[illegible] MAD 3104 Sec 1

The good doctor Bellenot
office 218 Love
office hours MW 12:30 - 2:15
Th 12:30 - 1:20

PRE REQUISITE: MAC 1132
(College Algebra and Trig)

TEXT: MOTT, KANDEL, BAKER: DISCRETE MATH FOR CS
parts of Chapters 1, 2, 4, 5 will be covered.

GRADES: The classic 90, 80, 70, 60 cut offs
based on
- 1 FINAL 32%
- 3 TESTS 48% (16% EACH)
- TP's 20% (See TP sheet)

ATTENDANCE & HOMEWORK: Attendance is REQUIRED. A person is considered absent if she/he does not turn in the homework with attempts on all the problems at the beginning of class. Homework is assigned daily and due the next class period. A person who misses more than 6 hours of class will receive an automatic fail. Late homework is not accepted.

TESTS: are in class and closed book and closed notes. Make up tests are very hard to obtain and are always harder tests. Tentative test dates: 1, Feb 2; 2, Mar 15; 3, Apr 12.

FAIR WARNINGS:

1. PROOFS ARE REQUIRED. A STUDENT WHO CAN'T PROVE THINGS WILL NOT PASS THIS CLASS.

2. We will "bounce around" in the text, most other sections will not.

3. A large percentage of DM 1 students do not pass.

EST PROBLEMS (Better known as TP's)

[Note this sheet does not apply to H.W.]

1. RULES

   A. They must be on 8½ by 11 paper.

   B. They must be written in ink.

   C. They must use only one side of each page.

   D. If there is more than one page, then the pages must be stapled or paper-clipped together.

   Failure to follow any rule costs a point each.

2. Grades

   A. Graded on a 0 to 10 basis.

   B. Graded on your reasoning, your ability to express your reasoning, neatness and your English.

3. Your TP average is computed using only your best n out of the m assigned where $\frac{n}{m}$ is roughly $\frac{2}{3}$.

4. Since TP's are assigned a week in advance of their due date, the solutions handed in are assumed to be carefully worked out. In ANY CASE they will be graded as if they were.

5. They must be your <u>OWN</u> work.

Dm 1

TP 3 Due 24 JAN 84 Tue

Given: $a_0 = 3$, $a_1 = 7$ and $a_n = 2a_{n-1} + 15a_{n-2}$ for $n \geq 2$

Proof by induction for $n \geq 0$

$$a_n = (-3)^n + 2 \cdot 5^n$$

↑ (TWO TIMES (5 TO N POWER))

TP4 DUE THURS 26 JAN 84

In a Graph G, Let P be a simple path from x to y and Q be a simple path from y to z, and R be the path formed by following P from x to y and then following Q from y to z (Called Concatenation)

A. Show R satisfies the definition of a Path

B. Give an counterexample to show R need not be a simple path

C. Prove that in G there is a simple path S' from x to y

TP5 ?

TP6 ?

DM 1 TP 5 due Tues 31 JAN 84

A vertex x in a connected graph G is said to be a cut-node (or cut-vertex) if the graph H (obtained by deleting x and the edges incident to x from G) is not connected.

Proof: Let G be a connected graph and x a node in G. The vertex x is a cut-node if and only if there are vertices y and z in G so that every path from y to z in G goes through x.

Thurs 2 Feb 84 is the date for TEST 1

Fri 3 Feb 84 is the last day to drop without dean's premission. The tests will be graded and in my office at 12:30.

TP 6 due Tues 7 Feb 84

You are to give an alternate proof of Th. 5.3.6 which also uses strong induction. ~~Use~~ Assume each connected graph with fewer than k vertices has a spanning tree. Let G be connected with k vertices and x any node of G. Let H be as in TP5. You MAY NOT assume x is not a cut-node. Note that H can have any number of connected pieces. ~~Show~~ [Hint: Show how a spanning forest for H can be used to construct a spanning tree for G]

DM 1

TP 8 Due 14 Feb 84 Tues

Prove A Graph is a tree if and only if it is connected and has fewer ~~ver~~ edges than vertices.

TP9 Due Thurs 16 Feb 84

Prove or Disprove:

A. Every connected graph has a node which is not a cut-node

B. Every connected graph has an edge which is not a cut-edge

TP10 Induction

DM 1

TP 11 due Thurs 23 Feb

Given: A connected graph G with n nodes and $n-1+k$ edges

Prove by induction (on k): For $k \geq 0$ G has at least k cycle subgraphs

TP12 due Sat 25 Feb

Prove or Disprove:

A. Every closed path with ~~more than~~ at least one edge in a tree, uses every edge in that path at least twice.

B. If G is a connected graph such that every closed path with at least one edge has at least one edge which is used at least twice, Then G is a tree

[illegible] disconnected [illegible]
[illegible] (component [illegible])
[illegible] is connected

[illegible] Digraph
1. If [illegible] and [illegible]
of [illegible]
edge [illegible]
2. If [illegible] and [illegible]
of [illegible] exactly one edge [illegible]

[illegible]
[illegible] Digraph
1. [illegible]
[illegible]
2. [illegible] of the [illegible]
[illegible]

DM1

TP 16 due 20 Mar 84 Tue.

