MAD 3104 — Discrete Math 1

Section 2, Spring 1995.

Instructor: Bellenot.

The good doctor's Office: 002-B Love, Office Hours: MWF 10:20–11:00 or by appointment. Email addressed bellenot@cs.fsu.edu, bellenot@math.fsu.edu, or even bellenot@fsu.edu will get to the good doctor, but the short address 'bellenot' works on math or cs machines.

Eligibility: A grade of C- or better in Pre-Calculus Mathematics (MAC 1140)

Text: Mott, Kandel, Baker *Discrete Mathematics for cs & math* 2^{nd} Edition.

Coverage: Parts of Chapters 1, 2, 4, and 5.

Final: At 12:30 – 2:30 am Thursday, Apr 27, 1995.

Tests: (3) Tentatively at Feb 1, Mar 1?(or 8?) and Apr 12. No Makeup tests.

Quizes: Every Wednesday (except test days). No Makeup quizes.

Grades: 90% A, 80%B, 70%C, 60%D.

Relative Weights F = 2T and T = Q (F is 1/3, each T is 1/6 and Q is 1/6).

Homework and Attendance are required. Indeed attendance will be taken by checking off homework. It is the student's responsibility to see that homework is delivered on time. (The homework needs to be turned in even when the student is absent.) Likewise, being absence is not a valid reason for not knowing the next assignment.

Four or more late or missing homeworks is an automatic FAIL.

Fair Warning: The good doctor teaches this course differently than other instructors. The order of material will be different and so will be the emphasis.

Relations

Problems: For the given A and R and each of the properties: A. reflexive, B. symmetric, C. anti-symmetric and D. transitive, decide if R has the property. If it has the property then prove it has that property or if it doesn't have the property then give a counterexample to show the property fails. (I.e. Prove or disprove.)

1. A is the set of real numbers and $aRb \iff a \leq b$.
2. A is the set of real numbers and $aRb \iff a < b$.
3. A is the set of real numbers and $aRb \iff 0 \leq a - b \leq 2$.
4. A is the set of real numbers and $aRb \iff |a - b| < 2$.
5. A is the set of odd positive integers and $aRb \iff a \neq b$ and a evenly divides b.
6. A is the set of real numbers and $aRb \iff a^2 - b^2 = 0$.
7. A is the set of positive integers and $aRb \iff a \quad \text{divides} \quad b$.
8. A is the set of integers and $aRb \iff a - b$ is odd.
9. A is the set of positive integers and $aRb \iff a \equiv 1 \mod b$.
10. A is the set of integers and $aRb \iff a \cdot b$ is even.
11. A is the set of points in the plane and $(a, b)R(c, d) \iff (a - c)^2 + (b - d)^2 \leq 5$.
12. A is the set of points in the plane and $(a, b)R(c, d) \iff a + b = c + d$.
13. A is the set of points in the plane and $(a, b)R(c, d) \iff |a - b| = |c - d|$.
14. A is the set of points in the plane and $(a, b)R(c, d) \iff a = c$.
15. A is the set of points in the plane and $(a, b)R(c, d) \iff a = d$.
16. A is the set of triangles in the plane and $tRs \iff$ triangle t has the same area as triangle s.
17. A is the set of triangles in the plane and $tRs \iff$ triangle t is similar to triangle s.
18. A is the set of triangles in the plane and $tRs \iff$ triangle t has either at least as much area as triangle s, or triangle t has at least as large perimeter as triangle s.
19. A is the set $\{1, 2, 3, \{1\}, \{1, 3\}, \{2\}\}$ and $aRb \iff a \in b$.
20. A is the set power set of $\{1, 2, 3\}$ and $aRb \iff a \subseteq b$.

Execises

1. Show the following statements are equivalent (for a simple graph G): [Hint: $a \iff b$ is Th 5.3.1, $a \implies c$ is Th 5.3.3, $a \iff d$ is Th 5.3.5, $a \iff e$ is Ex 7, $a \iff f$ is Ex 8]
   a. G is a tree (For vertices x and y of G there is a unique xy-path in G).
   b. G is connected and acyclic. (Acyclic means it has no cycles.)
   c. G is connected and $|E| = |V| - 1$.
   d. G is acyclic and $|E| = |V| - 1$.
   e. G is connected and each edge e is a bridge. (e is a bridge or cut edge means $G - e$ is disconnected.) [This says a tree is a minimal connected graph.]
   f. G is acyclic and if $x, y \in V(G)$ and $e = xy \notin E(G)$ then $G + e$ has a cycle. [This says a tree is a maximal acyclic graph.]

