Final DM2 Fall 86 name The good Doctor

Show ALL Work for credit. SS# 1

1-8: 10 pts; 9-16 15pts.

1.A. Find a F-augmenting path for the flow F on the transport network in figure 1. $S \xrightarrow{3} b \xrightarrow{3} d \xleftarrow{3} a \xrightarrow{2} c \xrightarrow{3} d$

B. On the transport network, show how to increase the flow using your path in part A.

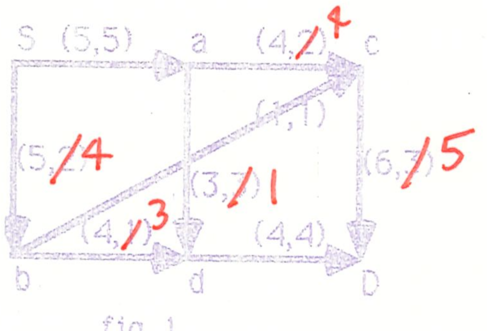


fig. 1

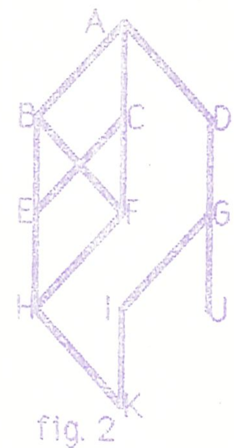


fig. 2

2.For the partial order "Hasse" graph in figure 2 answer:

A. The maximal elements are: A

B. The minimal elements are: J, K

C. Find glb (F, I): K

D. Give a topological enumeration: KJIHGEFDCBA & others

E. Find lub (E, F): ∄

3. The matrix (k=0) is an adjacency matrix of a digraph G. Use Warshall's algorithm to fill in the other matrices.

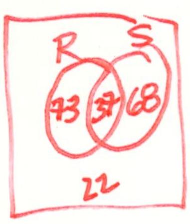
$$\begin{bmatrix} 0&0&0&0\\ 0&0&1&0\\ 1&0&0&1\\ 0&0&1&0 \end{bmatrix} \quad \begin{bmatrix} 0&0&0&0\\ 0&0&1&0\\ 1&0&0&1\\ 0&0&1&0 \end{bmatrix} \quad \begin{bmatrix} 0&0&0&0\\ 0&0&1&0\\ 1&0&0&1\\ 0&0&1&0 \end{bmatrix} \quad \begin{bmatrix} 0&0&0&0\\ 1&0&1&1\\ 1&0&0&1\\ 1&0&1&1 \end{bmatrix} \quad \begin{bmatrix} 0&0&0&0\\ 1&0&1&1\\ 1&0&1&1\\ 1&0&1&1 \end{bmatrix}$$

k = 0 k = 1 k = 2 k = 3 k = 4

4. Define the relation R on the set of non-zero rational numbers by xRy ⇔ x/y is an integer. For each of the property below say "yes" if R has that property else say "no" and give a counterexample.

reflexive	transitive	symmetric	anti-symmetric
yes	yes	no 2R1 not 1R2	no 1R−1 −1R1 but 1 ≠ −1

200
132
68



5.A.Company XYZ has 200 ditch diggers, 110 have round bottom shovels, 37 have both round and square bottom shovels and 22 don't have shovels. How many have square bottom shovels? $68+37=105$

B. How many ways are there to put 202 people into 4 rooms with at least one person in each room?

$$4^{202}-\binom{4}{1}3^{202}+\binom{4}{2}2^{202}-\binom{4}{3}1^{202}+\binom{4}{4}0^{202}$$

6. Solve $a_n = a_{n-1} + 6a_{n-2}$; $a_0 = 1$, $a_1 = 0$.

$x^2 - x + 6 = (x-3)(x+2)$

$a_n = A \cdot 3^n + B(-2)^n$

$1 = a_0 = A + B$

$0 = a_1 = 3A - 2B$

$2 \quad = 2A + 2B$

$2 \quad = 5A$

$A = 2/5 \quad B = 3/5$

$a_n = \frac{2}{5} \cdot 3^n + \frac{3}{5} \cdot (-2)^n$

7. A recurrence relation has characteristic polynomial $(x^2+1)(x-1)(x-2)(x+3)^3$

A. Write the general solution to the homogenuous problem.

$A i^n + B(-i)^n + C + D 2^n + E(-3)^n + Fn(-3)^n + G n^2 (-3)^n$

BCDE. For the given forcing function, write the correct guess for the form of the particular solution:

