MAD 3104 — Discrete Math 1

Section 3, Spring 1993.

Instructor: Bellenot.

The good doctor's Office: 002-B Love, Office Hours: MW 1:30–2:20 or by appointment.

Text: Mott, Kandel, Baker; 2nd Edition.

Coverage: Some or all of chapters 1, 2, 4 and 5.

Final: At 7:30 Monday Apr 26, 1993.

Tests: (3) Tentatively at Feb 3, Mar 3 and Apr 14. No Makeup tests.

Quizes: Every Wednesday (except test days). No Makeup quizes.

Grades: 90% A, 80%B, 70%C, 60%D.

Relative Weights F = 2T and T = Q (F is 1/3, each T is 1/6 and Q is 1/6).

Homework and Attendance are required.

Fair Warning: The good doctor teaches this course differently than other instructors. The order of material will be different and so will be the emphasis.

Relations

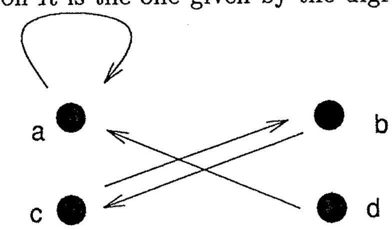
A list of relations for various problems.

1. R on $\mathbf{R}$ (the reals) defined by aRb is never true.
2. R on $\mathbf{N}$ (the natural numbers) defined by aRb is always true.
3. R on $\mathbf{R}$ defined by $aRb \Leftrightarrow a^2 + b^2 = 1$.
4. R on $\mathbf{N}$ defined by $aRb \Leftrightarrow a = b$ and a is even.
5. R on $\{1, 2, ...6\}$ defined by $aRb \Leftrightarrow a = 3$ or $b = 5$.
6. R on subsets of $\{1, 2, ...6\}$ defined by $aRb \Leftrightarrow a \cap b = \emptyset$.
7. R on triangles in the plane defined by $aRb \Leftrightarrow a$ is similar to b.
8. R on $\{0, ..2^{32}-1\}$ defined by $aRb \Leftrightarrow a+b \geq 2^{32}$. (Overflow on addition for unsigned 32 bit integers.)
9. R on people defined by $aRb \Leftrightarrow a$ and b were born on the same day of the week.
10. R on people defined by $aRb \Leftrightarrow a$ has more hair than b, either the number of hairs on $a \geq$ the number of hairs on b or the total length of hair on $a \geq$ the total length of hair on b. (A Hairy relation.)
11. R on names defined by $aRb \Leftrightarrow a$ has more letters than b.
12. R on names defined by $aRb \Leftrightarrow a$ comes before b in dictionary order.
13. R on C++ classes defined by $aRb \Leftrightarrow a$ is a subclass of b. (Note C++ allows multiple inheritance, a can be a subclass of more than one class, but not cycles of subclasses, if a is a subclass of b, then b and all of its superclasses are not subclasses of a.)

MAD 3104 Discrete Math 1 **Test 1** 1 Feb 1993

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

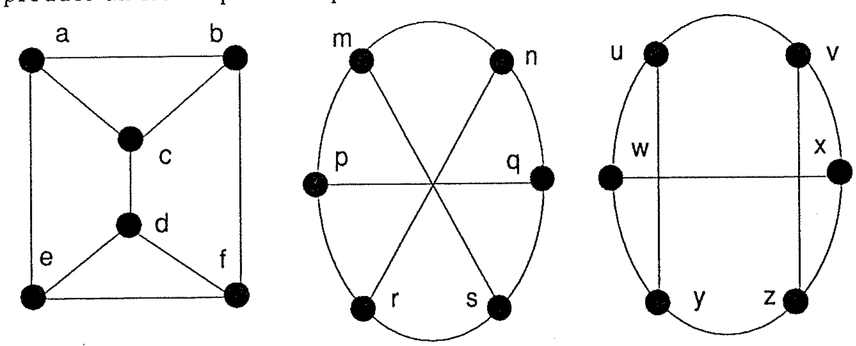
1. For $A = \{2,3,4,5,6\}$ and the relation R defined on A by $aRb \iff a = 3$ or $b = 5$.
   A. Draw the digraph of this relation R.
   B. Determine the order ($|V|$), the size ($|E|$) and give the in-degree and out-degree of each vertex of this digraph.
2. Give counterexamples to the statements below. The relation R is the one given by the digraph to the right.