$\binom{n}{j}$ is defined for $0 \leq j \leq n$

by $\binom{n}{j} = \frac{n!}{j!(n-j)!}$

Show for $1 \leq j \leq n$ $\binom{n}{j} = \binom{n-1}{j-1} + \binom{n-1}{j}$

TP17 due 22 Mar 84 Thus.

Without using the theorem G is conn $\Leftrightarrow$ G has a spanning tree

prove:

A. A minimal connected spanning subgraph of a connected graph G is a tree.

B. A ~~min~~ maximal tree subgraph of a connected graph G is a spanning subgraph

Note that these provided two new proofs of the theorem above.

TP 18 due Tues 27 Mar 84

Given $a_1 = 0$ and for $n \geq 2$ with $3 | n$ $a_n = a_{\left(\frac{n}{3}\right)} + 2$

Show by induction if n is a power of $3 \geq 3^0$

then $a_n = 2 \log_3 n$

TP 19 — due Thurs 29 Mar 84

(*)

G is a loop free connected graph.

K is a subset of edges of G so that

(1) The is a spanning tree T of G so that T and K have exactly the edge E in common.

(2) Any cycle subgraph of G has an even number of edges in common with K.

Let T_1, T_2 be the subtrees of T formed when the edge E is deleted from T

→ Show for any edge F in G:

F is in K ⟺ F goes from a node in T_1 to a node in T_2

TP 20 due Tues 3 Apr 84

Prove A set K which satisfies (*) in TP 19 is a cut-set

DM 1

TP 21 due Thurs Apr 5

Given $a_1 = 1$ and if $2 | n$ then $a_n = 3a_{n/2}$ $n > 1$

Prove by induction if n is a power of $2 \geq 2^0$ then $a_n = n^{\log_2 3}$

TP 22 due Tue Apr 10.

x & y are vertices in a connected graph G

A. Prove a path ~~from x to~~ with a minimal number of edges from x to y in G is a simple path.

B. Prove a minimal connected subgraph of G which contains both x & y is a path graph (p 400-401)

C. Give an example of something that satisfies B but not A.

TP 23 Due Tues 17 Apr 84

Given $a_1 = 4$ and for $n \geq 1$ define $a_{2n} = (a_n)^{2^n}$

(so a_n is defined for n a power of two

Prove by induction if n is a power of two $\geq 2^0$

then $a_n = 2^{2^n}$

TP 24 And Last Due Thurs 19 Apr 84.

88 chairs in a classroom are arranged in a rectangular array of 8 rows of 11 seats ~~here~~ each (or 11 columns of 8 seats ~~here~~ each). 50 Students take there places (1 student per chair)

A. Prove some row has at least 7 students

B. Prove some column must have at most 4 students

[illegible]

1. Convert [illegible]

(A - B) * (D + E)