B. $8 \cdot 4^n$ $A \cdot 4^n$ C. $10n$ $(An+B)n$ D. $6i^n$ Ani^n E. $n(-3)^n$ $(An+B)(-3)^n n^3$

8. Solve $a_n = 3a_{n/7} + 2n$; $a_1 = 5$;

$n = 7^k$

$b_0 = 5$

$b_k - 3b_{k-1} = 2 \cdot 7^k$

p. sol. $b_k = A \cdot 7^k$

$A \cdot 7^k - 3A \cdot 7^{k-1} = 2 \cdot 7^k$

$7A - 3A = 2 \cdot 7$

$4A = 14$

$A = \frac{7}{2}$

h. sol $A 3^k$

$b_k = A \cdot 3^k + \frac{7}{2} \cdot 7^k$

$5 = b_0 = A + \frac{7}{2}$

$\frac{3}{2} = A$

$b_k = \frac{3}{2} \cdot 3^k + \frac{7}{2} \cdot 7^k$

$$\boxed{\begin{aligned} a_n &= \tfrac{3}{2} \cdot 3^{\log_7 n} + \tfrac{7}{2} \cdot 7^{\log_7 n} \\ &= \tfrac{3}{2} \cdot n^{\log_7 3} + \tfrac{7}{2} \cdot n \end{aligned}}$$

9.A. Find the coefficient of x^{40} in:

x^{40} in $(1 + x + x^2 + \dots)^5 (x^5 + x^6 + \dots + x^9)^3 (x^3 + x^4 + x^5 + \dots)^4$

x^{40} in $\frac{1}{(1-x)^5} x^{15}(1 + \dots + x^4)^3 x^{12}(1 + x + \dots)^4$

x^{13} in $\frac{(1-x^5)^3}{(1-x)^{5+3+4}} = \left(1 - \binom{3}{1}x^5 + \binom{3}{2}x^{10} - \dots\right) \sum_{n=0}^{\infty} \binom{n+11}{n} x^n$

$$\boxed{\binom{13+11}{13} - \binom{3}{1}\binom{8+11}{8} + \binom{3}{2}\binom{3+11}{3}}$$

B. Write (but do NOT solve) a generating function and tell which coefficient we need to find in order to count the number of ways of putting 50 identical balls into 20 boxes so that odd numbered boxes have either 1, 3 or 7 balls, boxes 2,4,6,8 and 10 each have at least 2 balls and the rest of the boxes have no more than 3 balls.

x^{50} in $(x^1 + x^3 + x^7)^{10} (x^2 + x^3 + \dots)^5 (1 + x + x^2 + x^3)^5$

10. Solve by generating functions: $a_0 = 16$; and for $n > 0$, $a_n = 3a_{n-1} + 4 \cdot 5^n$.

$10 \cdot 5^n - 3 \cdot 10 \cdot 5^{n-1}$

$(10-6) \cdot 5^n$

$A(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + \sum_{n=1}^{\infty} a_n x^n$

$= 16 + \sum_{n=1}^{\infty} (3a_{n-1} + 4 \cdot 5^n) x^n$

$= 16 + 3x \sum_{n=1}^{\infty} a_{n-1} x^{n-1} + 4 \cdot \sum_{n=1}^{\infty} 5^n x^n$

$= 16 + 3x A x + 4\left[\frac{1}{1-5x} - 5^0 x^0\right]$

$A(1-3x) = 16 + \frac{4}{1-5x} - 4 = \frac{12(1-5x)+4}{1-5x} = \frac{16-60x}{1-5x}$

$\sum a_n x^n = A(x) = \frac{16-60x}{(1-3x)(1-5x)} = \frac{6}{1-3x} + \frac{10}{1-5x} = 6\sum 3^n x^n + 10 \sum 5^n x^n$

$C = \frac{16-60(\frac{1}{3})}{1-5(\frac{1}{3})} = \frac{-4}{-\frac{2}{3}} = 6 \quad D = \frac{16-60(\frac{1}{5})}{1-3(\frac{1}{5})} = \frac{4}{\frac{2}{5}} = 10$

$$\boxed{a_n = 6 \cdot 3^n + 10 \cdot 5^n}$$

11. Give examples:

A. A circuit which isn't a cycle.

B. A digraph of a relation which isn't reflexive or irreflexive.

C. A digraph of a symmetric relation which isn't transitive.

or

D. Find all the unilaterally connected components of

"4"

E. Give the run time complexity of a binary search in an array of length n.

O(log n)

12 & 13. Give counterexamples:

A. A non-trival closed path is a circuit.

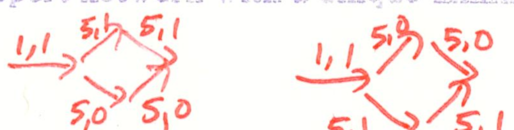
a, b, c

$a \to b \to c \to a \to b \to c \to a$

B. A maximal matching which isn't a complete matching.

C. A maximal directed path is a directed path of maximal length.

D. A transport network with a unique minimal cut has a unique maximal flow.



E. A transport network with a unique maximal flow has a unique minimal cut.

$\xrightarrow{1,1} \xrightarrow{1,1}$

{s}

{s a}

F. A partial order on more than 2 elements is never an equivalence relation.

P.O.