   A. R is reflexive.
   B. R is irreflexive.
   C. R is symmetric.
   D. R is anti-symmetric.
   E. R is transitive.
3. Give counterexamples to statements below.
   A. There is no graph with degree sequence $(2,2,2,3,3)$.
   B. Every prime is odd.
   C. A symmetric and transitive relation is reflexive.
   D. K_3 is the smallest non-null *multigraph* with only even vertices.
   E. Two graphs with the same degree sequence are isomorphic.
4. For each part, decide whether the logic is valid or invalid and draw a Venne diagram to support your answer.
   A. If it is true love, then you will marry.
   You will marry.
   Therefore, it is true love.
   B. If the lines ℓ_1 and ℓ_2 are two sides of a square, then ℓ_1 and ℓ_2 are parallel.
   The lines AB and BC are two sides of a square.
   Therefore, AB and BC are parallel.

In Problems 5 and 6: For the given A and R and each of the properties: A. reflexive, B. symmetric, C. anti-symmetric and D. transitive, decide if R has the property and if it doesn't have the property then give a counterexample to show the property fails.

5. A is the set of real numbers and $aRb \iff 0 \le a - b \le 2$.
6. A is the set of odd positive integers and $aRb \iff a \ne b$ and a evenly divides b.
7. Using the formula $\sum_{v \in V(G)} \deg v = 2|E|$
   A. Show a graph with 35 edges and 16 vertices has a vertex of degree at least 5.
   B. Show a graph with 35 edges and $\Delta \le 3$ has at least 24 vertices.
   C. Show a regular graph with 143 edges has either 286, 143, 26, or 22 vertices. (Be sure to rule out 11 and 13 vertices.)
8. Consider the pairs (Graph 1 and Graph 2) and (Graph 1 and Graph 3) of graphs below. In each case, either produce an isomorphism or prove none exists.

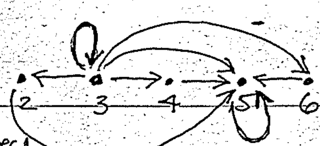


Graph 1 Graph 2 Graph 3

9. Prove by induction that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$.
10. Prove by induction that $n! > 2^n$ for $n \ge 4$.

Test 1 Answers

1. order = 5
size = 9



2,4,6 have in deg 1 out deg 1

3 has out deg 5 in deg 1 5 has out deg 1 in deg 5

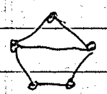
2. A Either b, c or d are counterexamples. bRb is false.

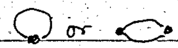
B. a is a counterexample since aRa.

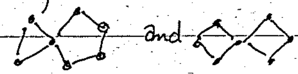
C. dRa but not aRd

D. bRc and cRb but b ≠ c.

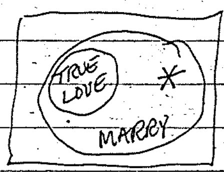
E. bRc and cRb but not bRb [or cRb and bRc but not cRc]

