DM 1 TP2 due Thurs 19 JAN 84

Disprove: A. For all integers $n \geq 1$, $n^2 - 20n + 103 > \sqrt{n}$.

B. A loop-free graph each of whoses vertices are of degree two is a cycle graph C_n (for some integer n)

C.D. If P and Q are different simple paths in a graph G going from node x to node y, and R is the path formed by following P from x to y and then Q backwards from y to x, then R is a cycle.

TP 1 induction

ENDTP

Solutions HW 1.

1.11 *3 initial case $n=1$ $LHS = 1^2 = 1$, $RHS = \frac{1\cdot 2\cdot 3}{6} = 1$

inductive step for some k assume the statement is true i.e.

$$\sum_{i=1}^{k} i^2 = 1^2 + 2^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

consider $$\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)^2}{6}$$

$$= \frac{(k+1)}{6}\left[k(2k+1) + 6(k+1)\right] = \frac{(k+1)}{6}\left[2k^2 + 7k + 6\right]$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$ which is $\frac{n(n+1)(2n+1)}{6}$ with n replaced by $k+1$

$\therefore$ the statement is true for $k+1$.

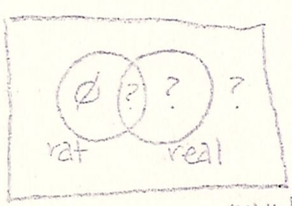
*8 initial case $n=1$ $LHS = 1\cdot 2 = 2$ $RHS = \frac{1\cdot 2\cdot 3}{3} = 2$

inductive step assume $\sum_{i=1}^{k} i(i+1) = \frac{k(k+1)(k+2)}{3}$

now $$\sum_{i=1}^{k+1} i(i+1) = \sum_{i=1}^{k} i(i+1) + (k+1)(k+2) = \frac{k(k+1)(k+2)}{3} + \frac{3(k+1)(k+2)}{3}$$

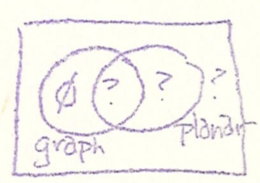
$$= \frac{(k+1)(k+2)}{3}\left[k+3\right] = \frac{(k+1)(k+2)(k+3)}{3}$$ thus the statement is true for $k+1$

1.9 *1 (u)

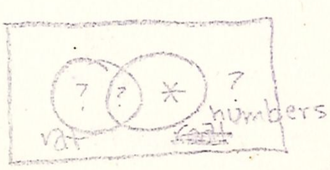


(t) doesn't fit

(s)

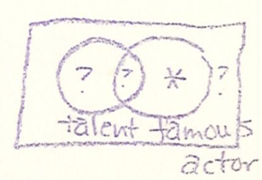


(q) (r)

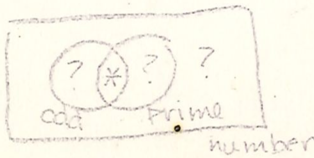


(p)

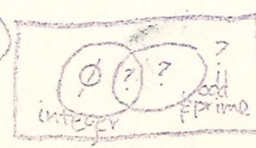
(o)



(n)

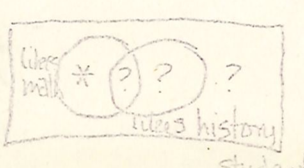


(m)(l)

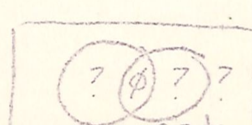


(k)?

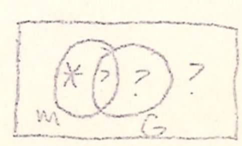
(j)



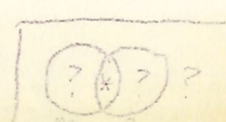
(i)



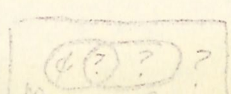
(g)



(f)



(e)



DM4 TP7 Due Thurs 9 Feb 84

Given $a_0 = 4$, $b_0 = 2$ and for $n \geq 1$ $a_n = 5a_{n-1} + b_{n-1}$, $b_n = a_{n-1} + 5b_{n-1}$

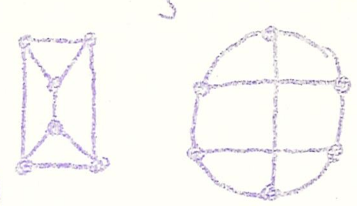
Prove by induction for $n \geq 0$ $a_n = 3 \cdot 6^n + 4^n$, $b_n = 3 \cdot 6^n - 4^n$

PT 1 (Practice Test)

1 Define A Connected Graph B. ~~Deg of~~ Degree of a vertex

2 Give examples A. A path which is not simple B. A circuit which isn't a cycle C. A simple path which starts and ends at the same vertex but isn't a cycle, D. A path with at least one edge which starts and ends at the same node which isn't a circuit.