2. Show the following statements are equivalent (G a simple graph):
   a. G is connected, but not a tree, and for vertices x and y of G there are at most two simple xy-paths in G.
   b. G is connected and has exactly one cycle.
   c. G is connected and $|E| = |V|$.
   d. G has an edge e so that $G - e$ is connected and acyclic.
   e. There is a tree T and an edge $e \notin E(T)$ so that $G = T + e$.

3. Show the following statements are equivalent (G a simple graph):
   a. There is an edge $e \notin E(G)$ so that $G + e$ is a tree.
   b. G is acylic and $|E| = |V| - 2$.
   c. G is acylic, disconnected and there is an edge so that $G + e$ is connected.
   d. G is the disjoint union of two trees. [G has two components both of which are trees.]
   e. If G is disconnected and for x, y vertices of G in different components, then for $e = xy$, $G + e$ is connected.

The following are from old tests.

4. Prove G is a forest $\iff$ every edge of G is a cut edge.

5. Prove by induction (on the number of vertices), if T is a tree, then $|V(T)| = |E(T)| + 1$.

6. Prove if G is acyclic and $|E(G)| = |V(G)| - 2$, then there is an edge $e \in E(\bar{G})$ so that $G + e$ is a tree.

7. Using the formula $\sum_{v \in V(G)} \deg v = 2|E(G)|$, or induction, or any other method, prove that a tree with a vertex of degree 3 has at least 3 vertices of degree 1. [*Hint:* the tree must have at least 4 vertices.]

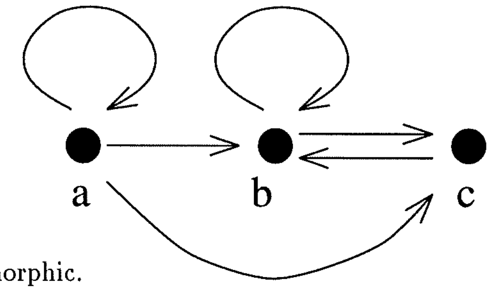
8. Prove if G is connected and $|E(G)| = |V(G)|$, then there is an edge $e \in E(G)$ so that $G - e$ is a tree.

MAD 3104 Discrete Math 1 **Test 1** 1 Feb 1995

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

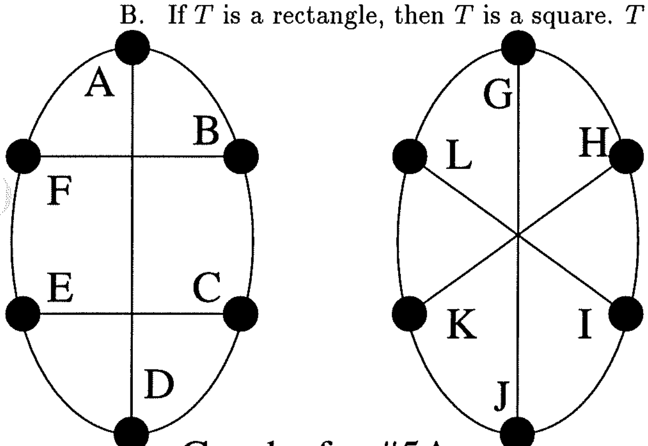
1. For $A = \{2,3,4,5,6\}$ and the relation R defined on A by $aRb \iff a = 3$ or $b = 5$.
 A. Draw the digraph of this relation R.
 B. Determine the order ($|V|$), the size ($|E|$) and give the in-degree and out-degree of each vertex of this digraph.

2. Give counterexamples to the statements below. The relation R is the one given by the digraph below.
 A. R is reflexive.
 B. R is irreflexive.
 C. R is symmetric.
 D. R is anti-symmetric.
 E. R is transitive.