G. For each choice of X being one of the words "weakly", "unilaterally" or "strongly", each X connected digraph G, has a vertex v, so that G-v is X connected.

v

X = strongly

H. If F is a maximal flow and k(X, X^c) = F(X, X^c), then (X, X^c) is a minimal S-D cut.

$S \xrightarrow{1,1} a \xrightarrow{1,1} b \xrightarrow{1,1} D$

X = {S, b}

I. A partially ordered set with only one maximal element has a maximum element.

a

J. The lexicographic ordering on strings of the usual 26 letters is a well ordering.

ab > aab > aaab ...

14. Prove by induction:

Given $a_n = \sqrt{6 + a_{n-1}}$ for $n \geq 1$, and $a_0 = \sqrt{3}$. Prove $a_n < 3$, for $n \geq 0$.

$a_0 = \sqrt{3} < 3$ ✓

assume $a_{n-1} < 3$

hence $a_{n-1} + 6 < 9$

& $a_n = \sqrt{6 + a_{n-1}} < \sqrt{9} = 3$ ✓

15. Prove a partial order is a total order if and only if its digraph is unilaterally connected.

If p.o. is total and a, b ~~in these~~ one elements then either $a \leq b$ or $b \leq a$ and there is a path of length one from a to b or b to a.

If p.o. is unlaterally conn given a, b there (sory) there is a directed path a to b. ~~thus~~ $(a,b) \in E^n \subseteq E$ since p.o. is tran. ∴ $a \leq b$.

16. Prove: If $G = (V, E)$ is a digraph and $n > 0$, then (x, y) is in E^n if and only if there is a directed path of length n from x to y in G.

$n = 1$

$(x,y) \in E \iff$ edge $x \to y$ in G

$\iff$ path of length 1 $x \to y$ in G

assume $(x,y) \in E^n \iff \exists$ path of length n $x \to y$ in G

$(x,y) \in E^{n+1} \iff$ some z $(x,z) \in E^n$ & $(z,y) \in E$

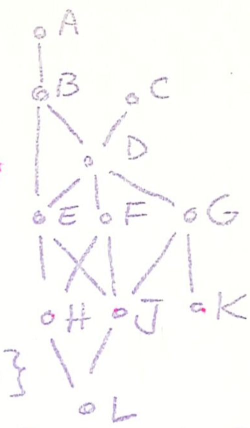
$\iff$ some z path length n $P\ x \to z$ & $z, y \in E$

$\iff$ ~~R~~ · ~~P~~$_x \to z$ followed by $z \to y$ is path

$\iff$ path of length n+1.

DM 2 Test 1 SHOW ALL WORK *1-4: 10pts NAME ______ ANSWERS
*5-8: 15pts NUMBER ______

1. For the p.o. "graph" to the right



A. List all maximal elements. A, C
B. List all minimal elements. L, K
C. List all upper bounds to the set $\{H, J, K\}$ D, C, B, A
D. Find lub $\{H, G\}$ D
E. Find glb $\{A, D\}$ D

2. The relation R is defined on the set $\{1, 2, 3, \ldots\}$ by $xRy \iff x+2 \le y$. For each property below say "yes" if R has that property else say "no" and give a counter example

A. Reflexive
no $0+2 \not\le 0$
$0 \not R 0$

B. Transitive
yes
$x+2 \le y$
$y+2 \le z$
$\Downarrow$
$x+4 \le z$

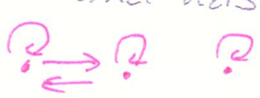
C. Symmetric
no
$0 R 4$
but not $4 R 0$

D. Anti-symmetric
yes
$x+2 \le y$ & $y+2 \le x$
$\Rightarrow x+4 \le x$
$4 \le 0$

3. DRAW Digraphs showing relations with the required properties
A. A relation which isn't transitive

B. A relation which is neither symmetric nor anti-symmetric

C. An equivalence relation on 3 or more vertices which isn't the relation "=" and has more than one equivalence class.



4. A recurrence relation has characteristic polynomial $(x^2+1)(x-1)(x+3)^2$ A. Write the general solution to the homo problem
$Ai^n + B(-i)^n + C + D(-3)^n + En(-3)^n$
BCDE For the given forcing fcn $f(n)$, write the correct guess for the particular solution
B. $8 \cdot 2^n$ $A2^n$ C. $100n$ $(An+B)n$ D. $6i^n$ $Ai^n n$ E. $(n+7)3^n$ $(An+B)3^n$

5. Solve $a_n + 2a_{n-1} = 10 \cdot 3^n$ $a_0 = 20$.
$x+2$
guess part $a_n = A \cdot 3^n$ $a_{n-1} = A \cdot 3^{n-1}$

$A3^n + 2A \cdot 3^{n-1} = 10 \cdot 3^n$

$3 \cdot A + 2A = 30$

$A = 6$

part $6 \cdot 3^n$

gener $a_n = A(-2)^n + 6 \cdot 3^n$

$20 = a_0 = A + 6$

$A = 14$

$a_n = 14 \cdot (-2)^n + 6 \cdot 3^n$

DM2T1P2

6\. A. Find a pair of elements in the p.o. graph in Prob 1 which do not have a lub!