3. Either produce an isomorphism or show why none exists

(or )

(Find a spanning tree for these graphs)

4. List ALL trees (exactly one for each isomorphism class) with 6 edges [or loop free graphs with 6 nodes & 9 edges)

5. For the statements to right
   A. Draw Venn Diagrams for each statement
   B. Is the conclusion valid or not WHY?

Apples are fruit
Food with seeds is fruit
∴ Apples are food with seeds

6. A. Write the negation of the sentence below so that the quantifiers precede the not
   For each boy there is a girl so that he is willing to work for her
   B. Explain why the sentence above is different from
   There is a girl so that each boy is willing to work for her

7. Give counterexamples
   A. A graph with a cut-edge has a cut-node
   B. A graph with a cut-node has a cut-edge
   C. Every tree has a vertex of degree one
   D. ~~There is~~ A graph with n nodes and $\frac{n(n-1)}{2}$ edges is connected

8 Prove by Induction $n! > 2^n$ for $n \geq 4$

9. Prove: A connected graph G is a tree if and only if G has no cycles

10. Explain why there is no loop-free graph connected graph with degree sequence (1,1,1,1,1,1,1,3,4)

DM 1. TP 15 due Mar 18 84 (after break)

[Note Test 2 is on Mar 15]

A. Show multiplication is well-defined in $\mathbb{Z}_n$ $(n \geq 1)$ that is if $r R_n s$ and $i R_n t$ then $r \cdot i \, R_n \, s \cdot t$
[HINT ADD & SUB: SL]

Some notation lets write $x | y$ for x divides y (with no remainder)
useful facts

(1) if $0 \leq i < n$ and $n | i$ then $i = 0$

(2) if n is prime and $n | ab$ then $n | a$ or $n | b$

B. Now suppose n is prime, in $\mathbb{Z}_n$: if $[\ell] \neq 0$ and $[j][\ell] = [k][\ell]$ then $[j] = [k]$

C. Again in $\mathbb{Z}_n$ for n prime: if $[\ell] \neq 0$ then for every $[k]$ there is a $[j]$ so that $[j][\ell] = [k]$
[HINT: Show $[0][\ell], [1][\ell], \ldots [n-1][\ell]$ are distinct]

Comment: B & C together show that $[j]/[k]$ is defined in $\mathbb{Z}_n$ if $[k] \neq [0]$ and n is prime.

HW 1. Compute $[3]+[5]$; $[3]-[5]$; $[3]\cdot[5]$; $[3]/[5]$ in $\mathbb{Z}_7$, in $\mathbb{Z}_{13}$ express your answers as $[i]$ where $0 \leq i < n$.

2. Show if $x \in \mathbb{Z}$ then $[x^2] = [0]$ or $[1]$ in $\mathbb{Z}_3$ and $[x^2] = [0], [1], [4], [9], [6]$ or $[5]$ in $\mathbb{Z}_{10}$
what are the possibilities in $\mathbb{Z}_5$?

3. A graph is said to be biconnected if it is connected and has no cut-nodes. A biconnected component of G is a maximal biconnected subgraph.

A. Find the biconnected components of

(1) (2)  (3) 

B. Why is there no equivalence relation on the nodes of a connected [illegible]

40. 14. If 13 players are each dealt 4 cards from a 52 card deck, what is the probability that each player gets one card of each suit?

16. What is the probability that an arrangement of a, b, c, d, e, f has (a) a & b together? (b) a before b?

24. What is the probability among the arrangements of GRACEFUL of having (a) No consecutive vowels? (b) F and G appear consecutively.

33. How many arrangements of INSTRUCTOR are there in which the successive vowels are three positions apart?

50, 21. How many ways are there to place nine different rings on the four fingers of your right hand if (a) the order of rings on a finger does not matter? (b) The order of rings on a finger is considered?

32. How many ~~ways~~ arrangements of n 0's and m 1's have k runs of 0's (A run is a consective set (1 or more) of the same digit.)

56, 3. In a bridge deal what is the probability that (a) West has 4 spades, 3 hearts, 3 diamonds and 3 clubs? (b) North and South have 5 spades each, West has 2 spades and East has 1 spade? (c) One player has all the aces? (d) All players have a (4,3,3,3) division of suits?

14. If n distinct objects are randomly put into n distinct boxes what is the probability that. (a) No box is empty? (b) Exactly one box is empty (c) Exactly two boxes are empty?

26. How many bridge deals are there in which North and South get all the Spades?

27. What is the probability in a bridge deal that each player gets a least two honors (An Honor is Ace, King, Queen, Jack of any suit)

33. How many ways are there to distribute 20 distinct flags onto 12 distinct flagpoles if (a) The order of the flags on each flagpole is counted? (b) No flagpole is empty and the order on each flagpole is counted?

38 How many nonnegative integer solutions are there to the pair of equations $x_1+x_2+x_3+x_4+x_5+x_6=20$ and $x_1+x_2+x_3=7$

HINTS: 16a 24b "glue" the letters together

21b 33 Pick the positions for identical objects first [14.

3. 26. 27 A bridge deals 13 cards hands to Four distinct players N E W & S. (Total No. $\underset{N}{\binom{52}{13}}\underset{E}{\binom{39}{13}}\underset{W}{\binom{26}{13}}\underset{S}{\binom{13}{13}}$)

Big Oh

Here we define what $O(f) = O(g)$ and what $O(f) < O(g)$ means for a special collection of functions.