3 A  B. 2 is an even prime. C the relation with digraph [no edges] 

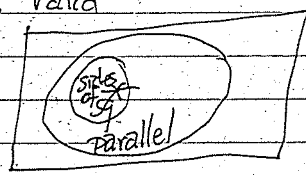
D.  E. ~~many~~

E. many possible examples. For one 

4 A. invalid



B. Valid



5A. Ref yes B. Sym no 2R0 but not 0R2

C. Anti-sym: yes D. Trans no 4R2 and 2R0 but not 4R0

6A Ref no 3R3 is false B. Sym no 3R9 but not 9R3

C. Anti-Sym yes D. Trans yes

7A. If every vertex v had $\deg v \le 4$

then $2|E| = \sum_{v \in V(G)} \deg v \le \sum_{v \in V(G)} 4 = 16 \cdot 4 = 64$

and the graph would have only 32 edges

B. If there were only 23 vertices and $\deg v \le 3$

then $2|E| = \sum_{v \in V(G)} \deg v < \sum_{v \in V(G)} 3 = 23 \cdot 3 = 69$

~~So the graph would have only 34 edges~~

7C Suppose the graph is k-regular then $2|E| = k|V| = \sum_{v \in V(G)} \deg v$

so $k|V| = 2|E| = 286 = 2 \cdot 11 \cdot 13$

Thus k can be 1, 2, 11, 13, 22, 26, 143, 286

if $k=1$ $|V| = 286$

$k=2$ $|V| = 143$

$k=11$ $|V| = 26$

$k=13$ $|V| = 22$

$k=22$ $|V| = 13$ } can't happen since max

$k=26$ $|V| = 11$ } deg is $|V|-1$

8. Graph 1 & Graph 2 are not isomorphic

graph 1 has triangles (every vertex is in a triangle)

and graph 2 has no triangles

an isomorphism between graph 1 a b c d e f

and graph 2 u y w x v z

9 startup

LHS = 1 ✓ RHS = 1^2 ✓

inductive step assume true for n=k: $1 + 3 + \dots + (2k-1) = k^2$

consider $\underbrace{1 + 3 + \dots + (2k-1)}_{k^2} + (2(k+1)-1) =$

$k^2 + (2k+2-1) = k^2 + 2k + 1 = (k+1)^2$

∴ it is true for n = k+1.

10. startup

LHS 4! = 24 RHS = $2^4 = 16$ LHS > RHS ✓

inductive step assume true for n=k: $k! > 2^k$

consider $(k+1)! = (k+1)k! > (k+1)2^k$

since $n \ge 4$ $(k+1) \ge 2$ so $(k+1)2^k \ge 2 \cdot 2^k = 2^{k+1}$

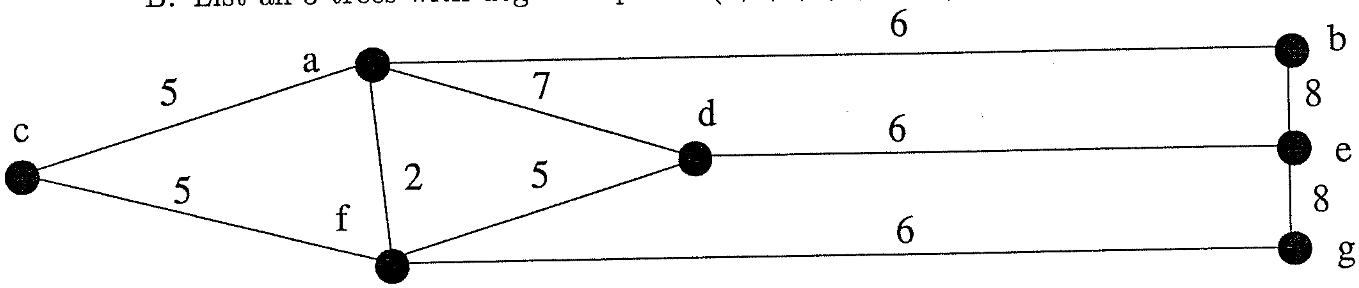
thus $(k+1)! > 2^{k+1}$

and it is true for n = k+1

MAD 3104 Discrete Math 1 **Test 2** 3 Mar 1993

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. Make sure your lists do not contain duplicate isomorphic copies.
   A. List all 6 trees with 5 edges.
   B. List all 5 trees with degree sequence $(1,1,1,1,2,2,3,3)$.



Graph 1

2. For Graph 1.
   A. Find the DFS spanning tree.
   B. Find the BFS spanning tree.
3. Give examples in Graph 1.
   A. A ce-path which is not simple.
   B. Two simple ce-paths P and Q such that P followed by Q backwards is a cycle.
   C. Two simple ce-paths P and Q such that P followed by Q backwards is a circuit but not a cycle.
   D. Two simple ce-paths P and Q such that P followed by Q backwards is not a circuit.
4. Using the formula $\sum_{v \in V(G)} \deg v = 2|E(G)|$
   A. Prove a graph with 35 edges and 16 vertices has a vertex of degree at least 5.
   B. Prove a graph with 45 edges and 24 vertices has a vertex of degree at most 3.
5. Prove by induction (on n) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
6. Find both minimal cost spanning trees for Graph 1.
7. Prove by induction (on the number of vertices), if T is a tree, then $|V(T)| = |E(T)|+1$.
8. Prove G is a forest $\iff$ every edge of G is a cut edge.
9. Prove if G is acyclic and $|E(G)| = |V(G)| - 2$, then there is an edge $e \in E(\bar{G})$ so that $G + e$ is a tree.
10. Using the formula $\sum_{v \in V(G)} \deg v = 2|E(G)|$, or induction, or any other method, prove that a tree with a vertex of degree 3 has at least 3 vertices of degree 1. [*Hint:* the tree must have at least 4 vertices.]