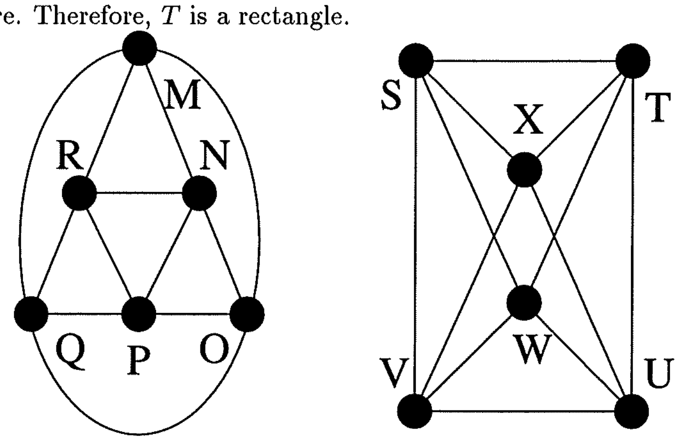


3. Give counterexamples to statements below.
 A. Every odd integer > 1 is prime.
 B. Every graph has at least one edge.
 C. There is no graph with degree sequence $(2,2,2,3,3)$.
 D. Any two graphs the degree sequence $(3,2,2,1,1,1)$ are isomorphic.

4. For each part, decide whether the logic is valid or invalid and draw a Venne diagram to support your answer.
 A. If $x+2=x$, then x is blue. $x+2=x$. Therefore, x is blue.
 B. If T is a rectangle, then T is a square. T is a square. Therefore, T is a rectangle.



Graphs for #5A



Graphs for #5B

5. Consider the graph pairs above.
 A. Draw the complement to each of the graphs for Problem 5A and either produce an isomorphism between the pair of graphs or prove none exists.
 B. The graphs for Problem 5B are isomorphic, produce an isomorphism.

In Problems 6 and 7: For the given A and R and each of the properties: A. reflexive, B. symmetric, C. anti-symmetric and D. transitive, decide if R has the property and if it doesn't have the property then give a counterexample to show the property fails.

6. A is the set of positive integers and $aRb \iff a+b$ is odd.

7. A is the set of points in the plane and $(a,b)R(c,d) \iff |a-c| \leq |b-d|$.

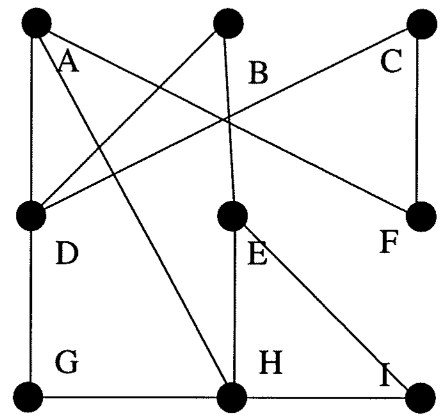
8. Negate the following statements and re-write them so that words like "not" or "no" are not used.
 A. For all triangles T, the area$(T) \geq$ perimeter(T).
 B. For some integers x, x is odd and x^2 is even.

9. A is the set of reals and $aRb \iff a + 10 \leq b$.
 A. Give counter-examples to show R is not reflexive and not symmetric.
 B. Give proofs to show R is anti-symmetric and transitive.

10. Prove by contradiction:
 A. Prove a graph with 35 edges and 16 vertices has a vertex of degree at least 5.
 B. Prove a graph with 45 edges and 24 vertices has a vertex of degree at most 3.

MAD 3104 Discrete Math 1 **Test 2** 1 Mar 1995

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.



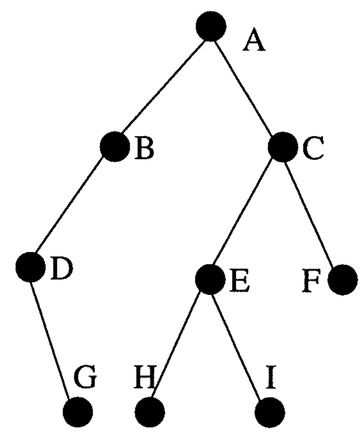
1. For graph to the right.
 A. Find the DFS spanning tree.
 B. Find the BFS spanning tree.

2. Draw the binary tree.
 A. With level-order vertices 1, 2, 3, 5, 6, 10, 13, 27, 54, 55.
 B. For the expression $((a+b)*(r-s)+7)/(u*v-x/y)$.
 C. BST for the data 60, 10, 80, 20, 50, 70, 40.