Let $f, g : \mathbb{N} \to \mathbb{R}$ so that eventually these functions are always positive. For example $f(n) = n - 100$ is one of these functions since for $n \geq 101$ $f(n) > 0$. Thus for two such functions $\frac{f(n)}{g(n)}$ is defined for n large enuff.

<u>Thm.</u> If f, g are functions as above and

if $\lim_{n \to \infty} \frac{f(n)}{g(n)}$ exists and is equal to L

then if $0 < L < \infty$ then $O(f) = O(g)$

if $L = 0$ then $O(f) < O(g)$

if $L = +\infty$ then $O(f) > O(g)$

<u>Remarks</u> 1. Althought this isn't a definition (It is in the text) you will have to look hard to find two functions f & g that fail to have a limit (but there is a pair in the text)

2. We will take this <u>Thm</u> ~~of~~ on its own. However if you know a little calculus the proof is easy. For example if $0 < L < \infty$ Let $\varepsilon > 0$ be given so that $L - \varepsilon > 0$ then is N so that $n \geq N$ implies

$$\left| \frac{f(n)}{g(n)} - L \right| < \varepsilon$$

$$-\varepsilon < \frac{f(n)}{g(n)} - L < \varepsilon$$

$$L - \varepsilon < \frac{f(n)}{g(n)} < L + \varepsilon$$

$$g(n)(L - \varepsilon) < f(n) < (L + \varepsilon) g(n)$$

Thus $n \geq N$ $f(n) < C g(n)$ with $C = L + \varepsilon$

and $g(n) < D f(n)$ with $D = \frac{1}{L - \varepsilon}$

3. If the above remarks cause you trouble worry no more about them

Examples

1 Show $O(1000n^2) = O(\frac{1}{10}n^2 + n - 127)$

Let $f = \frac{1}{10}n^2 + n - 127 \qquad g = 1000n^2$

then $\frac{f}{g} = \frac{\frac{1}{10}n^2 + n - 127}{1000n^2} = \frac{1}{10{,}000} + \frac{1}{1000n} - \frac{127}{1000n^2}$

as $n \to \infty$ both $\frac{1}{n}$ & $\frac{1}{n^2} \to 0$ so the limit $L = \frac{1}{10{,}000}$

Since $0 < L < \infty \qquad O(f) = O(g)$

2. Show $O(n^3 + n) = O(n^3 - n^2 + 27)$

$$\frac{n^3 - n^2 + 27}{n^3 + n} = \frac{(\frac{1}{n^3})}{(\frac{1}{n^3})} \frac{1 - \frac{1}{n} + \frac{27}{n^3}}{1 + \frac{1}{n^2}} = \frac{1 - \frac{1}{n} + \frac{27}{n^3}}{1 + \frac{1}{n^2}} \to 1$$

(this trick is called factor out the largest power of n)

3. Show $O(n) = O(\sqrt{n^2+1})$

$$\frac{\sqrt{n^2+1}}{n} = \frac{\frac{1}{n}}{\frac{1}{n}} \frac{\sqrt{1 + \frac{1}{n^2}}}{1} = \sqrt{1 + \frac{1}{n^2}} \longrightarrow 1$$

4. Show $O(100n + \sqrt{n}) < O(n^2)$

$$\frac{100n + \sqrt{n}}{n^2} = \frac{\frac{1}{n^2}}{\frac{1}{n^2}} \frac{\frac{100}{n} + \frac{1}{n\sqrt{n}}}{1} = \frac{100}{n} + \frac{1}{n\sqrt{n}} \longrightarrow 0$$

5. Show $O(\log_2 n) = O(\log_{10} n)$

if $y = \log_2 n$ then $2^y = n$ now $2^{\log_2 10} = 10$

so $n = 2^{\log_2 10 \frac{y}{\log_2 10}} = 10^{\frac{y}{\log_2 10}}$ hence $\log_{10} n = \frac{y}{\log_2 10}$

thus $\frac{\log_2 n}{\log_{10} n} = \frac{y}{\frac{y}{\log_2 10}} = \log_2 10 \to \log_2 10$

6. $O(\log_2 n) < O(n) < O(n \log_2 n)$

$\frac{n \log_2 n}{n} = \log_2 n \to \infty \qquad \frac{\log_2 n}{n} \to 0$ (HINT USE L'Hopitals)

(or $\log_{10} n$ is the number of digits use to write n in decimal which is much smaller than n.)

7. $O(2^n) < O(3^n) \qquad \frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n \longrightarrow 0$

8. $O(5^n+3^n) = O(4\cdot 5^n - 2^n)$

$$\frac{5^n+3^n}{4\cdot 5^n - 2^n} = \frac{\frac{1}{5^n}}{\frac{1}{5^n}} \frac{1+\left(\frac{3}{5}\right)^n}{\frac{1}{4}\cancel{\#}-\left(\frac{2}{5}\right)^n} \longrightarrow \frac{1}{\frac{1}{4}} = 4$$

9. $O(2^n n) < O(3^n)$ $\qquad \dfrac{2^n n}{3^n} = n\left(\dfrac{2}{3}\right)^n \to 0 \quad (L'H)$