Execises

1. G is a simple graph with 5 vertices and 7 edges.
   A. Show G has a vertex of degree at least 3.
   B. Show G could have no vertex of degree 3.
   C. Show G has a vertex of degree at most 2.
   D. Show G could have no vertex of degree 2.
   E. Show G has no isolated vertices.
   F. Show G could have no vertex of degree 4.

2. Give counter examples.
   A. A closed path is a circuit.
   B. If P and Q are distinct simple xy-paths, then P followed by Q backwards is a cycle.
   C. If P and Q are simple xy-paths, then P followed by Q backwards contains a cycle.
   D. If P is a non-trivial xy-path, then P contains a simple non-trivial xy-path.
   E. If P is a simple xy-path and Q is a simple yz-path, then P followed by Q is a simple xz-path.

3. Show the following statements are equivalent (for a simple graph G): [Hint: $a \iff b$ is Th 5.3.1, $a \implies c$ is Th 5.3.3, $a \iff d$ is Th 5.3.5, $a \iff e$ is Ex 7, $a \iff f$ is Ex 8]
   a. G is a tree (For vertices x and y of G there is a unique xy-path in G).
   b. G is connected and acyclic. (Acyclic means it has no cycles.)
   c. G is connected and $|E| = |V| - 1$.
   d. G is acyclic and $|E| = |V| - 1$.
   e. G is connected and each edge e is a bridge. (e is a bridge or cut edge means $G - e$ is disconnected.) [This says a tree is a minimal connected graph.]
   f. G is acyclic and if $x, y \in V(G)$ and $e = xy \notin E(G)$ then $G + e$ has a cycle. [This says a tree is a maximal acyclic graph.]

4. Show the following statements are equivalent (G a simple graph):
   a. G is connected, but not a tree, and for vertices x and y of G there are at most two simple xy-paths in G.
   b. G is connected and has exactly one cycle.
   c. G is connected and $|E| = |V|$.
   d. G has an edge e so that $G - e$ is connected and acyclic.
   e. There is a tree T and an edge $e \notin E(T)$ so that $G = T + e$.

5. Show the following statements are equivalent (G a simple graph):
   a. There is an edge $e \notin E(G)$ so that $G + e$ is a tree.
   b. G is acylic and $|E| = |V| - 2$.
   c. G is acylic, disconnected and there is an edge so that $G + e$ is connected.
   d. G is the disjoint union of two trees. [G has two components both of which are trees.]
   e. If G is disconnected and for x, y vertices of G in different components, then for $e = xy$, $G + e$ is connected.

MAD 3104 - DISCRETE MATHEMATICS I

PREREQUISITES. You must have passed MAC 1140 (College Algebra) with a grade of C- or better or have appropriate transfer or placement credit.

TEXT. Discrete Mathematics for Computer Scientists and Mathematicians, Second Edition, by Mott, Kandel and Baker

COURSE CONTENT. Chapters 1, 2, 4, and 5.

COURSE OBJECTIVES. The purpose of this course is to introduce students to the theory of deductive reasoning and to elementary counting principles and the principles of graph theory.

EXAM POLICY. No makeup tests or quizzes will be given. An absence from a unit test may be excused if the student presents sufficient evidence of extenuating circumstances. If an absence from a unit text is excused, the final exam grade will be used as the grade for the missed test. You will be allowed to drop two (2) quiz grades. Notes may not be used on tests or quizzes without specific authorization.

GRADING. There will be three unit tests, and each of these tests will count equally. The final examination will be weighted as 25% or 33%, whichever is to your advantage. A short quiz will be given each week except during a week in which an hour test is given. The cumulative quiz grade will be counted as one unit test in calculating final averages. Letter grades will be based on the following scale:

A: 90-100; B: 80-89; C: 65-79; D: 60-64

with +/- grades assigned at the discretion of the instructor.