3. Make pairwise nonisomorphic lists.
 A. List all 6 trees with 5 edges.
 B. List all 5 trees with degree sequence $(1,1,1,1,2,2,3,3)$.

4. Give examples in the graph in the upper right.
 A. A closed path which is not a circuit.
 B. A circuit which is not a cycle.

Give examples in the binary tree in the lower right.
 C. A vertex on level 3.
 D. A vertex where it is not height balanced.



5. For the binary tree to the right, list the vertices in:
 A. Preorder.
 B. Inorder.
 C. Postorder.

6. Prove by induction (on n) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

7. For each degree sequence below:
Either show the sequence is not graphic
Or construct a graph with the given degree sequence.
 A. (1,2,3,3,4,4,4)
 B. (1,1,2,4,4,6)
 C. (0,2,2,4,4,4)
 D. (0,3,3,4,4,4)

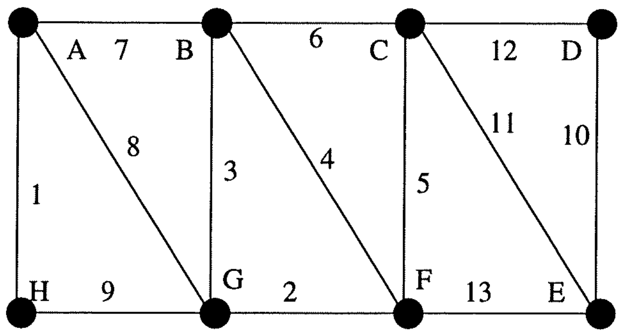
8. Given $a_0 = 3$, $a_1 = 0$ and $a_{n+1} = 6a_n - 8a_{n-1}$ for $n \geq 1$, prove by induction, for each integer $n \geq 0$, $a_n = 6 \cdot 2^n - 3 \cdot 4^n$.

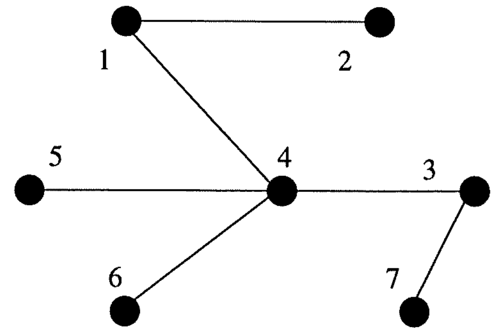
9. Prove by contradiction:
 A. Prove a graph with 41 edges and 20 vertices has a vertex of degree at least 5.
 B. Prove a graph with 49 edges and 25 vertices has a vertex of degree at most 3.

10. Prove by induction: For each integer $n \geq 0, 4^n > n^2$.

MAD 3104 Discrete Math 1 **Test 3** 12 Apr 1995

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.





1. For graph (above left), list the edges IN THE ORDER SELECTED by:
   A. Prim's Algorithm
   B. Kruskal's Algorithm

2. Prüfer codes:
   A. Give the Prüfer code for the tree (above right).
   B. Draw the tree with Prüfer code (4,4,3,5,2)

3 & 4. How many 8 digit license plates are there
   A. Altogether.
   B. Using only 4, 4, 4, 5, 8, 8, 9, and 0.
   C. With only even digits.
   D. With strictly increasing digits.
   E. With exactly 2 distinct digits using exactly 4 of each.

5 & 6. How many ways are there to put 33 balls into 7 distinct boxes if
   A. The balls are distinct.
   B. The balls are identical.
   C. The balls are identical, and box 5 has exactly 8 balls.
   D. The balls are identical, and each box is non-empty.

7 & 8. How many bridge hands are there with
   A. Only red cards (Diamonds and hearts are red, spades and clubs are not.)
   B. With 5 pairs. (No other pairs, no 3 of a kind.)
   C. Exactly one spade.
   D. At least one spade.
   E. With exactly one 2 and exactly 7 clubs.

8. Prove: If e is an edge not in G so that $G + e$ is a tree, then G is acyclic and $|V(G)| - 2 = |E(G)|$

9. Prove: G is connected and $|E| = |V|$ then G has an edge e so that $G - e$ is a tree.

10. Prove by induction on the number of vertices: If F is a forest with k components then $|E(F)| = |V(F)| - k$. [Extreme cases: 1. F has k isolated vertices and 2. F is a tree (i.e. $k = 1$).]