10. $O(n!) > O(2^n)$ $\qquad \dfrac{n!}{2^n} = \dfrac{n}{2}\cdot\dfrac{n-1}{2}, \cdots \dfrac{2}{2}\cdot\dfrac{1}{2} \longrightarrow \infty$

11. $O(n!) < O(n^n)$ $\qquad \dfrac{n!}{n^n} = \dfrac{n}{n}\cdot\dfrac{n-1}{n}\cdots\underbrace{\dfrac{\frac{1}{2}n}{n}\cdot\cdots\dfrac{1}{n}}_{\text{all terms} \le \frac{1}{2}} \le \left(\dfrac{1}{2}\right)^{\frac{n}{2}} \to 0$

12. $O\left(\left(\frac{n}{2}\right)^{\frac{n}{2}}\right) < O(n!)$ $\qquad \dfrac{n!}{\left(\frac{n}{2}\right)^{n/2}} = \dfrac{n}{\frac{n}{2}}\cdot\dfrac{n-1}{\frac{n}{2}}\cdots\dfrac{\frac{1}{2}n}{\frac{1}{2}n}\cdot\frac{1}{2}n-1\cdots 1 \to \infty$

13. $O(\log_2 n!) = O(n\log_2 n)$

for large n $\quad \frac{1}{2}n(\log_2 n - 1) = \frac{n}{2}\log_2\frac{n}{2} = \log_2\left(\frac{n}{2}\right)^{\frac{n}{2}}$

$< \log_2 n! < \log_2 n^n = n\log_2 n$

1. Put these big Oh's in increasing order $O(3^n n^2)$, $O(1)$
$O(n^2)$, $O(n\log n)$, $O(\sqrt{n})$, $O(3^n)$, $O(2^n)$, $O(4^n)$
$O(n)$, $O(n\sqrt{n})$, $O(n!)$, $O(\sqrt[3]{n^2+1})$, $O(n^{1000})$

2. Show $O(\sqrt{n^3+n}) = O(n\sqrt{n})$; $O(n^2-n+7) = O(n^2+n)$
$O(3^n+n^{100}) = O(2^n+3^n)$; $O(n^k) < O(3^n)$ for
$k = 1, 2, \ldots$

3. If it takes $\frac{1}{2}n^2$ time to sort n items ~~and 100~~ using method 1 and $100n$ time to sort n items using method 2 for which values of n would you use Method 1 > Method 2? What if Method 1 takes $1024n$ time and Method 2 takes $n\log_2 n$ time?

4. Make a Table rows labeled $n = 1, 2, 3, 4, 10, 20, 100, 200, 1000, 1000\,000, 10^{100}$ and columns labeled $\log_2 n$, $\log_{10} n$, $n\log_2 n$, n^2, n^3, 2^n, $n!$, n^n, $\sqrt{n}$.

DM1 TP7 Due Thurs 9 Feb 84

Given $a_0 = 4$, $b_0 = 2$ and for $n \geq 1$ $a_n = 5a_{n-1} + b_{n-1}$, $b_n = a_{n-1} + 5b_{n-1}$

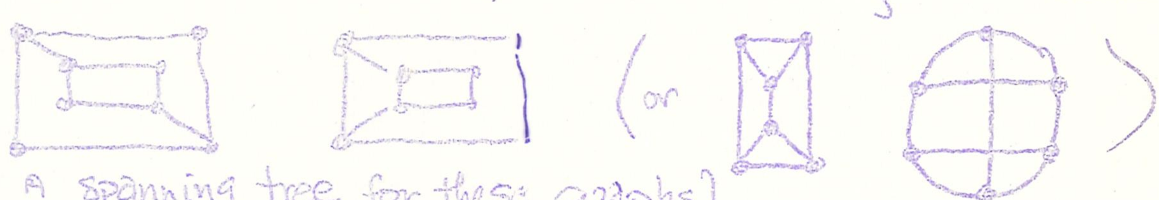
Prove by induction for $n \geq 0$ $a_n = 3 \cdot 6^n + 4^n$, $b_n = 3 \cdot 6^n - 4^n$

PT1 (Practice Test)

1 Define A Connected Graph B. ~~Deg of~~ Degree of a vertex

2 Give examples A. A path which is not simple B. A circuit which isn't a cycle C. A simple path which starts and ends at the same vertex but isn't a cycle, D. A path with at least one edge which starts and ends at the same node which isn't a circuit.

3. Either produce an isomorphism or show why none exists



(Find a spanning tree for these graphs)

4. List ALL trees (exactly one for each isomorphism class) with 6 edges [or loop free graphs with 6 nodes & 9 edges)

5. For the statements to right
A. Draw Venn Diagrams for each statement
B. Is the conclusion valid or not WHY?

Apples are fruit
Food with seeds is fruit
∴ Apples are food with seeds

6. A. Write the negation of the sentence below so that the quantifers precede the not
For each boy there is a girl so that he is willing to work for her
B. Explain why the sentence above is different from
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7. Give counterexamples
A. A graph with a cut-edge has a cut-node
B. A graph with a cut-node has a cut-edge
C. Every tree has a vertex of degree one
D. ~~There is~~ A graph with n nodes and $\frac{n(n-1)}{2}$ edges is connected

8 Prove by Induction $n! > 2^n$ for $n \geq 4$

9. Prove: A connected graph G is a tree if and only if G has no cycles

10. Explain why there is no loop-free connected graph with degree sequence (1,1,1,1,1,1,1,3,4)

DM1 PT3

1. How many license plates are there with A. 3 letters followed by 3 digits B. with either 2, 3, 4 or 5 letters? C. with 8 digits exactly 5 of them Ø's? D. with 6 letters exactly 2 of them vowels?

