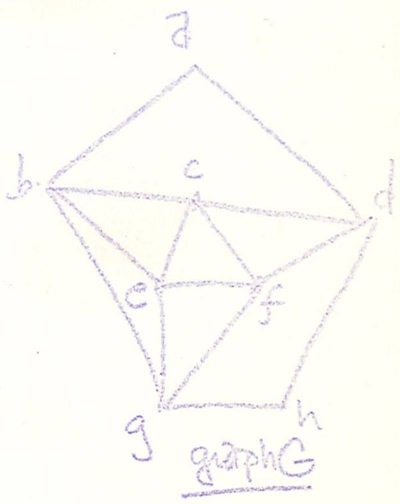
B. Multiply out $(1,2,3)(1,3,5,4)(4,2,5) =$

4A Show $O(2^n) < O(3^n)$

B Show $O(7n^{3/2} + n + 5) = O(\sqrt{n^3 + n^2})$

5. Define a relation R on the real numbers by
$x R y$ if and only if $x^2 + y^2 > 2xy$

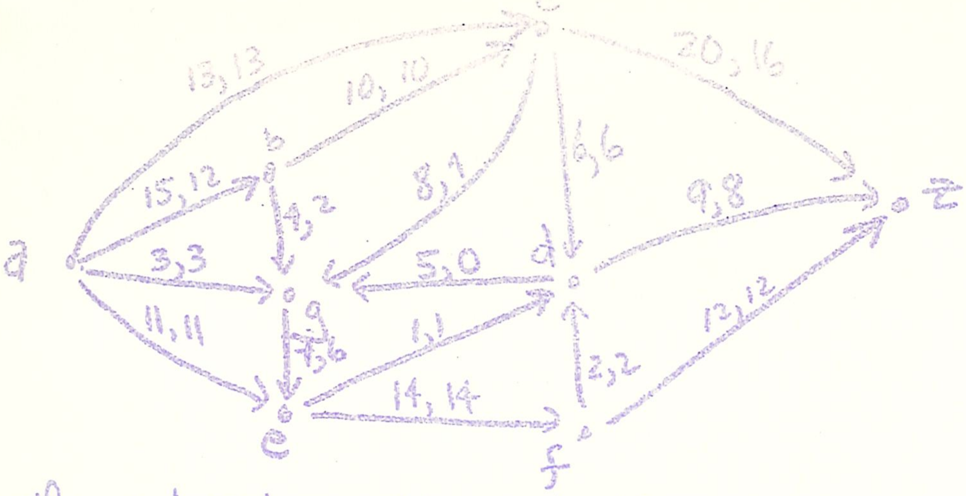
A. Is R reflexive?
B. Is R symmetric?
C. Is R transitive?
D. Let $x = 2$ find all y so that $x R y$ is false (on second thought don't do this the answer is $y = 2$)



graph G

6. In graph G find
A. an Euler circuit
B. a Hamiltonian circuit

7. Find $\chi(G)$ be sure to show
A. It isn't a smaller no.
B. It isn't a larger no.



8. For the network problem above
   A. Label the vertices per our algorithm.

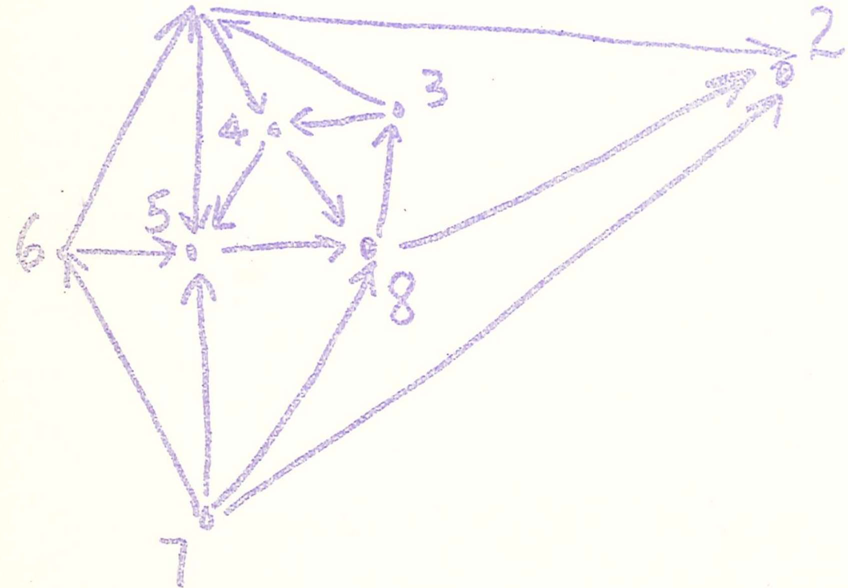
   B. Show how this changes the flow

9. In the network above IGNORE the directions on the edges and ~~you~~ use the first number as the cost of the edge
   A. Use Prim's Algorithm to find a minimal spanning tree (list the costs in order picked)