H, J

B. Find a counterexample to "a p.o. set with a least element is well-ordered"

C. Find a counterexample "if R & S are irreflexive, then RS is irreflexive"

D. Find two positive integers x, y so that lcm (x, y) isn't x, y or xy.

E. Subsets A & E so that $(A-E)\cup E \neq E$

7\. Given $a_1 = 5$ and $a_n = 9-2(1-\frac{1}{n})a_{n-1}$ for $n \geq 2$

Prove by induction $a_n = 3+\frac{2}{n}$ for $n \geq 1$.

$a_1 = 3+\frac{2}{1} = 5$

$$a_n = 9-2(1-\tfrac{1}{n})(3+\tfrac{2}{n-1}) = 9-2\left(3-\tfrac{3}{n}+\tfrac{2}{n-1}-\tfrac{2}{n(n-1)}\right)$$

$$= 9-6+\left(\frac{6(n-1)-4n+4}{n(n-1)}\right)$$

$$= 3+\frac{6n-6-4n+4}{n(n-1)} = 3+\frac{2(n-1)}{n(n-1)}$$

$$= 3+\frac{2}{n}$$

8\. Prove: if the finite p.o set A has exactly one minimal element s, then s is the least element of A. (You may use the fact that each finite p.o. set has a minimal element).

Let $x \in A$ Let $B = \{y \in A : y \leq x\}$

B has a minimal element b which is also a minimal element of A. ∴ $b = s$

and $s \leq x$. !

DM 2 Test 2 Show ALL work for credit By ______

23 people ; are worth 10 pts each, 5-8 15 pts each. No

80%

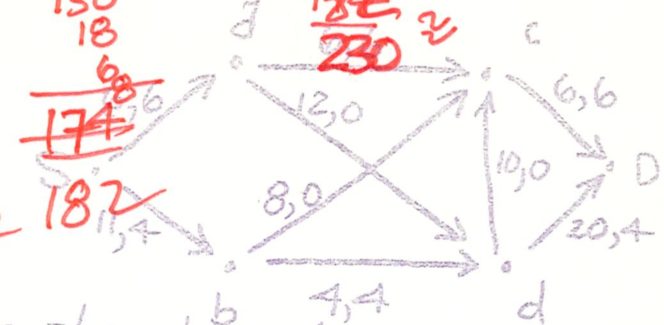
1. A. Find a F-augmenting path for the flow F on the transport network to right

(15) 2nd

15/2/1/0/0/0/0/0/3/0/2

150
18
~~6~~ 8
~~174~~
182

182
$\frac{\cancel{182}}{230}$ ≈



B. On the transport network show how ~~to~~ increase the flow using your path in A.

2. The matrix (k=0) is an adjacency matrix of a digraph G. Use Warshall's algorithm to fill in the other matrices

(9) 3rd

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

k=0 k=1 k=2 k=3 k=4

12/0/5/1/0/2/1/0/0/0/2

120
40
7
10
4
0
181

$\frac{181}{230}$ ≈ 80%

(6) 3. A. Construct the adjacency matrix for the digraph to the right

1st B. Give any topological enumeration of this digraph

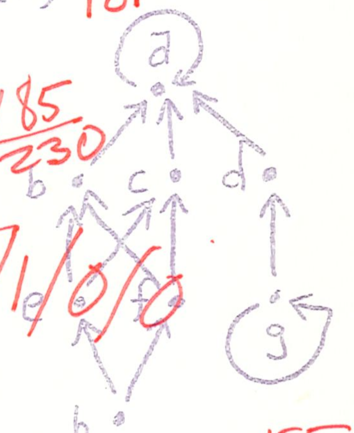
C. Give another topological enumeration different from ~~the~~ the one given in B

13/2/1/1/1/0/2/2/1/0/0

130
18
8
7
8
8

$\frac{185}{230}$

80%



4. Solve $a_n - 5a_{n-1} + 6a_{n-2} = 0$ $a_0 = 1$ $a_1 = 0$.

(5) 4th

12/0/0/2/1/0/0/3/3/0/2

120
14
6
9
6
0
155

$\frac{155}{230}$ ≈ 67

(0) 5. A. give an example of a maximal directed path which isn't a directed path of maximal length.