2. How many arrangements of MERRYCHRISTMASS are there with A. Altogether? B. with the Y between the M's (and no letters between them)? C. with the Y next to the C? D. with no consecutive vowels?

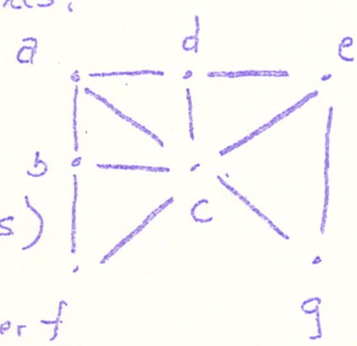
3. How many ways are there to pick 20 electronic games from Santa's 50 kinds with A. No repetitions? B. Unlimited repetitions C. With no more than 15 of any one kind?

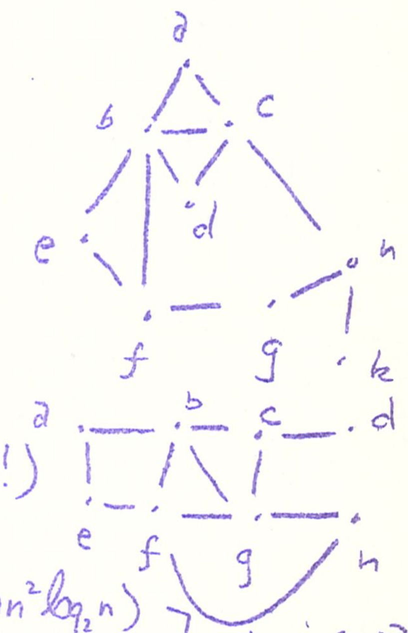
4. What is the probability that a 5-card hand has. A. All face cards? (FACE CARD = ACE, KING, QUEEN, JACK or TEN) B. At least one red card? (HEARTS & DIAMONDS ARE RED) C. Exactly one pair? D. At least one pair?

5. There are 4 roads from success to Scandal and 10 roads from Scandal to Ruin. Also there are 5 roads from sucess to Ruin which avoid Scandal. A. How many roads to Ruin are there from success (routes)? B. How many ways are there to go from sucess to ruin and back to sucess without repeating any portition of the route there on the way back.

6. How many ways are there to put 4 iden balls into 3 iden boxes? 4 dist balls into 3 iden boxes? 4 iden balls in 3 distinct boxes? 4 dist balls into 3 dist boxes?

7. Find spanning trees for the graphs to right by A DFS B BFS (note exactly one correct ans)





8. Arrange in increasing order

A. $O(2^n)$ $O(n \ln n)$ $O(n)$ $O(3^n)$ $O(n^{3/2})$

B. $O(\ln n)$ $O(2^n)$ $O(n^2)$ $O(n \ln n)$ $O(n!)$

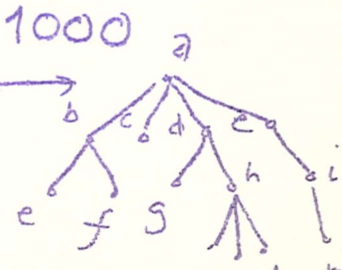
9. Show $O(n+7) = O(\sqrt{n^2+1})$

10. A. Arrange $O(2n^3)$ $O(3^n)$ $O(n!)$ $O(10n^2 \log_2 n)$

B. Arrange $2 \cdot 4^3$ 3^4 $4!$ $10 \cdot 4^2 \log_2 4$ — in increasing order

11. Draw the binary tree with sequence 111001001100 0

12. Do Post Order & Pre Order Transversal on the tree →



13. Give Counterexampes G conn loop free graph

A. if K has one edge in common with T a spanning tree of G then K is a cutset of G

B. if K has an even number of edges in common with each cycle of G then K is a cut set.

C. A graph with 3 biconnect components has 2 cut-nodes

14. Prove: A maximal cycle-free subgraph of a connected graph G is a spanning tree for G

15. Given $a_1 = 2$ $n \geq 2$ & $2/n$ implies $a_n = 4a_{(\frac{n}{2})} - 3$

Prove by induction for n a power of $2 \geq 2^0$ $a_n = n^2 + 1$

DM 1 TEST 1 Done By ____________ SS# ____________

Show All Work ALL Problems Worth 10 points

1. A. Draw all connected loop-free graphs with 4 edges
(No two graphs in your list can be isomorphic)

B. Negate the statement
"for each x, there is a y so that $y \geq 0$ and $y^2 = x$"
so that the quantifiers precede the not

2. A Draw a Venn Diagram
for the statement
"no A is B"
(include "?"'s)

B Draw a Venn Diagram
for the statement
"Some C is B"
(include "?"'s)

C. Mark the statements below V or INV if the statement
is a Valid or INValid consequence of the statements in A & B

1 "No C is A" 2 "Some C is not A" 3 "Some C is A"