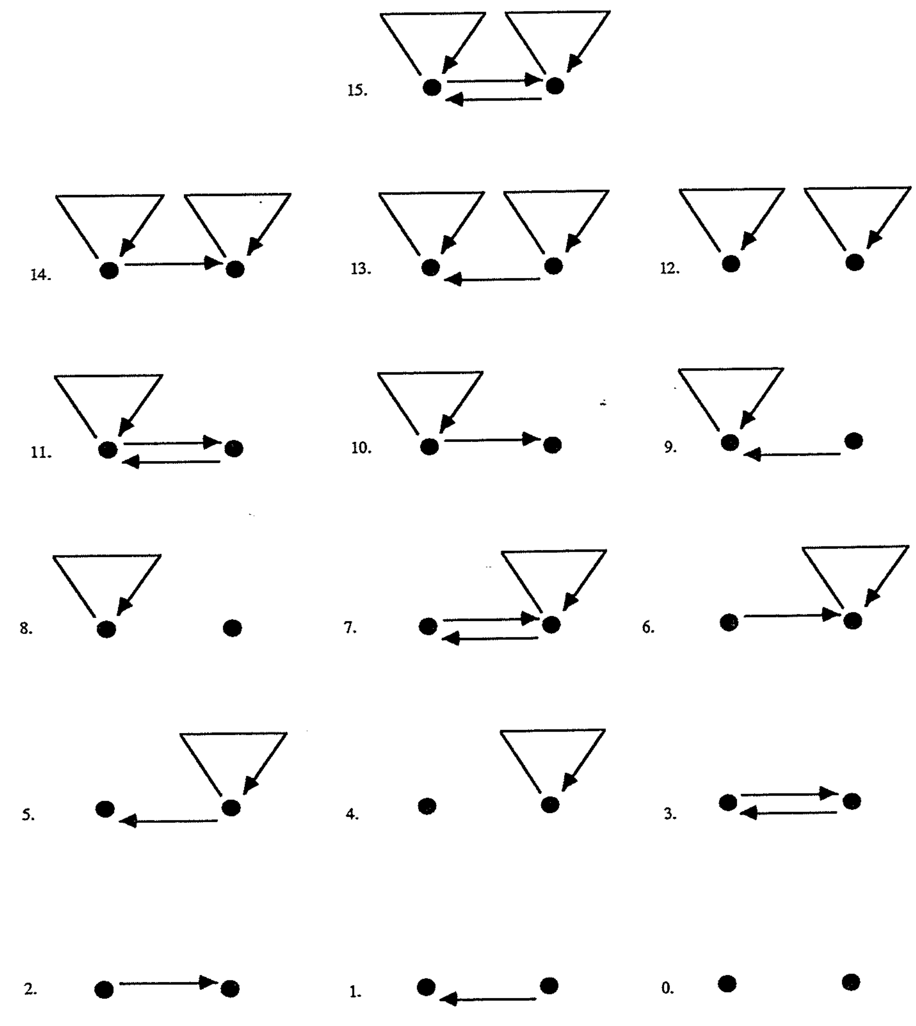
TEST 1: §§ 1.3-1.10
TEST 2: §§ 2.1-2.8
TEST 3: §§ 4.1-4.2, 5.1-5.4

FINAL EXAM will include §§5.5-5.6.

Relations

A list of relations for various problems.

1. R on $\mathbf{R}$ (the reals) defined by aRb is never true.
2. R on $\mathbb{N}$ (the natural numbers) defined by aRb is always true.
3. R on $\mathbf{R}$ defined by $aRb \Leftrightarrow a^2 + b^2 = 1$.
4. R on $\mathbb{N}$ defined by $aRb \Leftrightarrow a = b$ and a is even.
5. R on $\{1, 2, ...6\}$ defined by $aRb \Leftrightarrow a = 3$ or $b = 5$.
6. R on subsets of $\{1, 2, ...6\}$ defined by $aRb \Leftrightarrow a \cap b = \emptyset$.
7. R on triangles in the plane defined by $aRb \Leftrightarrow a$ is similar to b.
8. R on $\{0, ..2^{32} - 1\}$ defined by $aRb \Leftrightarrow a+b \geq 2^{32}$. (Overflow on addition for unsigned 32 bit integers.)
9. R on people defined by $aRb \Leftrightarrow a$ and b were born on the same day of the week.
10. R on people defined by $aRb \Leftrightarrow a$ has more hair than b, either the number of hairs on $a \geq$ the number of hairs on b or the total length of hair on $a \geq$ the total length of hair on b. (A Hairy relation.)
11. R on names defined by $aRb \Leftrightarrow a$ has more letters than b.
12. R on names defined by $aRb \Leftrightarrow a$ comes before b in dictionary order.
13. R on C++ classes defined by $aRb \Leftrightarrow a$ is a subclass of b. (Note C++ allows multiple inheritance, a can be a subclass of more than one class, but not cycles of subclasses, if a is a subclass of b, then b and all of its superclasses are not subclasses of a.)