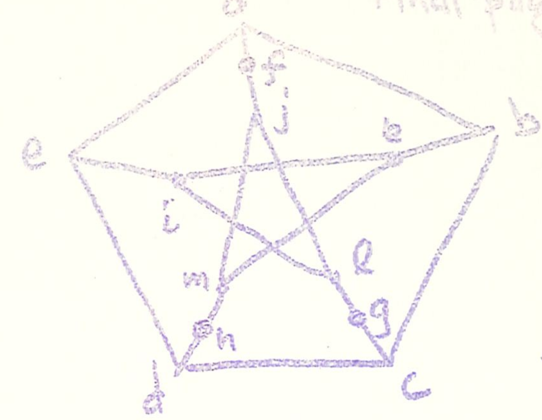
   B. Use Depth first search to find a spanning (labeled) tree

   C. Use Breadth first search to find a spanning (labeled) tree

10. Either give an isomorphism between the direct graph below & the network above or show there is none. (The iso must preserve the direction of edges too.)



11. Prove this graph has no hamiltonian circuit



12. Show that a connected planar graph with fewer than 30 edges has at least one vertex of degree 4 or less.

13. Let G be a connected graph
Prove G is a tree if and only if G has no circuits

14. List all connected planar graphs with 4 or fewer vertices. (Make sure your list does not contain isomorphic graphs)

15. Let G be a connected graph with each vertex having degree two. A little thought will convince you that G is just one circuit. Prove it.

16. Ye Olde Induction Problem:

Let G be a graph

We inductively define graphs $G_1, G_2, \ldots$ using the same vertices as G. The graph G_1 is G. After G_n is defined, define G_{n+1} on the vertices of G by if x, y are different vertices of G the edge (x,y) is in G if either (1) the edge (x,y) is in G_n or (2) there is a vertex z with $x \neq z \neq y$ and the edge (x,z) is in G_n and the edge (z,y) is in $G = G_1$.

<u>Prove</u> by induction on k, if $x \neq y$ are vertices in G and there is a path of k-edges from x to y in G, then the edge (x,y) is in G_k.

MAD 3105 FINAL triumphly done by ______________

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$(1,7)(2,9,8,4)(3,6)(5)$

B. Multiply out $(1,2,3)(1,3,5,4)(4,2,5) = (1,5,2,4)(3)$

4A Show $O(2^n) < O(3^n)$ $\quad \frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n \to 0$

B Show $O(7n^{3/2} + n + 5) = O(\sqrt{n^3 + n^2})$

$$\frac{\frac{1}{n^{3/2}}\sqrt{n^3+n^2}}{\frac{1}{n^{3/2}}\,7n^{3/2}+n+5} = \frac{\sqrt{1+1/n}}{7+1/n^{1/2}+5/n^{3/2}} \longrightarrow 1/7$$

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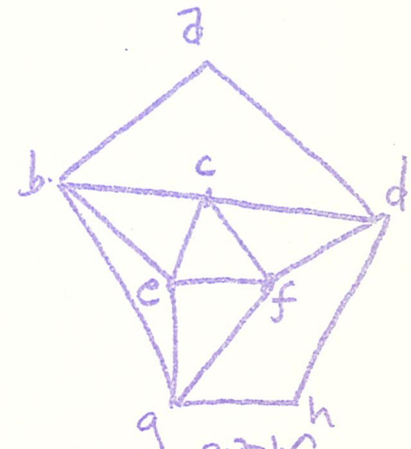

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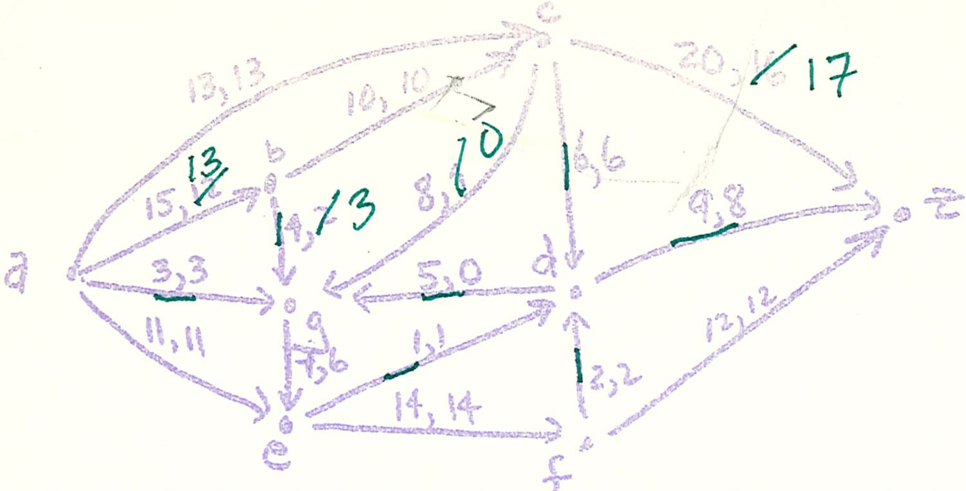