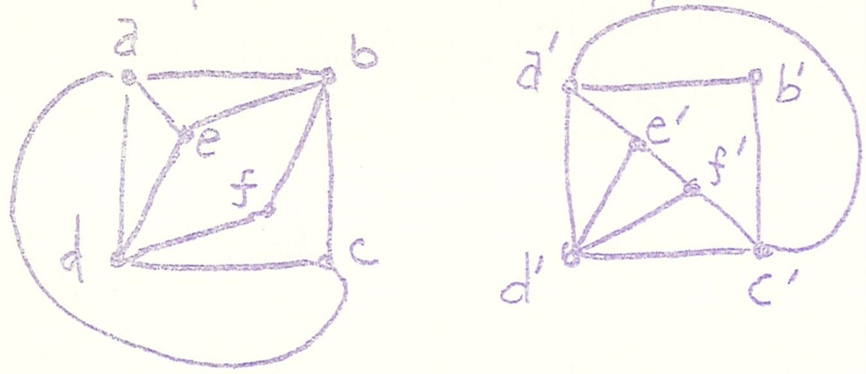
3. Give Counter Examples

A. A graph with no circuits is a tree.

B. No loop-free graph is isomorphic to its complement

C. A graph with a unique spanning tree is itself a tree.

4. Either produce an isomorphism or show why none exists



5. Give Examples

A. A path that repeats no edges but has revisit number = 3

B. A path that has at least one edge and starts and stops at the same vertex but ~~has~~ contains no cycle.

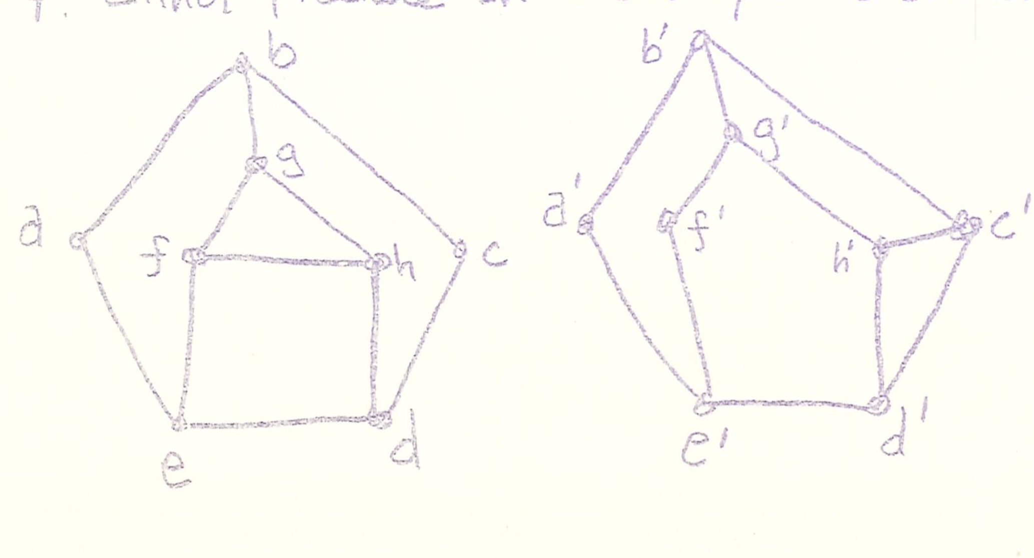
C. Two non-isomorphic loop-free graphs with degree sequence (1, 2, 2, 2, 3)

6. Define A. A cycle:

B. A loop:

C. A spanning tree:

7. Either produce an isomorphism or show why none exists



8. What is wanted here is three different proofs of the statement "if 4 divides x, then 2 divides x" you will find the statement "Since 2 divides 4" handy [All the proofs are 3 lines long]

A The Direct Proof:

B. The Indirect Proof:

C. The Proof by contradiction:

9. Prove If E is a cut-edge in the connected graph G then ~~E is an ed~~ for any spanning tree T of G, E is an edge of T.

10. Prove by Induction: Given $a_0 = 0$ & $a_n = a_{n-1} + 2n - 1$ for $n \geq 1$
Prove $a_n = n^2$ for $n \geq 0$

b. ... This 2D1 tables

1. How many rows does the table?

B. How big must ... be?

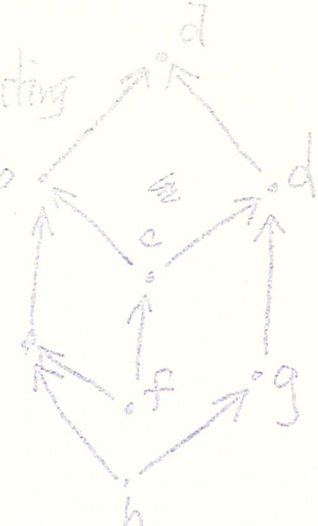
C. How wide should ... be?

Euler's theorem

4. Every ... the vertices of graph G has the same number of edges

3) The sum of ... (or ... edges) ... in a complete graph ... is ...

8. The Digraph to right is <u>Not</u> the one representing the relation R. But rather we define R by $xRy \Leftrightarrow x=y$ or there is a directed path (edges point the correct way) from x to y in the digraph to the right. It turns out that this R is a partial order.