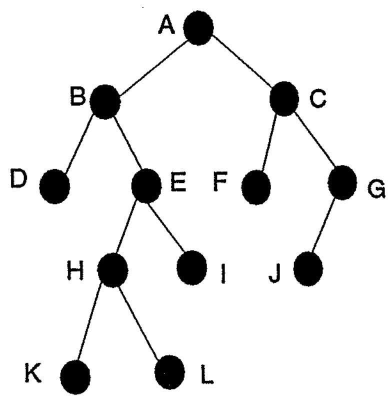


The Sixteen Relations on a Two Element Set

MAD 3104 Discrete Math 1 **Test 3** 14 Apr 1993

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. Draw the following binary trees.
   A. The BST (Binary Search Tree) for: 50, 70, 100, 30, 40, 80, 60.
   B. The binary tree with level-order vertices: 1, 2, 3, 4, 5, 6, 7, 10, 12, 25, 51, 102, 103



2. For the binary tree above, list the vertices in:
   A. Preorder
   B. Inorder
   C. Postorder
3. Negate the following and re-write them so that the word 'not' (nor any similar word) is not used.
   A. All solutions to $x^2 = 1$ are even.
   B. Some square roots are irrational.
   C. If G is pink, then H is not blue.

4. & 5. How many license plates are there with 7 letters?
   A. Altogether?
   B. That don't repeat any letters?
   C. That only use the letters AAEEEOU?
   D. That have exactly one 'Z'?
   E. That have at least one 'Z'?

6. How many ways are there to put 100 balls into 12 distinct boxes?
   A. If the balls are identical?
   B. If the balls are distinct?
   C. In either case prove that some box has at least 9 balls.
7. Prove if G is acyclic and $|E(G)| = |V(G)| - 2$, then there is an edge $e \in E(\bar{G})$ so that $G + e$ is a tree.

8. & 9. How many of 5 card poker hands are there?
   A. With 3 spades and 2 clubs?
   B. With 4 of a kind?
   C. Exactly 2 pair?
   D. At least one pair?
   E. With exactly one King and exactly 4 spades?

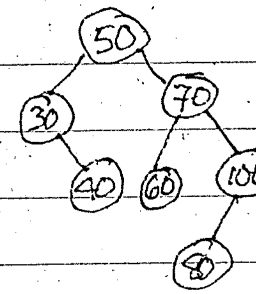
10. Let $B(n)$ be the number of binary trees with n vertices. Let $B(0) = 1$ (there is only one way to have an empty tree). Compute $B(1)$, $B(2)$ and $B(3)$ and prove for $n \geq 0$,

$$B(n+1) = \sum_{i=0}^{n} B(i)B(n-i).$$

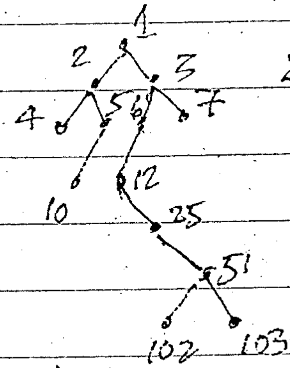
[Hint: Delete the root.]

Answers Test 3

1. A



B.