$\frac{230}{\cancel{115}}$

345

B. draw a digraph of a non-reflexive relation whose transitive closure is reflexive

5th

0/0/1/2/0/0/7/0/0/8/0/1/2/0/0/2

C. give a permutation of <1,2,3,4,5> which has a disorder of 3

13
24
63
48
4
6
158

$\frac{158}{\cancel{345}}$ ≈ 45

D. What is runtime complexity of interchange sort?

E. List ALL the unilaterally connected components of

(3)

#

139/345 ½ 40

6. Give Counterexamples:
A. A flow F on a transport network with zero flow on each out of S must have zero flow on every edge.

$\boxed{1}$ 0/1/0/1/0/1/4/2/0/5/1/2/2/1/3

14
12

B. A graph with two distinct undirected paths x to y must have a undirected cycle.

10
36
16
30
5

7^{th}

C. If each cycle in G has odd length, then each circuit in G has odd length.

8
6
2
139

D. If each vertex v in G has in deg(v) = out deg(v) = 1 then G is strongly connected

E. An F augmenting path P for a flow F must be a simple path

7\. A. Rewrite $3^{\log_9 n}$ as n^k with k as simple as possible
BCD Solve $a_n = 2a_{n/3} + n$ $a_1 = 4$

0/0/1/3/2/2/1/4/1/0/0/1/1/1/0/6

13
36
22
20
9
32
7
4
3
2
148

$\boxed{5}$ 6^{th}

148/345 42

8\. Prove: If G is a digraph (V, E) and $n \geq 1$, then $(x,y) \in E^n \iff$ (if and only if) there is a directed path of length n from x to y.

$\boxed{6}$ 0/0/1/0/0/2/2/0/0/2/1/1/1/1/6/6

8^{th}

83/345 25%

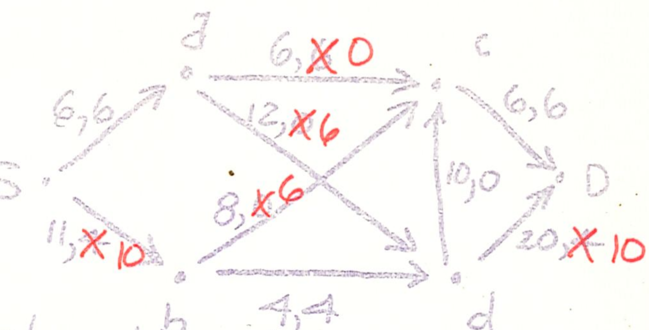
13
20
18
12
5
4
3
2
6
83

DM2 Test 2 Show ALL work for credit By ______________

1–4 are worth 10 pts each, 5–8 15 pts each. No ______________

1. A. Find a F-augmenting path for the flow F on the transport network to right

$S \xrightarrow{7} b \xrightarrow{8} c \xleftarrow{6} a \xrightarrow{12} d \xrightarrow{16} D$



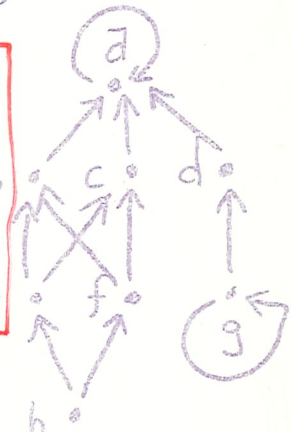
B. On the transport network show how ~~to~~ increase the flow using your path in A.

2. The matrix (k=0) is an adjacency matrix of a digraph G. Use Warshall's algorithm to fill in the other matrices

$$\underset{k=0}{\begin{bmatrix} 0&1&0&0\\ 0&0&0&1\\ 0&1&0&0\\ 1&0&0&0 \end{bmatrix}} \quad \underset{k=1}{\begin{bmatrix} 0&1&0&0\\ 0&0&0&1\\ 0&1&0&0\\ 1&1&0&0 \end{bmatrix}} \quad \underset{k=2}{\begin{bmatrix} 0&1&0&1\\ 0&0&0&1\\ 0&1&0&1\\ 1&1&0&1 \end{bmatrix}} \quad \underset{k=3}{\begin{bmatrix} 0&1&0&1\\ 0&0&0&1\\ 0&1&0&1\\ 1&1&0&1 \end{bmatrix}} \quad \underset{k=4}{\begin{bmatrix} 1&1&0&1\\ 1&1&0&1\\ 1&1&0&1\\ 1&1&0&1 \end{bmatrix}}$$

3. A. Construct the adjacency matrix for the digraph to the right

$$\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&0&0&1&0&0&1&0\\ 0&0&0&0&1&1&0&0 \end{bmatrix}$$



B. Give any topological enumeration of this digraph

C. Give another topological enumeration different from ~~the~~ the one given in B

4. Solve $a_n - 5a_{n-1} + 6a_{n-2} = 0$ $a_0 = 1$ $a_1 = 0$.

$x^2 - 5x + 6$

$(x-2)(x-3)$

$a_n = A \cdot 2^n + B \cdot 3^n$

$a_n = 3 \cdot 2^n - 2 \cdot 3^n$

$1 = a_0 = A + B$

$0 = a_1 = 2A + 3B$

$-2 \quad = -2A - 2B$

$-2 = B$

$A = 3$

5. A. give an example of a maximal directed path which isn't a directed path of maximal length.

$a \quad b \to c$ $P = a$.