A. List all maximal elements for R

B. List all minimal elements for R

C. Find $\inf(b, g)$

D. Find $\sup(f, g)$

9. Prove: An edge E which is not in any cycle of the connected graph G is a cut-edge of G

10. Prove by induction: Given $a_1 = 0$ and for $n \geq 2$

$$a_n = \begin{cases} a_{(\frac{n}{2})} + 1 & n \text{ even} \\ a_{(\frac{n+1}{2})} + 1 & n \text{ odd} \end{cases}, \quad \text{Show for } n \geq 1 \quad a_n = \lceil \log_2 n \rceil$$

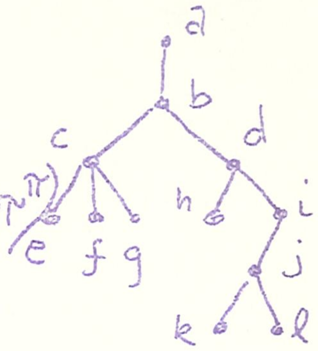
[Hints. 1 $\lceil \log_2 n \rceil = k \Leftrightarrow 2^{k-1} < n \leq 2^k$ 2. Strong induction]

DM1 TEST 3 SHOW ALL WORK FOR CREDIT 10pts EACH BY ____________

1 For the tree to right list the nodes in

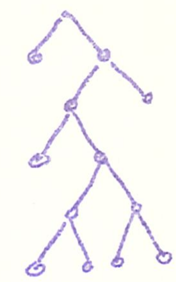
A. Pre-Order

B. Post-Order



2. A. Arrange $O(\pi\sqrt{n})$, $O(\pi^n)$, $O(n^\pi)$, $O(n\log_\pi n)$, $O(\pi^\pi)$ in increasing order

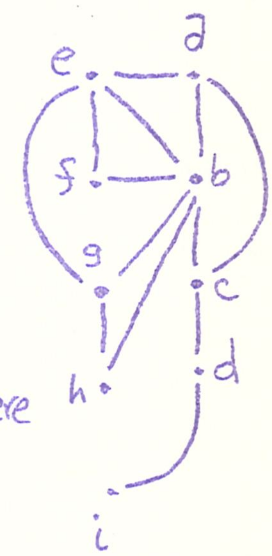
B. Give the zero-one sequence for the binary tree to right



3. Find spanning trees for the graph below the binary tree by (note only one correct answer)

A DFS

B BFS



4&5 How many arrangements of SUMMERVACATION are there

A. Altogether?

B. With the R next to the V?

C. With No consecutive vowels?

D. with at least two vowels adjacent?

6&7 How many non-negative integer solutions are there to

A. $\Sigma_{i=1}^{50} x_i = 20$?

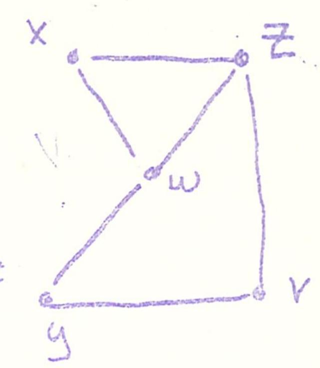
B. with each $x_i \leq 1$?

C. with $x_1 \geq 3$, $x_2 \geq 2$, $x_3 \geq 4$ and the rest ≥ 0?

D. how many to $\Sigma_{i=1}^{50} x_i \leq 20$?

DM1T3P2

8 Give Counterexamples. The graph G is



A. A Minimal spanning subgraph of G is a tree.

B. A minimal connected subgraph containing x, y & z of G has 3 edges

C. A maximal subgraph each of whose vertices has degree ≤ 2 of G is a cycle.

9. Prove: A maximal disconnected subgraph of a connected graph G has exactly two connected components

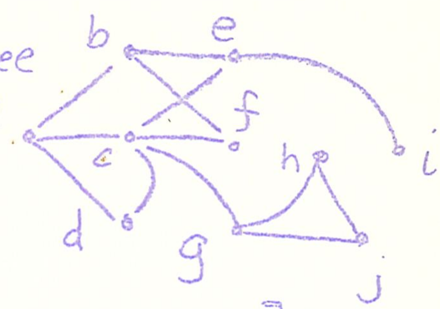
10. Given $a_1 = 0$ $\quad a_n = 5a_{(\frac{n}{2})} + 4$ for n a power of two ≥ 2