2A. ABDEHKLICFGJ
B. DBKHLEIAFCJG
C. DKLHIEBFJGCA

3\. A. Not (All solutions to $x^2=1$ are even) $\Leftrightarrow$ some solutions to $x^2=1$ are odd

B\. Not (Some square roots are irrational) $\Leftrightarrow$ all square roots are rational

C\. If G is pink, then H is not blue $\Leftrightarrow$ G is not pink or H is not blue

so negation is "G is pink and H is blue"

4&5. A. 26^7 B. $26\cdot25\cdot24\cdot23\cdot22\cdot21\cdot20$ C. $\frac{7!}{2!\,3!\,1!\,1!} = \binom{7}{2}\binom{5}{3}\binom{2}{1}\binom{1}{1}$

D\. $7\cdot25^6$ E. 26^7-25^7

6\. A. $\binom{100+12-1}{12-1}$ B. 12^{100}, C. If every box had ≤ 8 balls then there would be only $8\cdot12=96<100$ balls. A contradiction.

7\. G is not connected. (Since G is acyclic, if it was connected then it would be a tree and $|E(G)|=|V(G)|-1$ and not $|E(G)|=|V(G)|-2$.) Let e be an edge connecting two components of G. G+e is acyclic still and $|E(G+e)|=|E(G)|+1=|V(G)|-2+1=|V(G+e)|-1$ so G+e is a tree.

8&9 A. $\binom{13}{3}\binom{13}{2}$ B. $\binom{13}{1}\binom{4}{4}\binom{48}{1}$ C. $\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1}$

D\. $\binom{52}{5}-\binom{13}{5}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}$ E. $\frac{1}{K♤}\binom{12}{3}\binom{39-3}{1}+\binom{3}{1}\binom{12}{4}$

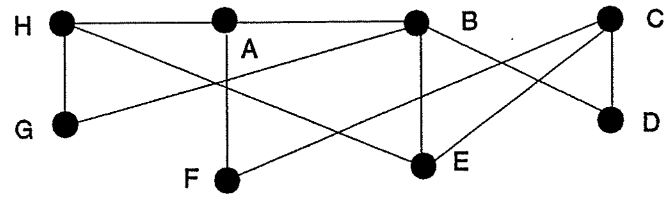
10\. B(1): "·" so B(1) = 1; B(2): "/" or "\\" so B(2) = 2; B(3): ... B(3) = 5. Each binary tree with n+1 vertices has two subtrees. if the left subtree has i vertices then the right subtree has n−i vertices. Note i ranges from 0 to n. Any binary tree with i vertices can be the left subtree & similarly for right. So

$B(n+1) = B(0)B(n) + B(1)B(n-1) + \dots + B(n-1)B(1) + B(n)B(0) = \sum_{i=0}^{n} B(i)B(n-i)$

MAD 3104 Discrete Math 1 **Final** 26 April 1993

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

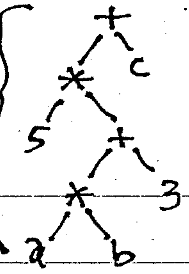
1. Binary trees.
   A. List all binary trees with 2 edges.
   B. Rewrite $5 * (a * b + 3) + c$ in postfix.



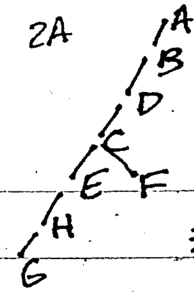
2. For the graph above.
   A. Find the DFS spanning tree.
   B. Find the BFS spanning tree.
3. The vertex A in a binary tree has level-order number $2^{k+3}+7$, what are the following numbers (simplify your answers).
   A. The height of the vertex A?
   B. The level-order number of the left child of A?
   C. The level-order number of the right child of A?
   D. The level-order number of the parent of A?
   E. The level-order number of the parent of the parent of A?
4. Give counterexamples to statements below.
   A. Every tree has a vertex of degree one.
   B. No graph has an odd number of even vertices.
   C. Forests are disconnected.
   D. Every tree with ten vertices has a vertex with degree three or more.
   E. If $|V(G)| = |E(G)|$, then for every $e \in E(G), G - e$ is a tree.
5. How many license plates are there with twelve letters using the just the ten greek letters Δ, Π, Ω, Σ, Θ, Λ, Γ, Ξ, Ψ or Φ?
   A. That do not start with a Ω?
   B. That have exactly one Δ?
   C. That have at least one Δ?
6. For the relation R, defined on the set of integers by $aRb \iff |a-b| < 2$, and for each of the properties: A. reflexive, B. symmetric, C. anti-symmetric and D. transitive, decide if R has the property and if it doesn't have the property then give a counterexample to show the property fails.
7. Prove by **INDUCTION** (without using $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$) that
$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3}.$$
8. Using the formula $\sum_{v \in V(G)} \deg v = 2|E|$
   A. Prove a graph with 35 edges and 16 vertices has a vertex of degree at least 5.
   B. Prove a graph with 45 edges and 24 vertices has a vertex of degree at most 3.
9. How many ways are there to put 25 balls into 10 distinct boxes?
   A. If the balls are identical?
   B. If the balls are distinct?
   C. In either case prove that some box has at least 3 balls.
10. How many five card poker hands are there?
   A. At least one pair?
   B. With at most two suits?
   C. With exactly two suits?
11. Prove by induction (on the number of vertices) If T is a tree, then $|V(T)| = |E(T)| + 1$.
12. Prove if G is connected and $|E(G)| = |V(G)|$, then there is an edge $e \in E(G)$ so that $G - e$ is a tree.