B. draw a digraph of a non-reflexive relation whose transitive closure is reflexive $\rightleftarrows$

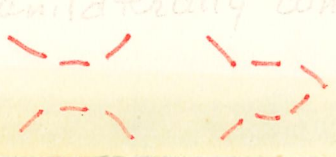
C. give a permutation of ⟨1, 2, 3, 4, 5⟩ which has a disorder of 3

⟨3 2 1 4 5⟩   ⟨4 1 2 3 5⟩

D. What is runtime complexity of interchange sort?

$O(n^2)$

E. List all the unilaterally connected components of



4 of them.

6. Give Counterexamples:

A. A flow F on a transport network with zero flow on each out of S must have zero flow on every edge.

$S \xrightarrow{10,0} \circlearrowright_{1,1}^{1,1} \xrightarrow{10,0} D$

B. A graph with two distinct undirected paths x to y must have a undirected cycle.

$x \xrightarrow{e} y$

$P_1 \quad x \xrightarrow{e} y \xleftarrow{e} x \xrightarrow{e} y$

$P_2 \quad x \rightarrow y$

C. If each cycle in G has odd length, then each circuit in G has odd length.

figure 8 has even

D. If each vertex v in G has in deg(v) = out deg(v) = 1 then G is strongly connected

 !

E. An F augmenting path P for a flow F must be a simple path

$S \xrightarrow{10,0} a \xrightarrow{10,0} D$

$S \to a \to b \to a \to D$

7. A. Rewrite $3^{\log_9 n}$ as n^k with k as simple as possible

$3^{\log_9 n} = 9^{\log_9 3 \log_9 n}$

$= 9^{\log_9 n \log_9 3} = n^{1/2}$

B,C,D,E. Solve $a_n = 2a_{n/3} + n \quad a_1 = 4$

$b_k - 2b_{k-1} = 3^k \quad b_0 = 4$

p sol $A \cdot 3^k$

$A \cdot 3^k - 2A \cdot 3^{k-1} = 3^k$

$3A - 2A = 3$

$A = 3$

g sol

$b_k = A \cdot 2^k + 3 \cdot 3^k$

$4 = b_0 = A + 3$

$A = 1$

$b_k = 2^k + 3 \cdot 3^k$

$a_n = 2^{\log_3 n} + 3n$

$= n^{\log_3 2} + 3n$

8. Prove: If G is a digraph (V, E) and $n \geq 1$, then $(x,y) \in E^n \Longleftrightarrow$ (if and only if) there is a directed path of length n from x to y.

induct $n = 1$ $\quad (x,y) \in E^1 \Leftrightarrow (x,y)$ is path of length 1 (an edge)

assume true for n

$(x,y) \in E^{n+1} \Leftrightarrow \exists z \ (x,z) \in E^n \ \& \ (z,y) \in E$

$\Leftrightarrow$ there exist path P of length n x to z & edge z to y

$\Leftrightarrow$ there exist path Q x to y of length n+1 & (z,y) is last edge on Q

$\Leftrightarrow \exists$ path of length n+1 x to y

Show ALL Work for credit. SS# ____________

1-8: 10 pts; 9-16 15pts.

1.A. Find a F-augmenting path for the flow F on the transport network in figure 1.

14/2/0/0/1/0 →

B. On the transport network, show how to increase the flow using your path in part A.

140
18
6
164

#1.

$\frac{164}{170} = \boxed{96.5\%}$

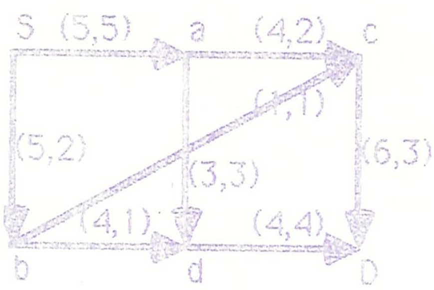


fig. 1

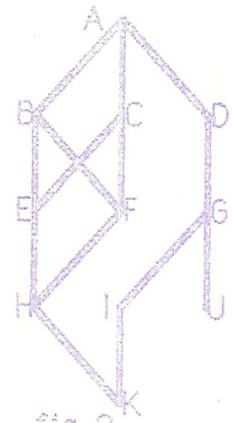


fig. 2

#4

30
64
14
18
5
131

2.For the partial order "Hasse" graph in figure 2 answer:

