TP's (note these rules do NOT apply to HW)

1. Rules
   - A. They must be on $8\frac{1}{2} \times 11$ paper
   - B. They must be written in ink
   - C. They must use one side of each page
   - D. If there is more than one page, then the pages must be stapled or paper-clipped together.

Failure to follow any rule cost a point each

2. Grades
   - A. Graded on a 0 to 10 basis
   - B. Graded on your reasoning, your ability to express your reasoning, neatness and your English.
3. TP average is computed using only your best n out of the m assigned where $\frac{n}{m} \approx \frac{2}{3}$
4. Since TP's are assigned a week in advance of their due date, the solutions are graded as if they are carefully worked out.
5. They must be your OWN work

TP 1 due 3 sept 86 Wed

Given: $a_0 = 10$ and for $n \geq 1$ $a_n = 3a_{n-1} + 2 \cdot 3^n$

Prove by induction $a_n = 10 \cdot 3^n + 2n3^n$

MD2

TP2 due Fri 5 Sept

The relation R is defined on the set $\mathbb{R}$ of all real numbers by $xRy \iff x+y \geq -10$.

For each of the following statements provide either a proof or disprove the statement (i.e. give a counterexample)

A. R is transitive
B. R is reflexive
C. R is irreflexive
D. R is Symmetric
E. R is anti-symmetric
F. R is Asymmetric

TP 3 due Mon 8 Sept

Given $a_0 = \sqrt{2}$ & for $n \geq 1$ $a_n = \sqrt{2 + a_{n-1}}$

Prove by induction for $n \geq 0$ $a_n < 2$.

TP 4 due Wed 10 Sept

A. Given R is a partial order on the set A and SR is defined by

$$x \, SR \, y \iff xRy \ \& \ x \neq y$$

<u>Show</u> the relation SR ("strictly R") is irreflexive, asymmetric and transitive

B Given R is a irreflexive, asymmetric and transitive relation on A and RE is defined by

$$x \, RE \, y \iff xRy \text{ or } x = y$$

<u>Show</u> the relation RE ("R or equal") is a partial order on A.

TP 5 due Fri 12 Sept.

<u>Given</u> $a_0 = 10$ and $a_1 = 0$ and for $n \geq 2$

$$a_n + a_{n-1} - 6a_{n-2} = 0$$

<u>Prove by induction</u> $a_n = 6 \cdot 2^n + 4 \cdot (-3)^n$ for $n \geq 0$

Let $P, \leq$ be a poset and let $B \subset P$

1. An element $b \in B$ is called the <u>least</u> or <u>minimum</u> element (resp. <u>greatest</u> or <u>maximum element</u>) of B if for $x \in B$ then $x \geq b$ (resp $x \leq b$)

<u>Thm:</u> B can have at most one least element (or greatest element)

2. An element $b \in B$ is <u>minimal</u> (resp <u>maximal</u>) if $x \in B$ and $x \leq b \Rightarrow x = b$ (resp $x \in B$ & $x \geq b \Rightarrow x = b$)

" A maximum element is maximal but not conversely

3. An element $a \in P$ is an <u>upper bound</u> for B if $b \in B \Rightarrow a \geq b$

An element $c \in P$ is an <u>lower bound</u> for B if $b \in B \Rightarrow c \leq b$.

4. If the set of lower bounds of B has a greatest element, that element is called the <u>greatest lower bound</u> or glb (or inf)

similarly B may have a <u>least upper bound</u> (lub or sup)

5. A <u>lattice</u> is a poset in which each pair of elements has a least upper bound and a greatest lower bound.

TP 6 . due 15 sept

A. Prove by induction: Each finite poset has a maximal element.

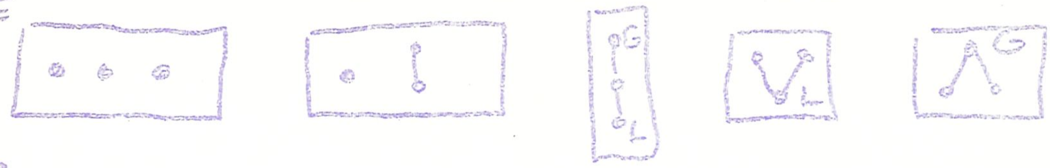
B. Each finite lattice has a maximum element.