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$a(-,\infty)$, $b(a^+,3)$ $g(b^+,2)$ $c(g^-,1)$ $z(c^+,1)$ | $e(g^+,1)$

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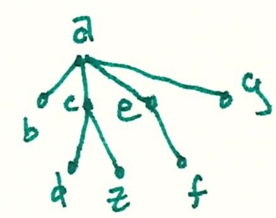

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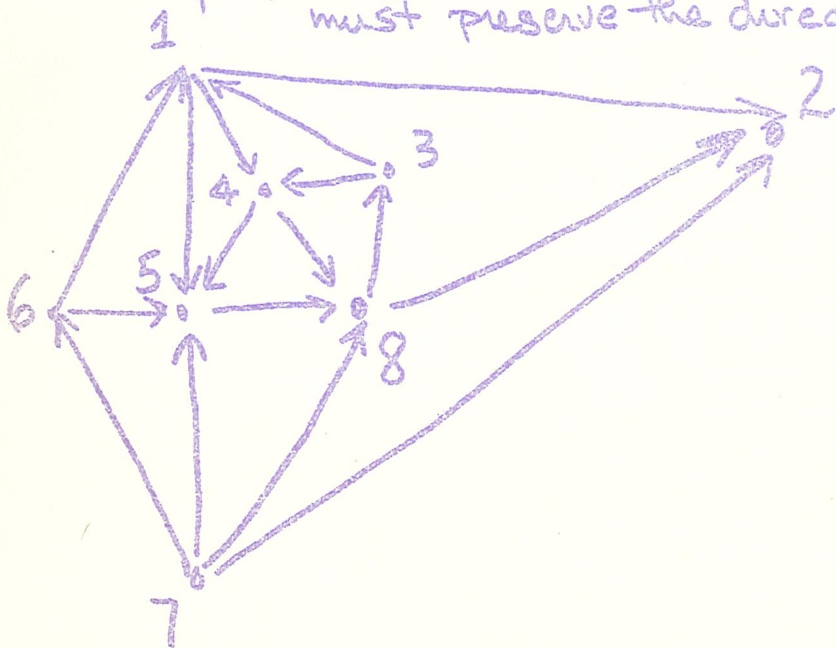

	OUT	IN	
1	3	2	c
2	0	3	z
3	2	1	b
4	2	2	e
5	1	4	g
6	2	1	f
7	4	0	a
8	2	3	d

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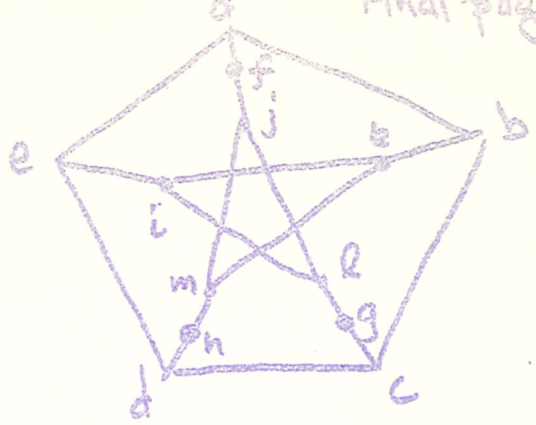

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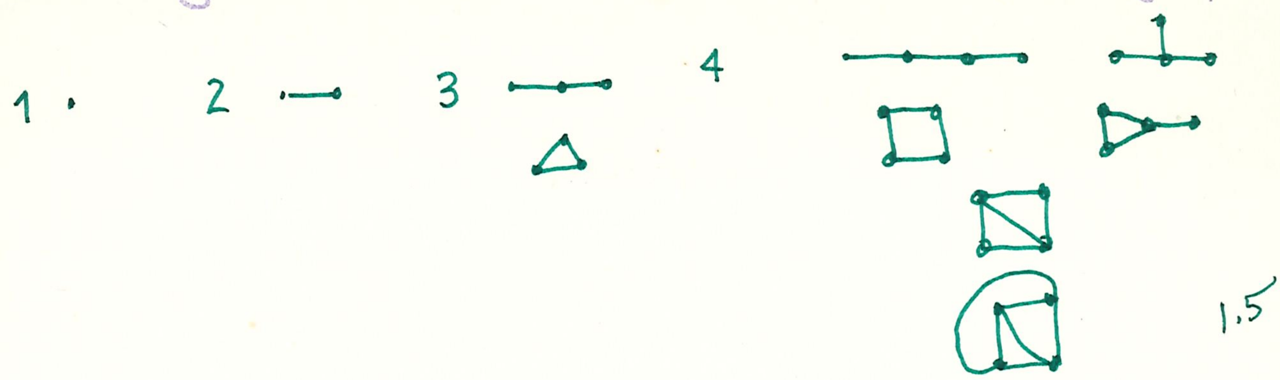

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