A. The maximal elements are: ____________

B. The minimal elements are: ____________

C. Find glb {F, I}: ____________

D. Give a topological enumeration: ____________

E. Find lub {E, F}: ____________

$\frac{131}{170} = \boxed{77.1\%}$

3/0/8/2/3/1/0 →

3. The matrix (k=0) is an adjacency matrix of a digraph G. Use Warshall's algorithm to fill in the other matrices.

10/0/3/1/0/0/0/0/3/0/0

100
24
7
6
137

#2

k = 0	k = 1	k = 2	k = 3	k = 4
$\begin{bmatrix} 0&0&0&0 \\ 0&0&1&0 \\ 1&0&0&1 \\ 0&0&1&0 \end{bmatrix}$	[ ]	[ ]	[ ]	[ ]

$\frac{137}{170} = \boxed{80.6\%}$

4. Define the relation R on the set of non-zero rational numbers by xRy ⇔ x/y is an integer. For each of the property below say "yes" if R has that property else say "no" and give a counterexample.

reflexive	transitive	symmetric	anti-symmetric

0/0/0/14/0/0/1/2/0/0/0

98
4
6
108

#9

$\frac{108}{170} = \boxed{63.5\%}$

5.A.Company XYZ has 200 ditch diggers, 110 have round bottom shovels, 37 have both round and square bottom shovels and 22 don't have shovels. How many have square bottom shovels?

40
16
42
6
4
3
4
115

4/0/2/6/1/0/1/1/2/0/0

B. How many ways are there to put 202 people into 4 rooms with at least one person in each room?

#5 tie

$A_i = \{x_i = 0\}$

$A_1^c \cap \dots \cap A_4^c$

164
131
137
108
540
115
655

$\frac{115}{170} = \boxed{67.6\%}$

655 ≈ $\boxed{77.1\%}$

90 8 ~~35~~ 0 3 — 136

9/0/1/~~4~~ 5/0/0/0/1/0/0/1 — #3

$\boxed{\frac{136}{170} \approx 80.0\%}$

7. A recurrence relation has characteristic polynomial $(x^2+1)(x-1)^4(x-2)(x+3)^3$

A. Write the general solution to the homogenuous problem.

BCDE. For the given forcing function, write the correct quess for the form of the particular solution:

B. $8 \cdot 4^n$____________ C. $10n$____________ D. $6i^n$____________ E. $n(-3)^n$____________

3/1/3/1/2/3/1/5/1/0/2 — #10

30 9 24 7 12 15 4 3 2 — 106

$\boxed{\frac{106}{170} \approx 62.4\%}$

8. Solve $a_n = 3a_{n/7} + 2n$; $a_1 = 5$;

80 16 6 5 6 2 — 115

8/0/2/0/1/1/0/2/1/0/2 — #6 tie 5

$\boxed{\frac{115}{170} \approx 67.6\%}$

9.A. Find the coefficient of x^{40} in:

$(1 + x + x^2 + \ldots)^5 (x^5 + x^6 + \ldots + x^{93})^3 (x^3 + x^4 + x^5 + \ldots)^4$

3/0/2/1/3/3/1/0/0/1/0/1/0/0/1/1

45 26 12 33 30 9 6 4 1 — 166 — #8

$\boxed{\frac{166}{255} \approx 65.1\%}$

B. Write (but do NOT solve) a generating function and tell which coefficient we need to find in order to count the number of ways of putting 50 identical balls into 20 boxes so that odd numbered boxes have either 1, 3 or 7 balls, boxes 2,4,6,8 and 10 each have at least 2 balls and the rest of the boxes have no more than 3 balls.

10. Solve by generating functions: $a_0 = 16$; and for $n>0$, $a_n = 3a_{n-1} + 4 \cdot 5^n$.

5/0/0/3/1/1/0/0/0/0/0/0/2/1/1/3

75 36 11 10 6 2 1 — 141 — #11

$\boxed{\frac{141}{255} \approx 55.3\%}$

136	170
106	170
115	170
166	255
141	255
664	1020

$\boxed{65.1\%}$

11. Give examples:
A. A circuit which isn't a cycle.

B. A digraph of a relation which isn't reflexive or irreflexive.

C. A digraph of a symmetric relation which isn't transitive.

D. Find all the unilaterally connected components of

E. Give the run time complexity of a binary search in an array of length n.

12 & 13. Give counterexamples:
A. A non-trival closed path is a circuit.

B. A maximal matching ~~which~~ is ~~not~~ a complete matching.

C. A maximal directed path is a directed path of maximal length.

D. A transport network with a unique minimal cut has a unique maximal flow.

E. A transport network with a unique maximal flow has a unique minimal cut.

F. A partial order on more than 2 elements is never an equivalence relation.

#11, 12 & 13

G. For each choice of X being one of the words "weakly", "unilaterally" or "strongly", each X connected digraph G, has a vertex v, so that G-v is X connected.