1A. / ⟨ ^ ⟩ \ \

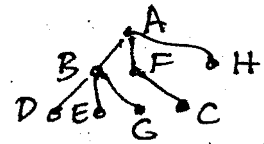
B. 5 a b * 3 + * c +



2A



2B



4A "." B. "△" C. "." D"---------" E. ▽ 1 ← e

3A. K+3 3B. $2^{k+4}+14$ 3C. $2^{k+4}+15$ 3D. $2^{k+2}+3$ 3E. $2^{k+1}+$

5A. $9\cdot 10^{11}$ B. $12\cdot 9^{11}$ C. $10^{12}-9^{12}$

6A. yes ~~B.~~ no 1R2 & 2R1 but 1≠2 B. yes D. no 1R2 & 2R3 but 1R̸3

7. Start up n=1 LHS $1\cdot 2=2$ RHS $= 1\cdot 2\cdot 3/3 = 2$ ✓

Ind step assume $1\cdot 2+\dots+n(n+1) = \frac{n(n+1)(n+2)}{3}$ and consider

$\underbrace{1\cdot 2+\dots+n(n+1)}_{\text{IND HYP}} + (n+1)(n+2) = \frac{n(n+1)(n+2)}{3} + (n+1)(n+2)\cdot\frac{3}{3}$ ~~factor out (n+1)~~

$= \frac{(n+1)(n+2)}{3}[n+3]$ done [last step factored out $\frac{(n+1)(n+2)}{3}$]

8. A suppose not then for all v deg v ≤ 4 so

$70 = 2(35) = 2|E| = \sum \deg v \le \sum 4 = 4|V| = 4(16) = 64$ or $70 \le 64$

B suppose not then for all v deg v ≥ 4 so

$90 = 2(45) = 2|E| = \sum \deg v \ge \sum 4 = 4|V| = 4(24) = 96$ or $90 \ge 96$

contradictions

9 A. $\binom{25+10-1}{10-1}$ B 10^{25} C. Suppose not, then each box has ≤ 2 balls so there ≤ 2 × 10 = 20 balls A contraction.

10. A. $\binom{52}{5} - \binom{13}{5}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}$ B. $\binom{4}{2}\binom{26}{5}$ (the two suits) C. $\binom{4}{2}\binom{26}{5} - \binom{4}{1}\binom{13}{5}$ (one suiters)

11. Start up |V| = 1, T is "." so |E| = 0 and |V(T)| = |E(T)| + 1

Ind Step Suppose trees with n vertices satisfy |V| = |E| + 1 and let T be a tree with n+1 vertices. Let x be a vertex of degree one in T. T−x is a tree with n vertices and one less edge than T so |V(T)| = |V(T−x)| + 1 = |E(T−x)| + 1 + 1 = |E(T)|. (IND HYP)

12. G has a cycle. (Otherwise it would be connected & acyclic hence a tree & |V(G)| = |E(G)| + 1) Let e be a cycle edge. G−e is still connected (cycle edges are not cut edges) and |V(G−e)| = |V(G)| = |E(G)| = |E(G)−e)| + 1 so G−e is connected and |V(G−e)| = |E(G−e)| + 1 so G−e is a tree.