MAD 3105 Discrete Math 2 **Final** 16 Dec 1993

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

n log n

log n

n ?

1A. What big O (i.e. $O(f(n))$) best describes the run time of the following algorithms: i. Sorting n items by Merge Sort. ii. Searching in a sorted array of n items by Binary Search. iii. Sorting n items by Interchange Sort.

1B. Arrange in increasing order: $O(n^2 \log n), O(n!), O(n^3), O(n^2), O(3^n), O(n^{100}), O(n\sqrt{n}), O(n^2\sqrt{n}), O(2^n)$.

2. Solve: $a_n - 5a_{n-1} + 6a_{n-2} = 0; a_0 = 7, a_1 = 16$.

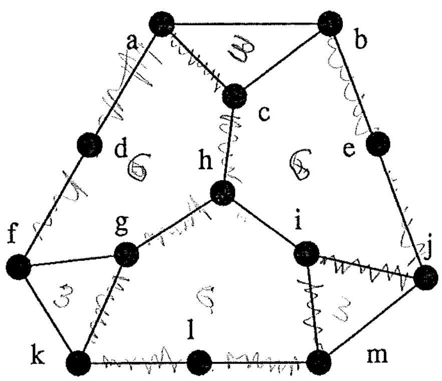
3. For the relation R on the set of integers defined by $xRy \iff x + 3 < y$. Either prove R has the given property or give a counter-example to show it does not have that property.
   A. Reflexive. B. Symmetric. C. Anti-symmetric. D. Transistive.

4. Prove the following relation R is an equivalence relation and describe the equivalence class of (6,4). The relation R is defined on the points of the plane by $(a, b)R(c, d) \iff b - a^2 = d - c^2$. $c^2 - a^2 = d - b$

5. Give the *correct guesses* to the form of the particular solution of the linear non-homogenuous recurrence relation which has the given forcing function. The characteristic polynomial has roots 1, 1, 2 and 5.
   A. $-5 \cdot 3^n$. B. $(2n - 5)4^n$. C. $2 \cdot 5^n$. D. $n^2 - 1$.

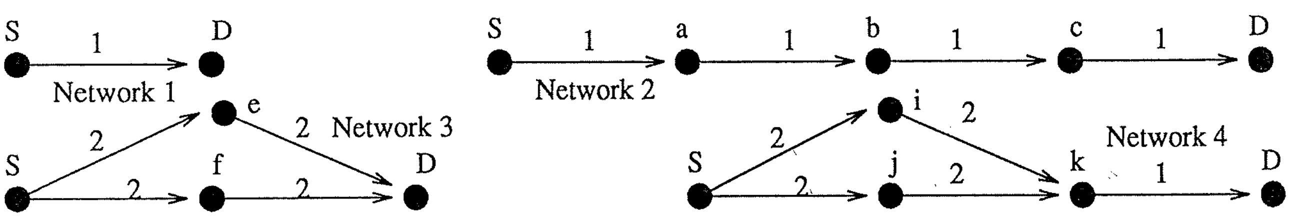
6. Solve: $a_n - a_{n-1} = 2n - 1; a_0 = 5$.

7. Solve: $a_n = 2a_{n/2} + n; a_1 = 1$. (Simplify your answer.)



8. Find $\chi(G)$ for G to the right. Prove G has this chromatic number.

9. Show that G to the right does not have a Hamiltonian cycle:
   A. Without using Grinberg's Theorem.
   B. Using Grinberg's Theorem. *Hint:* Show one of the degree six regions must be outside any Hamiltonian cycle.



10. For the transport networks above:
    A. Which have a unique maximal flow?
    B. Which have a unique minimal cut?
    C. Which have the property that every non-zero integer-valued flow is maximal?
    D. Which have the property that every cut is minimal?

11. Using $|E| \le 3|V| - 6$ and $2|E| = \sum_{v \in V} \deg(v)$. Show plane connected (simple) graphs have a vertex of degree 5 or less. Then show a plane connected (simple) graph with strictly fewer than 30 edges has a vertex of degree 4 or less.

12. There is exactly one 4-regular graph with 5 vertices, K_5, and exactly one 4-regular graph with 6 vertices, the edge graph of the octahedron. For both of these graphs, either give a plane drawing of the graph, or carefully prove the graph is non-planar.