H. If F is a maximal flow and $k(X, X^c) = F(X, X^c)$, then (X, X^c) is a minimal S-D cut.

I. A partially ordered set with only one maximal element has a maximum element.

J. The lexicographic ordering on strings of the usual 26 letters is a well ordering.

-6	-12	-18	-21	-24	-26	-30	-33	-42
1	1	5	2	3	1	1	2	1

39
33
135
48
63
19
15
24
3

379/765!

49.5%

14. Prove by induction:

Given $a_n = \sqrt{6 + a_{n-1}}$ for $n \ge 1$, and $a_0 = \sqrt{6}$. Prove $a_n < 3$, for $n \ge 0$.

#7

5/1/0/2/0/2/2/1/1/0/0/1/0/0/0/2

$\boxed{\frac{170}{255} \approx 66.7\%}$

75
14
24
20
18
8
7
4
170

15. Prove a partial order is a total order if and only if its digraph is unilaterally connected.

#16

0/0/0/0/0/4/0/1/1/0/2/0/0/1/1/~~7~~

$\boxed{\frac{68}{255} \approx 26.7\%}$

40
8
7
10
2
1
68

16. Prove: If G = (V, E) is a digraph and n > 0, then (x, y) is in E^n if and only if there is a directed path of length n from x to y in G.

#15

2/1/0/0/0/0/0/0/0/0/5/0/4/0/0/5

30
14
25
12
81

$\boxed{\frac{81}{255} \approx 31.8\%}$

170
68
81
319

$\frac{319}{765} \approx 41.7\%$

1. For the p.o. "graph" to the right ~~1/8/6/1/2~~ 12/0/8/0/6/0/1/0/2/0/0

A. List all maximal elements. ______

B. List all minimal elements ______

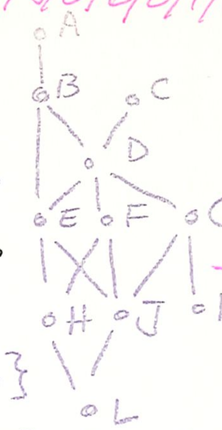
C. List all upper bounds to the set $\{H, J, K\}$ ______

D. Find lub $\{H, G\}$ ______

E. Find glb $\{A, D\}$ ______

1

78.6%



120
64
36
4
4
228

2. The relation R is defined on the set $\{1, 2, 3, \dots\}$ by $xRy \iff x+2 \le y$. For each property below say "yes" if R has that property else say "no" and give a counter example

A. Reflexive	B. Transitive	C Symmetric	D. Anti-symmetric

14/0/0/6/1/6/0/2

77.2%

2

140
42
6
30
6
224

3. Draw Digraphs showing relations with the required properties

A. A relation which isn't transitive

4/0/0/7/0/0/16/0/0/0/2

40
49
48
177

B. A relation which is neither symmetric nor anti-symmetric

5

61.0%

C. An equivalence relation on 3 or more vertices which isn't the relation "=" and has more than one equivalence class.

4. A recurrence relation has characteristic polynomial $(x^2+1)(x-1)(x+3)^2$ A. Write the general solution to the homo problem

2/0/11/0/8/0/4/0/2/0/2

6

60.7%

BCDE For the given forcing fcn f(n), write the correct guess for the particular solution

B. $8 \cdot 2^n$ ______ C. $100n$ ______ D. $6i^n$ ______ E. $(n+7)3^n$

20
88
48
16
4
176

5. Solve $a_n + 2a_{n-1} = 10 \cdot 3^n$ $a_0 = 20$.

7/0/3/1/0/3/1/0/1/0/3/3/5/0/0/2

7

435

56.1%

105
39
12
30
9
7
15
12
15
244

6. A. Find a pair of elements in the p.o. graph in Prob 1 which do not have a lub!

7/0/0/5/0/0/ 9/8/0/4/0/0/ 3/0/0/1

B. Find a counterexample to "a p.o. set with a least element is well-ordered"

105
60
81
24
9
279

C. Find a counterexample "if R & S are irreflexive, then RS is irreflexive"

64.1%

3

D. Find two positive integers x, y so that lcm (x, y) isn't x, y or xy.

E. Subsets A & E so that $(A-E) \cup E \neq E$

7. Given $a_1 = 5$ and $a_n = 9 - 2(1-\frac{1}{n})a_{n-1}$ for $n \geq 2$

Prove by induction $a_n = 3 + \frac{2}{n}$ for $n \geq 1$.

12/0/0/1/1/0/0/1/0/3/3/6/9/9/1/0

4

61.8%

180
12
11
8
18
15
24
1
269

8. Prove: if the finite p.o set A has exactly one minimal element s, then s is the least element of A. (You may use the fact that each finite p.o. set has a minimal element).

1 @ 5 & 28 @ 0

01.1%

8