prove by induction for n a power of $2 \geq 2^0$ then

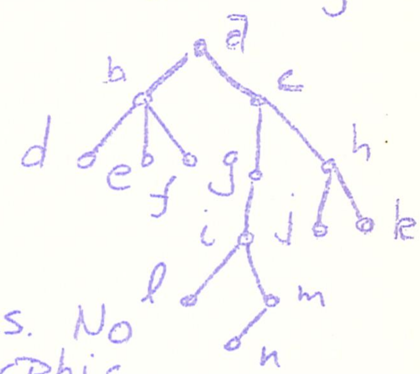
$a_n = n^{\log_2 5} - 1$

PM1 FINAL SHOW ALL WORK 10pts each BY ________________

1. For the graph to right find a spanning tree
by A DFS B BST



C. List the vertices of the tree to right in Postorder



2. A. Draw all (undirected) trees with 5 edges. No two trees in your list can be isomorphic

B. Negate "There is a y so that for each x $y > 0$ and $y \neq x^2$" so that your answer does not contain the word "not"

3. A Change $(a+b)\cdot(c+d/e)+f$ to prefix

B. Build the BST for
40, 60, 20, 45, 30, 25, 90, 100, 70, 50, 75

4. A. Find GCD(1462, 132) by the Euclidean Algorithm (Show work!)

B. In $\mathbb{Z}_{13}$ find i with $0 \le i < 13$
so that $[i] = \dfrac{[2]}{[9]}$

5. Draw a Venn Diagram for A & B
A. All x is y

B. no z is x

C. For Each of the below label the statement Valid or InValid consequence of statements A & B above
1. no y is z 2. no x is z 3. Some y is z

6. In the directed tree T every parent has exactly 4 children. T has 100 leaves.

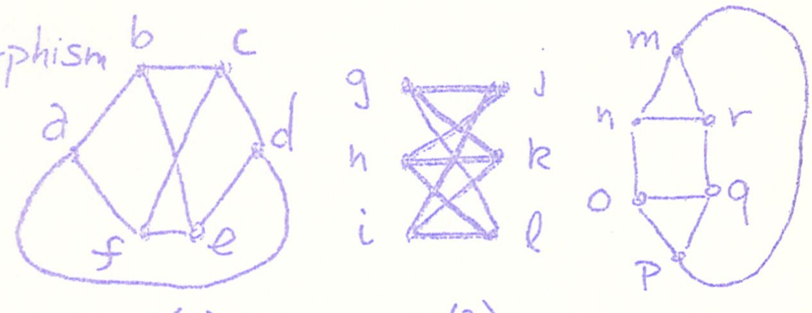
   A. How many nodes does T have?

   B. How high must T be?

   C. How high could T be?

7. Either produce an isomorphism or show why none exists

   A between graph (1) & graph (2)



(1) (2) (3)

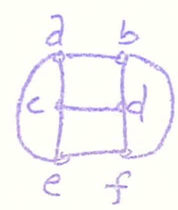
   B between graph (2) & graph (3)

   C between graph (1) & graph (3)

8. Give examples

   A. Two non-isomorphic loop-free graphs with degree sequence (1, 1, 2, 2, 3, 3)

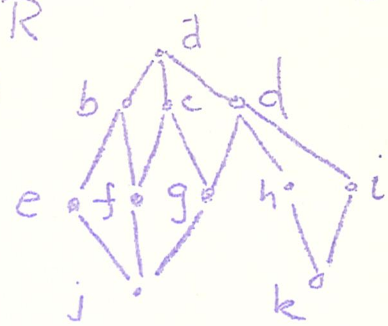
   B. A digraph of a non-reflexive relation which isn't irreflexive either.

   C. A maximal disconnected subgraph of  with 5 edges

9. Define R on {1, 2, 3, ...} by $xRy \iff x \mid (y+1)$
For each property below either say no and give a counterexample or say yes

A Reflexive? | B. Irreflexive? | C Transitive? | D. Anti-symmetric

10. For the Hasse diagram of the Partial Order R ($xRy \Leftrightarrow$ there is upward path $x \to y$ or $x = y$)



A. List all maximal elements
B. List all minimal elements
C. List all elements "less than or equal to" d
D. Find inf (b, g)
E. Find sup (j, h)

11. How Many 5 card hands have
A Exactly 3 spades and exactly 1 club?

B. At least one spade?

12. What is the PROBABILITY of an arrangement of ABBCDEEFG
A. of having the C next to the D?

B. of having no consecutive vowels?

13 & 14. How many ways can you put 100 balls into 239 distinct boxes
A. If the balls are distinct and repetition is allowed?

B. If the balls are identical and no repetition is allowed?

C. If the balls are identical and repetition is allowed?

D. If the balls are identical and no box can have more than 75 balls

15. How many 4-member committees can be formed from a group of 5 French 6 Cubians and 4 English
A. With exactly one French person?

B. With at least as many French as Cubians and at least as many Cubians as English?

16. Prove: A minimal connected subgraph containing the nodes x, y & z of the connected graph G is a tree.

17 & 18 Give Counterexamples.

A. Every connected graph with a cut-edge has a cut-node

B. A tree with 5 or more edges has either at least 4 vertices of degree 1 or all vertices of degree ≤ 2.

C. An equivalence relation on a set with at least 2 elements is never a partial order.

D. If $f(n)$ and $g(n)$ are positive functions for all $n \geq 1$ and $O(f) = O(n)$ and $O(g) = O(n^2)$ then $f(n) \leq g(n)$ for all $n \geq 1$

E. In any binary tree with 3 or more nodes preorder is always different from inorder

19. Prove. If G is a connected loop-free graph which is not a tree then G has at least two spanning trees.

20. Given $a_0 = 0$ and $a_n = 2a_{n-1} + 2^{n+2}$ for $n \geq 1$
Prove by induction $a_n = 4n2^n$ for $n \geq 0$