C + B + (D * E + F) * P

2. Evaluate using a stack (show all work)

5 6 + 2 3 * + 30 + 4 /

2 7 8 9 + 1 + 9 / 7

3. Give the BST for

70, 60, 40, 65, [illegible], 35, 80, 90, 85, 63

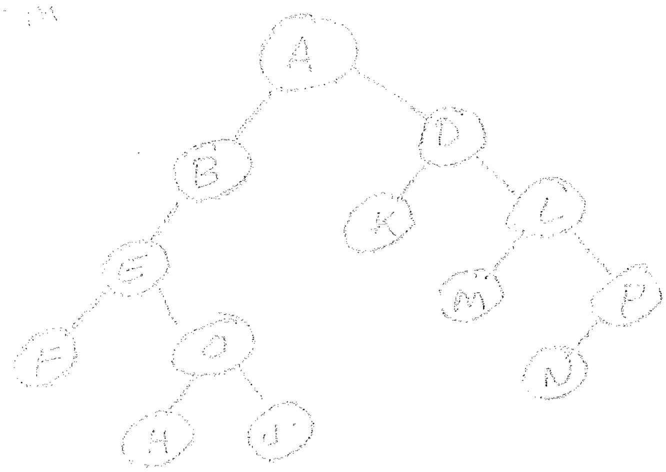
7, 12, 14, 9, [illegible], 2, 1

4. Traverse the tree to right in

PreOrder

InOrder

PostOrder



5 For the tree to right give

Father of J

Ancestor of L

Left subtree of B

right son of D

Height of tree

right subtree of B

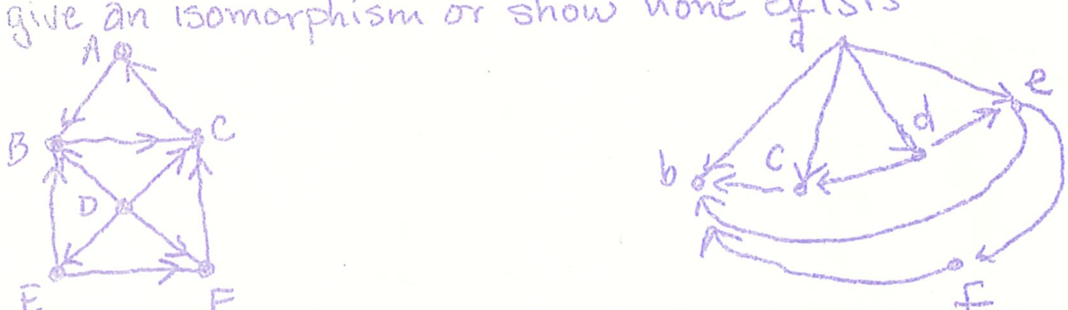
DM1 PT2

1A How many ways can you choose a chairman and 4 other members of a committee from M Math types and CS compsci types if the Chairman must be a Compsci type & the committee must have at least one Math type

B. How many lincense plates have three letters followed by four digits where neither the last letter is "O" (oh) nor the first digit is 0 (zero)

C. How many 7-card hands have 1. Exactly A pair & a 3 of a kind 2. At least one pair 3. Exactly 4 cards in one suit with no pair

2. Either give an isomorphism or show none exists



3. For the relation $xRy \Leftrightarrow x+3 \le 2y$ on $\{0,1,2\ldots\}$ and the properties ~~before~~ below either ~~state~~ R has that property or give a counterexample to show it doesn't: Reflexive, Irreflexive, Symmetric, Asymmetric, Anti-symmetric, Transitive.

4. In $\mathbb{Z}_{13}$ what is $[3]-[7]$, $[-10]\cdot[3]$, $[5]/[3]$?

5. Prove: $d = \gcd(a,b)$ $(a,b \ge 1)$ if and only if the smallest positive integer r which can be written $r = ma+nb$ $(m,n \in \mathbb{Z})$ is d.

6. Let X be a finite set and R a partial order on X

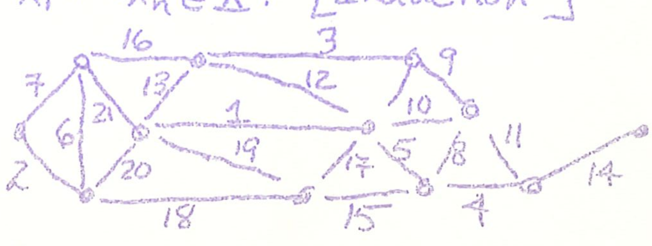
A. Prove: For each $x \in X$, there is a minimal $y \in X$ with yRx

B. Prove: If X has only one minimal element x, then x is a minimum

C. Prove: If for every $a,b \in X$, $\inf(a,b) = g\ell b(a,b)$ exists, then X has a minimun element.

D. Prove: If $\inf(a,b)$ exists for every $a,b \in X$ then for $n \ge 2$ $\inf(x_1, \ldots x_n)$ exists for $x_1 \ldots x_n \in X$. [Induction]

7. List the Costs of the Edges picked by Kruskal's Algorithm in the order choosen for the graph to the right



8. A certain directed tree every node has outdegree 3 or outdegree 0. This tree has 101 parent nodes. How many leaves? How many nodes? How high must it be? How high can it be?

9. For the Binary Tree to the right A. List the nodes in 1. Preorder, 2 inorder 3 postorder

B. Give the level order number for each node

C. There is a way of giving a unique zero-one sequence to any binary tree as follows. When traversing the tree in pre order put down a 1 if the node is a parent and 0 if it is a leave (i.e. = 1010

10. Write "the tree" for $(a+b)c-d/(e+f)$ and write this expression in prefix & postfix.

11. Give counterexamples.

A. A relation that is not reflexive is irreflexive

B. A graph (conn loop-free) with n nodes and n+2 edges has 3 cycles

C. A node in a cycle subgraph C in a conn graph G is not a cut ~~edge~~ node of G.

D. If x is not a cut-node of G and H is G with x & its incident edges removed and y is not a cut node of H then y is not a cut-node of G

12. Prove

A. in 11D the statement is true if we had the requirement that $\deg(x) \geq 2$.

B. A connected graph with n nodes & n-1 edges is a tree

C. A graph with n nodes, n-1 edges and no circuits is a tree

D. <u>By induction</u> a connected graph with n nodes has at least n-1 edges. Assume each graph has a non-cut node. Do NOT use the word TREE in your proof.

13. If $\mathfrak{X} = \{A : A \subset \mathcal{U}\}$ and $ARB \Leftrightarrow A \subset B$ find the minimum and maximum elements of $\mathfrak{X}$. find $\inf(A,B)$ and $\sup(A,B)$ or least upper bound. Show $\subset$ is not a total order

14. On $\mathbb{R}^2$ defined $(a,b) \leq (c,d) \Leftrightarrow a \leq c \;\&\; b \leq d$. Show this isn't a total order but is a partial order. Find $\inf((7,-5),(3,10))$