A. TP & more answers

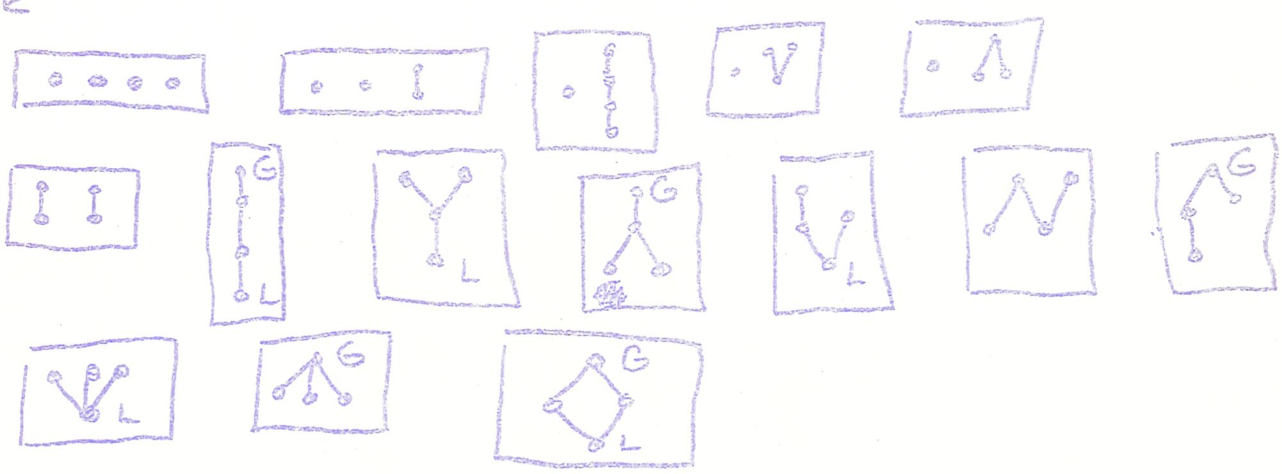
23. ONE

TWO

THREE



FOUR



20 a. $\mathbb{R}, \leq$ ($\mathbb{R}$ itself has no least element also $\{x \in \mathbb{R} : 0 < x\}$

b.  ... etc

c. $\mathbb{R}, \leq$ $\mathbb{R}$ or $\{x \in \mathbb{R} : x < 0\}$

d. $\mathbb{R}, \leq$ $B = \{x \in \mathbb{R} : x < 0\}$ has lub 0 but $0 \notin B$

e. $\mathbb{Q}, \leq$ (rationals) $B = \{x \in \mathbb{Q} : x < \sqrt{2}\}$
the upper bounds to B are $U = \{x \in \mathbb{Q} : x > \sqrt{2}\}$
(since $\sqrt{2}$ is irrational) and U has no least element.

TP 7 Due 19 Sept. For $n \geq 1$ $a_n = (-1)^{n+1}\frac{1}{n}$
and $S_n = \sum_{i=1}^{n} a_i$. Prove for n even ≥ 2
$S_n < S_{n+2} < S_{n+1}$ and for n odd ≥ 1
$S_n > S_{n+2} > S_{n+1}$, By induction.

TP8 due mon 22 sept

Given: R is a ~~relation~~ partial order on A. Relations S and T are defined on A as follows

$$xSy \iff xRy \text{ and } x \neq y$$

$$xTy \iff xSy \text{ or } x = y.$$

[We already know that T is a partial order on A.]

Prove $R = T$

TP9 due wed 24 sept

Given $a_0 = 5$ $a_1 = 16$ ~~&~~ for $n \geq 2$ $a_n = 4a_{n-1} - 4a_{n-2} + 6\cdot 2$

$(a_n = 4a_{n-1} - 4a_{n-2} + 6\cdot 2^n)$

Prove by induction $a_n = 5\cdot 2^n + 3n^2\cdot 2^n$ for $n \geq 0$

TP

due mon 29 sept

complete the following proof of the theorem

Each finite p.o. set has a maximal element

proof: By induction on the number of elements in the p.o. set.

start-up: Suppose A is any p.o. set ordered by $\leq$ and A has one element b.

Now b is maximal since "$x \in A \Rightarrow x = b$" is true so that "$x \in A$ and $x \geq b \Rightarrow x = b$" is certainly true.

inductive step

we assume each p.o. set with n or fewer elements has a maximal element.

Now let $A, \leq$ be a p.o. set with $n+1$ elements.

you
fill in
this part

A has a maximal element $\square$